Application of Essential Work of Fracture Methodology to Polymer Fracture

Kai Duan, Xiaozhi Hu*
School of Mechanical Engineering
University of Western Australia
Perth, Australia

Yiu-Wing Mai
Center for Advanced Materials Technology
School of Aerospace, Mechanical and Mechatronic Engineering
University of Sydney, Australia

ABSTRACT

The applicability and limitation of the essential work of fracture model originally proposed for double-notched polymer and metal specimens with full ligament yielding prior to failure are studied using two sets of ABS polymer results measured with three-point-bend specimens without full ligament yielding. The model is then extended and related to a simple fracture mechanics model on the local fracture energy distribution, which is originally developed for quasi-brittle fracture of concrete-like materials. Both models lead to the conclusion that the height of the crack-tip plastic zone controls the fracture toughness of materials, and the specific fracture energy dissipation along a crack path.

INTRODUCTION

The essential work of fracture (EWF) model [e.g. 1,2] has been successfully used to analyze the specific fracture energy of double-notched polymer and metal specimens with full ligament yielding prior to failure. Recently, EWF has also been used to the plane strain fracture of ABS three-point-bend (3-p-b) specimens [3] that clearly do not satisfy the full ligament yielding condition prior to the final failure.

In this paper, we are going to discuss the applicability and limitation of EFW, and show why EFW can still be used even if the full ligament yielding is not satisfied. The key argument is that the specific fracture energy G_f dealt with by EWF can always be determined experimentally regardless the ligament condition.

SPECIFIC FRACTURE ENERGY G_f

Interestingly, the quasi-brittle fracture of concrete-like materials is also frequently characterized by the specific fracture energy G_f as used for yield failure of polymers and

*Fax: 61-8-9380-1024, email: xhu@mech.uwa.edu.au

metals. RILEM has set up a standard for G_f measurements of concrete [4]. Similar to the case of EWF for polymers and metals, a stable load (P) and load-point displacement (δ) curve is required so that G_f can be evaluated from the total energy consumption over the initial ligament area.

$$G_f = \frac{1}{B \cdot (W - a)} \cdot \int P \cdot d\delta \qquad (1)$$

where B is the specimen thickness, W is the width and a is the initial crack (or notch) length. In the case of a double-notched specimen, the total width ($2W$) and the total crack length from two notches ($2a$), as commonly denoted, should be used.

Clearly, the specific fracture energy G_f as used by RILEM and EWF is only an average fracture energy measurement, which is equivalent to assuming a constant fracture energy distribution over the entire fracture area. Such a simplified assumption has to be limited to special material and specimen conditions. To extend the applicability of the specific fracture energy G_f adopted by RILEM and EWF, a local fracture energy distribution concept has been proposed [5,6]. The local specific fracture energy g_f is used to describe the energy dissipation along the crack path, over the fracture area of $B{\cdot}(W\text{-}a)$. Both G_f and g_f are related according the energy conservation principle.

$$G_f = \frac{1}{(W - a)} \cdot \int_0^{W-a} g_f \cdot dx \qquad (2)$$

If the local fracture energy distribution g_f is indeed constant over the crack path, $0 < x < W\text{-}a$, as assumed, then $G_f = g_f =$ constant, and the RILEM definition which is also used by EWF is valid. Clearly, the local specific fracture energy concept extends the applicability of the RILEM definition and EWF.

EWF: THE RELATIONSHIP BETWEEN G_f AND LIGAMENT

A typical double-notch specimen used by EWF is shown in Figure 1(a). For the deep notch geometry, a circular plastic zone with the radius of ($W\text{-}a$) prior to fracture is adequate. EWF shows that the following linear relationship exists for polymers and metals.

$$\begin{aligned} G_f &= g_0 + \text{constant} \cdot (W - a) \\ &= g_0 + g_p \end{aligned} \qquad (3)$$

where g_0 is the essential work of fracture corresponding to zero plastic zone size. The non-essential work of fracture, g_p, is related to the plastic zone size, which can be evaluated from the volume of the plastic zone for a constant plastic work density ρ_p.

$$\begin{aligned} g_p &= \rho_p \cdot \left(\frac{\text{thickness} \cdot \text{length} \cdot \text{height}}{\text{thickness} \cdot \text{length}} \right)_{\text{plastic zone}} \\ &= \rho_p \cdot \left(\text{constant} \cdot \frac{B \cdot (2W - 2a) \cdot (2W - 2a)}{B \cdot (2W - 2a)} \right)_{\text{plastic zone}} \\ &= \rho_p \cdot (\text{constant} \cdot \text{height})_{\text{plastic zone}} \\ &= \text{constant} \cdot (W - a) \end{aligned} \qquad (4)$$

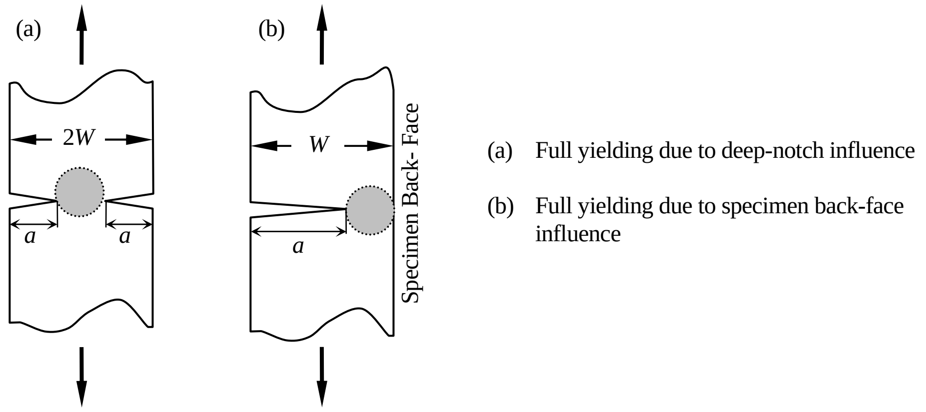

FIGURE 1 (a) Common EWF specimen showing the influence of deep double notches on the plastic zone and then G_f, and (b) deep single notched EWF specimen showing the influence of specimen back-face on the plastic zone and then G_f.

As shown by equations (3) and (4), the linear relationship between G_f and ligament $2{\cdot}(W\text{-}a)$ actually proves that G_f is directly related to the crack-tip plastic zone height measured by $2{\cdot}(W\text{-}a)$.

The yield condition shown in Figure 1(a) can also be satisfied by a single edge notched specimen either under direct tension or bending (e.g. 3-p-b) as illustrated in Figure 1(b), where the full ligament length is given by (W-a) and equations (3) and (4) are still valid. Obviously, the full ligament yield can only be achieved for a very deep notch or crack. In this case, the crack-tip condition can also be considered under the strong influence of the specimen back-face boundary. When the boundary is far away from the crack tip, full ligament yield is not possible. However, it has been shown that EWF could still be used to ABS 3-p-b specimens without full ligament yield [3], which shows EWF can be extended to cases without full ligament yield.

BOUNDARY EFFECT MODEL AND LOCAL ENERGY DISTRIBUTION

Recognition of the boundary influence from Figure 1(b) is important. The classic EWF can be considered modeling the specimen back boundary influence, a case dealt with frequently for quasi-brittle fracture of concrete-like materials. Similar to G_f of polymers and metals modeled by EWF, G_f of quasi-brittle materials like concrete is also found to be ligament and specimen size dependent. The fracture process zone (FPZ) in front of a crack tip in a quasi-brittle material like concrete is similar to a crack-tip plastic zone in a ductile polymer or metal. If a concrete specimen has a deep notch or crack as shown in Figure 1(b), FPZ covers the whole ligament showing the strongest back boundary influence. If a crack tip is far away from the specimen back-face boundary, a fully developed FPZ without any back-face boundary influence is expected. The RILEM G_f definition does not require the condition that FPZ has to cover the entire ligament area.

A simple boundary effect model has recently been developed [7-9], based on the local fracture energy concept proposed by Hu and his colleagues [5,6]. The full ligament region in a specimen with a single edge notch or crack has been separated into the inner and boundary zones illustrated in Figure 2(b). In the inner zone, the crack-tip FPZ is far away from the specimen back boundary, and can thus remains constant or shows no boundary effect. In the boundary zone, development of the crack-tip FPZ is restricted by the ligament as FPZ is too close to the specimen back-face boundary. This is akin to the full ligament yield in a polymer or metal specimen modeled by EWF.

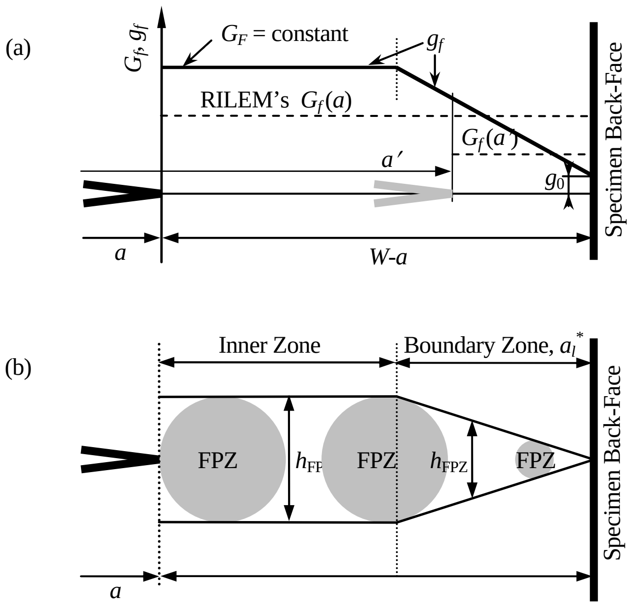

FIGURE 2 Separation of inner and boundary zones by a_l* in a specimen of width W and crack a. (a) Corresponding bi-linear local energy g_f distribution in comparison with the RILEM G_f as the average fracture energy. (b) Variation of FPZ and its height h_{FPZ} in the inner and boundary zones.

If a linear function is assumed for the local specific fracture energy g_f as shown in Figure 2(a), it can be obtained from equation (2) that:

$$G_f = \begin{cases} g_0 + (G_F - g_0)\cdot\dfrac{(W-a)}{2a_l^*} & W\text{ - }a \le a_l^* \\ G_F - (G_F - g_0)\cdot\dfrac{\cdot\, a_l^*}{2(W-a)} & W\text{ - }a > a_l^* \end{cases} \quad (5)$$

where G_F is the maximum stable specific fracture energy in the inner zone. For a large specimen with a crack-tip away from all the specimen boundaries, G_F is identical to the critical strain energy release rate G_{IC}. The transitional ligament length a_l^* is used to separate the inner and boundary zones as illustrated in Figure 2. Clearly, in the boundary zone where (W-a) < a_l^*, equation (5) is identical to EWF or equation (3). Therefore, the linear relationship between G_f and ligament (W-a) as proven by EWF implies a linear local fracture energy distribution over the ligament area.

ANALYSIS OF ABS 3-p-b RESULTS

The specific fracture energy G_f of ABS-740 was measured using 3-p-b specimens [3]. The span is 56 mm, depth W is 14 mm, and thickness B = 4 and 7 mm respectively. Two testing temperatures were 20 and 80 °C.

The 20 °C results were measured using only the specimens with thickness B = 7 mm, and are shown in Figure 3(a). Even after excluding the two solid points that were questionable as discussed by the original authors, the relationship between G_f and (W-a) is clearly not always linear. Applying equation (5) to the results, it is found that g_0 =

3.35 N/mm, G_F = 20.6 N/mm, and the transitional ligament length a_l^* separating the inner and boundary zones is 5.8 mm. The curve from equation (5) is also provided. It should be noted the saturated specific fracture energy G_F is higher than the experimental G_f values.

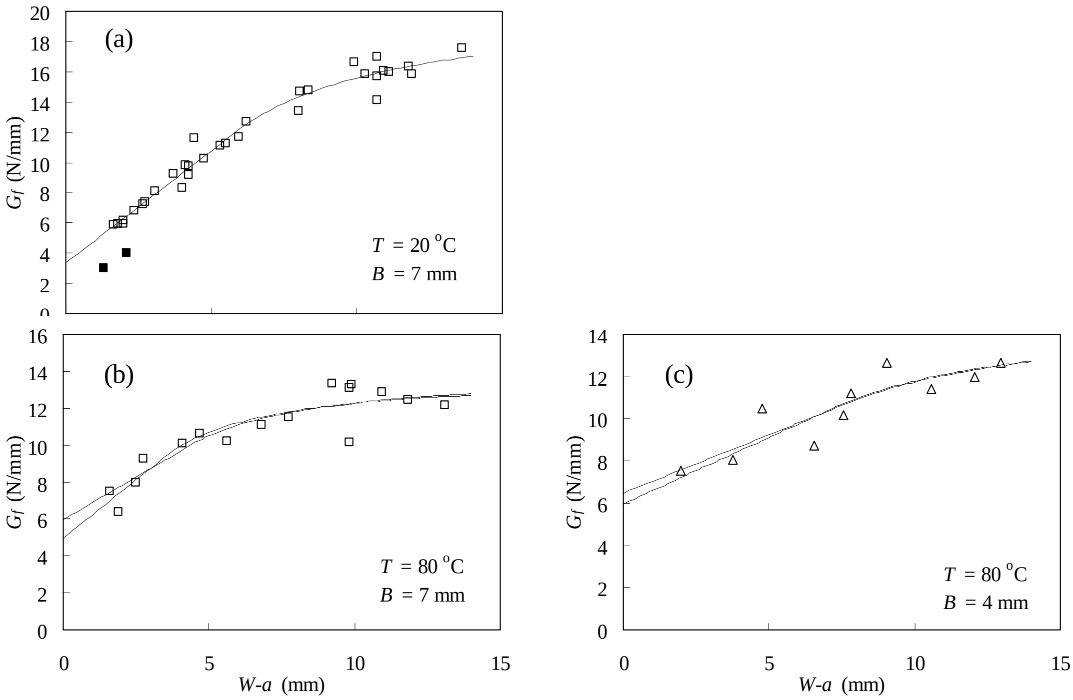

FIGURE 3 The comparison of the G_f predicted using Eq. (5) with experimental data [3]. The solid and dashed curves in (b) and (c) are obtained using different g_0 values and an identical g_0 value as listed in Table 1.

The 80 °C results were measured using the specimens with thickness of 4 and 7 mm, and are shown in Figures 3(b) and 3(c). Again, the relationship between G_f and (W-a) is clearly not linear for the results in Figure 3(b). Two slightly different approaches are adopted in the present study. First, the two sets of experimental results with B = 4 and 7 mm can be used separately to determined their own g_0 and G_F values. Second, the same g_0 can be assumed for both sets of experimental results if both specimens with B = 4 and 7 mm satisfy either the plane stress or strain condition at the same time.

The results from equation (5) are listed in Table 1. The saturated G_F is almost identical for both approaches for a given thickness of either 4 or 7 mm. The essential work of fracture g_0 estimated appears to be thickness dependent if we assume the specimens with B = 4 and 7 mm are influenced by the plane stress condition. A higher g_0 for the specimens with thickness B = 4 mm is consistent with the well-known plane stress fracture toughness behavior. The thickness dependence of G_F is also consistent with the explanation. The single g_0 assumed to be applicable to both specimens of B = 4 and 7 mm is 5.96 N/mm, between the two separate g_0 values of 4.97 and 6.48 N/mm. The curves from equation (5) are plotted in Figures 3(b) and 3(c) for the two different approaches.

TABLE 1. 80 °C results from Equation (5)

	Different g_0		Identical g_0	
B (mm)	7	4	7	4
g_0 (N/mm)	4.97	6.48	5.96	
G_F (N/mm)	13.80	15.12	14.06	15.11
a_l^* (mm)	3.50	7.82	4.40	7.27

DISCUSSIONS AND CONCLUDING REMARKS

The non-linear G_f measurements of ABS polymer following the EWF methodology have been explained by introducing the concept of local fracture energy g_f distribution, which extends the applicability of EWF. As a result, the full ligament yield in a polymer specimen prior to final failure is no longer necessary. A linear G_f and ligament (W-a) relation described by EWF implies a linear local fracture energy g_f distribution in the specimen back-face boundary region, which is directly related to the plastic zone height at the crack tip.

The conclusion that the crack-tip plastic zone height controls the fracture energy dissipation is significant, as the same linear G_f and ligament (W-a) relation can also be taken as the G_f and plastic zone height relation. This conclusion has led to a simple fracture mechanics model [10] recently proposed for the adhesive thickness effect on fracture toughness of adhesive joints, which is a case where the crack-tip plastic zone height is identical to the adhesive thickness.

ACKNOWLEDGEMENTS

The financial support from the Australian Research Council (ARC) through a Discovery Projects grant is acknowledged.

REFERENCES

1. Cotterell, B. & Reddell, J.K. 1977. "The Essential Work of Plane Stress Ductile Fracture," *Int. J. Fract.*, 13: 267-277.
2. Atkins, A.G. & Mai, Y.W. 1985. *Elastic and Plastic Fracture: Metals, Polymers, Ceramics, Composites, Biological Materials*. Ellis Horwood Ltd., Chichester.
3. Luna, P., Bernal, C., Cisilino, A., Frontini, P., Cotterell, B. and Mai, Y. -W. 2003. "The Application of the Essential Work of Fracture Methodology to the Plane Strain Fracture of ABS 3-Point Bend Specimens," *Polymer*, 44: 1145-1150.
4. RILEM TC-50 FMC, 1985. "Determination of the Fracture Energy of Mortar and Concrete by Means of Three-Point Bend Tests on Notched Beams," *Mater. Struct.*, 18:287-90.
5. Hu, X. Z. 1990. *Fracture Process Zone and Strain Softening in Cementitious Materials*. ETH Building Materials Reports No.1, ETH Switzerland. (AEDIFICATIO Publishers, 1995).
6. Hu X.Z, Wittmann F.H., 1992. "Fracture Energy and Fracture Process Zone," *Mater. Struct*. 25: 319-326.
7. Duan, K., Hu, X.Z., Wittmann, F.H. 2002. "Explanation of Size Effect in Concrete Fracture Using Non-Uniform Energy Distribution," *Mater. Struct*., 35:326-331.
8. Duan, K., Hu, X.Z., Wittmann, F.H. 2003. "Boundary Effect on Concrete Fracture and Non-Constant Fracture Energy Distribution," *Eng. Fract. Mech*., 70:2257-2268.
9. Duan, K., Hu, X.Z., Wittmann, F.H., 2003. "Thickness Effect on Fracture Energy of Cementitious Materials," *Cem. Concr. Res*., 33(4):499-507.
10. Duan, K., Hu, X.Z., Mai, Y.W. 2003. "Substrate Constraint and Adhesive Thickness Effects on Fracture Toughness of Adhesive Joints", J. Adhesion Science and Technology, in press.