inter • noise 98
Christchurch NZ November 16-18

I-INCE Classification No. 43.5

ESTIMATION OF STRUCTURE-BORNE NOISE IN PLATES FOR FAR FIELD CONDITIONS USING THE STRUCTURAL INTENSITY METHOD

N K Mandal, R A Rahman and M S Leong

Institute of Noise and Vibration, University of Technology Malaysia
Jalan Semarak, 54100 Kuala Lumpur, Malaysia

1. INTRODUCTION

Vibration power flow measurement is an important tool in identifying significant paths of vibration transmission from sources through a structure. The measurement of vibration level itself may give limited information since stationary waves may be present giving rise to large amplitude with no power transmitted. Structural intensity is a vector quantity where the power flow pattern may be identified by measuring the structural intensities on the structure. This paper examines the use of structural intensity in the theoretical estimation of vibration power flow in naturally orthotropic plates in the frequency domain under far field conditions. The theoretical power flow models could be used in technically orthotropic plates, and corrugated plates for example, by defining the elastic rigidity constant using the method of elastic equivalence [2].

2. COMPLEX POWER IN THE FAR FIELD

The bending stiffness of the flexural wave equation in isotropic plates is obtained as a common factor in the classical plate equation [3]. It is however not possible to obtain such a common term for bending stiffness in orthotropic plate because of different rigidity constants in the flexural wave equation [2]. For free vibration analysis in orthotropic plates, the plate governing equation could be modified to obtain an equivalent non-dimensional representation by changing the variables [4, 5]. As an approximate analysis of bending wave power in orthotropic plates, it is possible to introduce dimensionless parameter [2] to modify the plate governing equation. Structural intensity for far field conditions would be denoted by I and not P hereafter in this paper to differentiate it from the general case [1].

The governing orthotropic plate equations are available in the literature, Troitskey [2], for example. It is possible to apply Newton's law in order to obtain a relationship between the shear forces and the transverse motion w for free vibration. If the analysis is restricted to free harmonic time variations, the flexural wave equation for naturally orthotropic plates could be expressed as

$$D_x \frac{\partial^4 w}{\partial x^4} + 2H \frac{\partial^4 w}{\partial x^2 \partial y^2} + D_y \frac{\partial^4 w}{\partial y^4} = m'' \omega^2 w \qquad (1)$$

where, D_x, D_y are the bending rigidity, H is the torsional rigidity, m'' is the mass per unit area of the plate, and ω is the natural frequency. The dimensionless parameters [2] used here for the purpose of simplifying the equation (1) are

$$\xi = \frac{x}{a}, \qquad \eta = \frac{y}{b} \quad \text{and} \quad \gamma_{ort} = \frac{a}{b}\sqrt[4]{\frac{D_y}{D_x}}$$

where γ_{ort} is the edge length ratio, a and b are the sides of the plate in the x and y direction respectively such that $b \geq a$ and $\gamma_{ort} \leq 1$ (Fig. 1). The latter condition also states that $D_x \geq D_y$. Therefore, by partial differentiation of the spatial derivatives of the flexural wave equation (1), and by introducing $H = \sqrt{D_x D_y}$ [2] in the flexural wave equation for free vibration in orthotropic plates, a modified general plate equation could be obtained as,

$$\frac{\partial^4 w}{\partial \xi^4} + 2\gamma_{ort}^2 \frac{\partial^4 w}{\partial \xi^2 \partial \eta^2} + \gamma_{ort}^4 \frac{\partial^4 w}{\partial \eta^4} = a^4 \left(\frac{\omega^2 m''}{D_x}\right) w \qquad (2)$$

If the bending wave number $k = \sqrt[4]{\frac{\omega^2 m''}{D_x}}$ is introduced, the above equation is reduced to

$$\left(\frac{\partial^2}{\partial \xi^2} + \gamma_{ort}^2 \frac{\partial^2}{\partial \eta^2}\right)^2 w = a^4 k^4 w \qquad (3)$$

The flexural wave equation may be further expressed in a simpler form as

$$(\nabla')^2 w - (a^2 k^2)^2 w = 0 \qquad (4)$$

The above flexural wave equation (4) represents the free vibration of orthotropic plates such that $\nabla' = \left(\frac{\partial^2}{\partial \xi^2} + \gamma_{ort}^2 \frac{\partial^2}{\partial \eta^2}\right)$. This term is referred as the modified Laplace operator.

Equation (4) may be replaced by two different second order equations in operator form; i.e.

$$(\nabla' + a^2 k^2) w = 0 \qquad (5a)$$

$$(\nabla' - a^2 k^2) w = 0 \qquad (5b)$$

Equation (5a) represents the condition of far field where free propagative waves exit. Equation (5b) is the condition of near field [3] as the disturbances decay exponentially from the sources and boundaries. A complete solution of the governing flexural wave equation (4), for example, is not possible for plates in general [6]. A typical solution could however be achieved using a Hankel function of the second kind [3].

If the co-ordinates x, y are transformed to ξ, η, and by introducing the dimensionless parameters defined earlier, the x-component of shear force [2] under far field conditions could be expressed in a new form by incorporating the Fourier transform

$$\overline{Q}_x = \frac{D_x k^2}{j\omega} \frac{\partial \overline{v}}{\partial x}. \tag{6}$$

The x component of structural intensity (complex power) in orthotropic plates from shear force alone could be expressed according to equation [1] as,

$$I_{xS}(f) = < - \overline{v}^* \overline{Q}_x >$$

The linear velocity and shear force quantities in the above equation could be estimated by the finite difference approximation using a two point transducer array measurement as shown in Fig. 1.

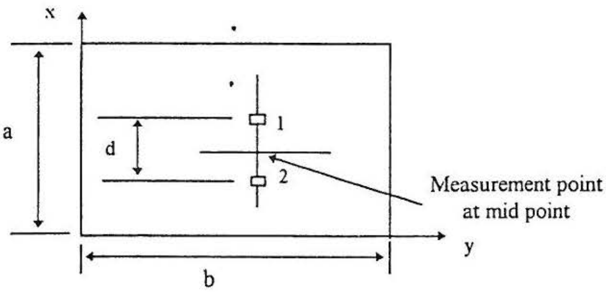

Fig. 1 The co-ordinate systems of the plate with two point transducer array.

The linear velocity and shear force are given by:

$$\overline{v} = \frac{1}{2}(\overline{v}_1 + \overline{v}_2) \quad \text{and}$$

$$\overline{Q}_x = \frac{D_x k^2}{j\omega}\left(\frac{\overline{v}_1 - \overline{v}_2}{d}\right)$$

Substituting values of linear velocity and shear force, and by evaluating the ensemble averages term by term, the final complex form of structural intensity in the x-direction by shear force component could be expressed as

$$I_{xS}(f) = \frac{D_x k^2}{2 j\omega d}[(G_{22} - G_{11}) + 2j\,\mathrm{Im}\{G_{12}\}] \tag{7}$$

where d is the distance between two successive points, ω is the angular frequency, G_{12} is the cross-spectrum of the velocity signals at points 1 and 2, and G_{22} and G_{11} are the two auto-spectrum of the velocity signals. The real part of the above complex power flow equation (7) defines the power transmitting by shear force in the x direction under the far field conditions of the orthotropic plates, and could be expressed as

$$I_{xS}(f) = \frac{\sqrt{D_x m''}}{d}\,\mathrm{Im}\{G_{12}\} \tag{8}$$

As the contribution of vibration power from shear force and moments are equal in the far field [6], the total active power could be simply obtained by doubling the power from either the shear force component or the moment component. The total active power from shear force part in the far field in the x-direction (Fig. 1) is therefore obtained as

$$I_x(f) = \frac{2\sqrt{D_x m''}}{d} \mathrm{Im}\{G_{12}\} \tag{9}$$

Only one cross-spectrum of velocity signal is necessary to quantify the total power. This is a new definition of structural intensity which defines the power flow per unit width of the plate. This expression is consistent with the conventional description of structural intensity being a power flow per unit width of the plate (W/m).

The y component of total structural intensity (active power) in the far field could be obtained similarly by changing the transducer array,

$$I_y(f) = \frac{2\sqrt{D_y m''}}{d} \mathrm{Im}\{G_{12}\} \tag{10}$$

The practical form of the intensity equation (9) could be obtained using cross-spectrum of acceleration signals as,

$$I_x(f) = \frac{2\sqrt{D_x m''}}{d\omega^2} \mathrm{Im}\{G_{12}\} \tag{11}$$

where G_{12} is the cross-spectrum of the acceleration signals at points 1 and 2 (Fig. 1).

3. CONCLUSION

The structural intensity technique was used to formulate the bending wave power in naturally orthotropic plates under far field conditions in the frequency domain. A new but similar description of structural intensity was proposed. This new formulation defines the vibrational power flow per unit width of the plates. Only one cross-spectra was used to obtain the intensity vectors in a point on the plate in a particular direction. This is similar to the conventional "2-transducer" method as commonly used in vibration power flow measurements.

4. REFERENCES

[1] N. K. Mandal, R. A. Rahman and M. S. Leong, Proc. Inter-Noise '98., 'Measurement of bending wave power in plate-like structures under general field conditions using cross-spectral density method'.

[2] M. S. Troitskey, Stiffened plates: bending, stability and vibration (Elsevier Scientific Publishing Company, Amsterdam, 1976).

[3] L. Cremer and M. Heckl, Structure-borne sound: structural vibration and sound radiation at audio frequencies (Springer-Verlag, Berlin, 1988).

[4] J. R. Kuttler and V. G. Sigillito, J. Sound Vib., 'Upper and lower bounds for the frequency of clamped orthotropic plates', **73**(2); 247-259, (1980).

[5] D. J. Gorman, J. Sound Vib., 'Accurate free vibration analysis of clamped orthotropic plates by the method of superposition', **140**(3); 391-411, (1990).

[6] D. U. Noiseus, J. Acou. Soc. Am., 'Measurements of power flow in uniform beams and plates', **47**(1), 238-247, (1970).