Analysis and Design of Event-triggered Networked Control Systems

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Declaration

The research results presented in this thesis have not been previously submitted either in whole or in part for the award of a degree at Central Queensland University (Australia) or any other institution of higher education. I certify that the material presented in this thesis is original except where due reference is made in the text.

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Abstract

Networked control systems (NCSs) have been receiving increasing research attention in the last decade due to their attractive advantages such as flexible installation, low cost and easy maintenance, which lead to the wide applications in industry, agriculture, aerospace, remote surgery, and so on. A fundamental property of NCSs is the introduced network channels, which are bandwidth limited. This thesis is mainly concerned with how to effectively save the limited network resources, network bandwidth and/or battery power, while some desired control performance can be maintained. For this purpose, event-triggered transmission schemes are considered for NCSs to reduce some unnecessary network transmissions. The idea of the event-triggered transmission scheme is that the current sampled data is released for transmission whenever the error between the current and the latest transmitted sampled data exceeds a pre-designed threshold. An input delay method together with sampled-data error bounds induced from event-triggered transmission schemes is employed to model the inter-event dynamics. In this thesis, the event-triggered transmission schemes are studied in linear NCSs with signal quantization, network-induced delays, packet dropouts, respectively, followed by its application in a class of nonlinear systems represented by Takagi-Sugeno (T-S) fuzzy models.

- Signal quantization is an inevitable procedure in NCSs and it determines how many bits it takes to represent measurement in the digital form for transmission via a digital network channel. $\mathcal{L}_2$ stability analysis and controller design are considered for an NCS with quantized measurement under an event-

triggered transmission scheme. Furthermore, an interactive design method is presented to design an event-triggered transmission scheme and a finite-level dynamical quantizer in an integrated framework, where an output feedback controller is designed to ensure that the state of the closed-loop system is uniformly ultimately bounded.

- A novel decentralized event-triggered transmission scheme based on asynchronous sampling is proposed for NCSs, where system state can only be measured by several spatially distributed sensors instead of a centralized sensor node. Input delays resulted from the transmission scheme and network-induced delays are well considered by using a switching Lyapunov-Krasovskii functional. An $\mathcal{L}_2$ decentralized controller design method is developed for the decentralized event-triggered control system. The obtained criteria can be used to co-design of the parameters of the event-triggered transmission scheme and $\mathcal{L}_2$ controller gains.

- Packet dropouts are very challenging in an event-triggered NCS since each event-triggered measurement is important for feedback control, and there exists some relationship between two consecutive event-triggered signals because of the sampled-data errors involved in the event-triggered transmission scheme. A compensation scheme of packet dropouts is proposed for an event-triggered NCS in this thesis.

- When an event-triggered transmission scheme is adopted in a nonlinear networked system described by a T-S fuzzy model, the traditional parallel distribution compensation (PDC) method does not apply since the fuzzy controller could not receive enough information about premise variables of the plant due to the event-triggered transmission scheme. A fuzzy dynamical output feedback controller is proposed to regularly generate the control input, which makes the controlled system stable with a certain H_∞ disturbance attenuation level.

Several simulation examples are given to illustrate the effectiveness of the proposed approaches. It is shown in all the cases mentioned above with an appropriate event-triggered transmission scheme, the average transmission interval can be increased substantially while some certain level of control performance is maintained.

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Chapter 1

Introduction

1.1 Networked control systems

As digital communication and networking technologies develop incrementally in both theory and applications, communication networks are introduced in an increasing number of practical control systems to connect system components such as sensors, controllers, actuators. These kinds of spatially distributed control systems are later formulated as networked control systems (NCSs) [1]. See Figure 1.1 for a basic model of a networked control system.

NCSs have received extensive research attention in the last decade due to the following reasons. On the one hand, compared with traditional control systems, NCSs have many attractive merits such as flexible installation, low-cost deployment and easy maintenance, which result in that they have been applied to an increasing number of areas including aerospace [2], mobile sensor networks [3], haptics collaboration over internet [4], industry process control [5], agriculture control [6], remote surgery [7], etc. On the other hand, the use of communication networks in NCSs inevitably brings some challenging issues including network-induced delays, data packet dropouts and limited network bandwidth. Any one of these issues may deteriorate the NCSs' control performances or even lead to instability of the system. Many results on analysis and synthesis in traditional control systems need to be re-examined in non-ideal network environments. What makes the problem more

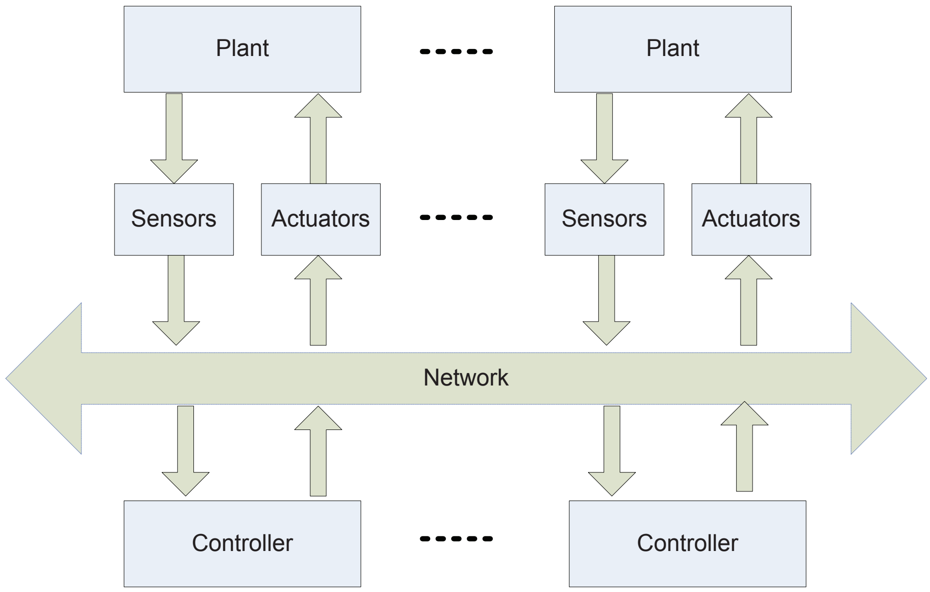

Figure 1.1: A basic model of a networked control system

challenging is that a comprehensive study on NCSs is a kind of interdisciplinary research area of control, communication and information theory.

It is noted that the fundamental difference between NCSs and traditional control systems is the introduced network channels, which make information transmissions in NCSs flexible and complicated. On the one hand, system components can be conveniently connected through the pervasive sensor networks and a number of applications appear due to this kind of flexible transmission of information in NCSs. On the other hand, several complicated problems emerge as signals are transmitted via network channels in NCSs.

i) *Limited network bandwidth.* All the network channels are bandwidth limited, that is, the amount of information that can be transmitted through a given network channel within a certain time period is limited. Although today's networks usually have much increased bandwidth (e.g. Ethernet (from 10Mb/s to 100Gb/s)), the demand for increased network bandwidth is endlessly growing. When the traffic load of a network becomes too heavy, the network quality of service (QoS) may deteriorate, which usually leads to increased network-induced delays and data packet

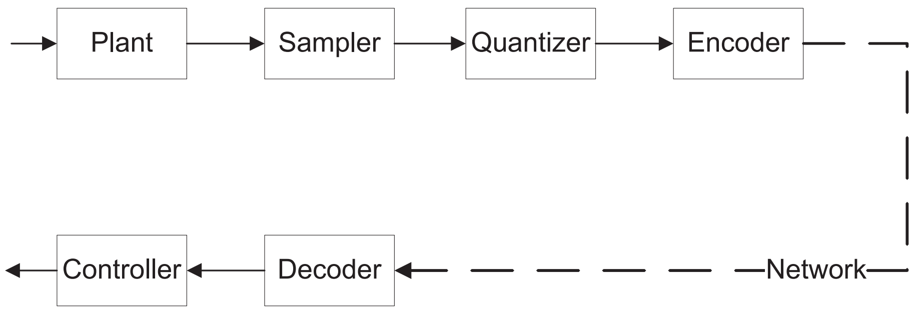

Figure 1.2: A general diagram of data transmission in an NCS

dropouts that may degrade the control performance of NCSs.

ii) *Sampling and quantization.* Figure 1.2 shows a general flow chart of data transmission from a plant to a controller in an NCS. The system output is continuous in time and it should be sampled for the use of feedback. After the sampling process, the discrete-time measurement is still continuous in amplitude. Any measurement to be transmitted through a network channel has to be quantized and encoded into a digital signal. Signal quantization can be seen as a process of sampling in amplitude [9,104]. In addition, the number of quantization levels directly determines the bit rate (bits per measurement) required of this network transmission in a general NCS.

iii) *Network-induced delays and data packet dropouts.* Network-induced delays emerge in an NCS primarily due to delays in signal processing (e.g. encoding, decoding) and signal transmission through the network channel. Another emerged issue in NCSs is the phenomenon that data packets may get lost during the transmission through networks especially through wireless networks. Packet dropouts usually result from network congestions or from transmission errors due to noises, signal distortion, jitter, etc. in physical network channels. Also, data packets out-of-order on account of time-varying transmission delays may lead to packet dropouts because the transmitted packets are usually time-stamped and the outdated data packets are discarded intently in many systems. It is noted that the presence of network-induced delays and/or packet dropouts in a control system can severely deteriorate the system's control performance, which can not be guaranteed if the delays or dropouts exceed some specified limits [5, 8, 124], for example, the maximum allowable delay bound (MADB) [17], the maximum packet-loss upper bound [95].

iv) *Network resilience.* Network mediums are usually not very reliable and they are susceptible to unexpected external disturbances such as malicious attacks, unmeant mistakes (e.g. misconfigurations, hardware faults), environmental conditions including hazardous environments, natural disasters. Such disturbances can significantly deteriorate network transmissions in NCSs. And resilience in NCSs is concerned with the ability of NCSs to achieve and maintain a desired control performance in an unreliable network environment subject to the disturbances. Therefore, network resilience is related to a range of areas such as network security, network survivability, fault-tolerance control, and this problem will not be considered in this thesis.

One can see that it makes a sense to reduce the required network bandwidth of a given NCS, by which network QoS can be improved and the effect of non-ideal network transmission can be alleviated. For a given NCS, the required network bandwidth is determined by two factors: the transmission frequency or how often the measured signals are transmitted through the network; and the number of bits per measurement or how accurately the transmitted value is represented in the digital form [26]. Considerable efforts have been made towards these two directions. As can be seen from the flow chart of data transmission in Figure 1.2, the signals are transmitted at the same rate with data sampling. In networked control systems, data sampling usually takes a time-triggered manner as in sampled-data systems, where it is assumed that arbitrarily many signals flow between system components accurately. Specially, the periodic sampling is frequently employed in NCSs since it is easy to be studied in theory and implemented in practice due to the well-developed theory in traditional control systems [27–29]. Following this way, in order to reduce the required network bandwidth of an NCS, many efforts have been made to increase the average sampling period [26, 31]. However, an interesting question is raised in the research on NCSs: *Is it necessary to transmit all the sampled data for feedback control to maintain some certain control performance in a given NCS ?*

This question is raised for NCSs with mainly the following considerations: i) It takes some amount of the limited network resources (network bandwidth and/or battery power) to transmit any sampled data through a network channel in an NCS while this cost does not need to be considered in traditional sampled-data systems; ii) The time-triggered sampling is executed only according to the elapse of time, regardless of the dynamic evolution of the system; therefore, it is possible that some unnecessary sampled signals are transmitted for feedback. For example, the measurement error between two consecutive sampled signals may be so small that the control inputs induced by them can be very similar. In this case, transmission of the latter measurement is unnecessary for feedback and is a waste of network resources.

To deal with the problem, this thesis considers *event-triggered transmission schemes* for NCSs, by which the unnecessary transmissions are expected to be substantially reduced while some desired control performance is maintained. In this way, the sampled signals are transmitted through a network channel only when necessary and it is expected that the limited network resources can be saved effectively. For this purpose, the following issues are required to be considered:

- *How to design the event-triggered transmission schemes for NCSs*?
- *How to carry out controller design for event-triggered NCSs under consideration of signal quantization, network-induced delays and/or packet dropouts*?

Now, we first review some existing results on time-triggered NCSs. As mentioned above, sampling and quantization are two different directions of efforts to save network bandwidth in time-triggered NCSs. Much attention has been paid to increase the maximum allowable sampling period, by which the transmission frequency can be accordingly reduced and network resources saving can be expected. Based on bounded time-varying sampling periods, the effect of a sampling period on the system performance is examined in [25] and an algorithm is presented to optimize the sampling frequencies to maintain the system stability under the limited computing resources available. In order to reduce the required network bandwidth, time-varying sampling is considered for a model-based NCS [26], where sampling intervals are varying either on a time interval or are driven by an identically independently distributed stochastic process or a finite-state Markov chain, and simulation examples show that larger sampling intervals can be achieved with the latter two methods while a stochastic stability is guaranteed. Based on an input delay approach and a discontinuous time-dependent Lyapunov functional, an improved maximum allowable upper bound on sampling intervals which guarantees the system stability is presented in [31]. It is noted that the results obtained by the time-triggered sampling schemes are conservative in the sense that maximum sampling intervals are usually obtained based on the worst case in a system, which implies that some data sampling/transmissions in other cases may be unnecessary.

While the sampling rate and transmission schemes determine *when* to transmit system data; the other factor that influences the required network bandwidth of an NCS, the number of bits per measurement, determines *what* to be transmitted through networks. As is shown in Figure 1.2, sampled data should firstly be quantized and encoded before being transmitted through the digital communication channel. A quantizer is inherently a nonlinear device, which makes a partition of the signal space and maps each of the partition cells to a representing signal (or a quantization level). The required data rate (bits per measurement) of the network channel is closely related to the number of quantization levels. Several quantizers have been designed to achieve different communication and control performances [101, 105].

The formulations of these kinds of problems include control with limited information [103,104], feedback control under data rate constraints [105], digital finite communication bandwidth control [106]. A fundamental issue in the quantized control is what the minimum data rate (quantization resolution) is required to stabilize an unstable system if there exists such a bound below which the unstable system will not be stabilized no matter what quantizer and controller strategies are used. Focusing on this point, a large number of results are obtained in recent years (see e.g. [24,104–106,108–110]). It is demonstrated in [104] that the coarsest memoryless quantizer that quadratically stabilizes an SISO system is the logarithmic quantizer and the optimal logarithmic base depends only on the unstable eigenvalues of the system. This result is generalized to multiple-input-multiple-output linear systems in [112], where it is illustrated that the sector bound approach is nonconservative for studying several quantized feedback design problems with a logarithmic quantizer. Based on the memoryless logarithmic quantizer, a dynamic logarithmic quantizer is proposed in [113], where a dynamic scaling parameter is introduced to scale back the sampled data when it is outside the quantization region (or the quantizer is saturated). By a primitive quantizer with data rate distribution and a set volume approach, a necessary and sufficient condition of stabilizing discrete-time linear systems is obtained in [108]. A similar result on data rate required to stabilize partially-observed, perturbed linear systems is obtained in [114] by employing the entropy power inequality of information theory and a piecewise uniform adaptive quantizer under some mild assumptions including the disturbance is bounded. In the case when the disturbances is unbounded and the data rate of the channel is time-varying, some necessary and sufficient conditions about the data rate required to stabilize the system are given in the form of expectation of the data rate [110], where the proof of the necessity part relies upon the entropy power inequality and the same adaptive quantizer as in [114]. Based on these results, You *et al.* [24] studied the case when the communication channel is unreliable and with Markovian packet dropouts, the minimum data rate required for mean square stabilization of the system is presented. It is worth mentioning that these data rate bounds are the minimum data rate values required in theory. However, the corresponding control strategies presented for achieving these bounds are still difficult to be applied in practice due to several problems such as the strong memory of encoder/decoder, powerful computation and repeating quantization.

It is of significance to study how network artifacts affect the control performance of NCSs since they are unavoidable in practice. Network-induced delays and data packet dropouts have been extensively studied as two basic of such artifacts in NCSs (see, for example, [1, 10, 13–24]). Many efforts have been devoted to analysis and synthesis of the NCSs with well modeled network-induced delays and/or packet dropouts, which are usually bounded to grantee some certain control performances. Network-induced delays are usually modeled in NCSs as a positive constant [4] or a time-varying variable which can be interval random varying [30] or stochastically varying with known probability distribution [27]. By considering network traffic conditions, it is shown that network-induced delays are non-uniformly distributed [11] and multifractal [12] in IP-based networks. According to the characteristics, A kind of probabilistic interval distribution of the communication delay is modeled and studied in [14]. Approaches developed in delay systems are employed to study the effect of network-induced delays on NCSs as in [10, 13–17, 30]. For example, a delay decomposition method is formulated for linear delay systems in [71] and applied to H_∞ control of NCSs in [30]. The problem of obtaining a maximum allowable delay bound in networked control systems is studied in [17] by the linear matrix inequalities (LMIs) approach and a network scheduling method is proposed based on the delay bound to allocate the network bandwidth and the sampling period.

Packet dropouts occur occasionally in NCSs and this phenomenon is frequently modeled as a random process with limited successive packet dropouts [19] or specially as a Bernoulli process [1,21–23] or Markov process [24] according to different network structures. The packet dropouts process of the channel is modeled as an independent and identically distributed process in [22], where a minimum data rate is given in terms of the unstable eigenvalues and the packet dropout rate for stabilization of the system. For mean square stabilization of an SISO system with stochastic packet dropouts, a fundamental limit on the maximum allowable dropouts probability is established in terms of the unstable eigenvalues.

1.2 Time-triggered control

The vast majority of today's control systems are controlled with digital devices such as personal computers, microprocessors, which have overwhelming advantages over analogue devices on computation, flexibility, cost, etc. The computer-based controllers can only work with digital signals, while a system usually evolves continuously and the system output is an analogue signal. Therefore, the analogue output of a continuous-time system has to be converted in the digital form before being used by a digital controller. An important step in the analog-to-digital conversion (ADC) process is sampling, through which a continuous-time signal is transformed to a discrete-time signal. Such a continuous-time system controlled with a digital device is traditionally known as a sampled-data system.

A natural choice of sampling schemes is the periodic sampling scheme due to its easy implementation in practical situations. In a general sampled-data system, the sampling rate determines the frequency of the feedback data flow, e.g. how frequently the system output is sampled, the control input is updated. Accordingly, the sampled measurement is periodically quantized, transmitted. This kind of control strategy is referred to as the time-triggered control [46].

The time-triggered control has been the dominant control approach for several decades in the system control area. The analysis and synthesis for time-triggered control systems have become easy due to the well-developed theory for sampled-data control systems. One critical issue in the time-triggered control approach is how to select the sampling rate, which determines the frequency of the complete feedback data flow. It is clear that there exists signal distortion during the ADC process, which may weaken the effect of feedback control. Therefore, a fast sampling rate is favored for achieving and maintaining a desired control performance in a continuous-time system. In fact, there usually exists a tradeoff between sampling rate and some control performance [31–34]. The effect of sampling rate on the stability of a nonlinear sampled-data system is investigated in [32, 33], where it is shown that for an underlying continuous-time system which is stable, there is a minimum sampling rate such that the system can remain stable if it is sampled faster than the critical rate. A similar relationship between the sampling rate and attraction domain of a sampled-data system is presented [32]. Many efforts have been made to investigate the allowable sampling rates for sampled-data systems [31, 35–38]. For example, a sampling interval dependent sufficient condition is obtained for sampled-data stabilization of linear systems by using an input delay approach and a descriptor system approach [35]. An impulsive system method is introduced to study exponential stability of sampled-data systems [37], where the upper bounds on sampling intervals (including both constant and variable sampling intervals) can be obtained through the given stability criteria. A refined input delay approach is proposed for sampled-data control of linear systems with bounded nonuniform sampling by using time-dependent Lyapunov functionals [31], where the obtained upper bound on sampling intervals can be greater than the analytical delay bound that guarantees stability.

As a special implementation of sampled-data systems, networked control systems (NCSs), generally adopt the time-triggered control approach due to the easy implementation and well-developed theory on sampled-data systems. As is introduced in the previous section, a large number of results on analysis and design of NCSs based on the time-triggered control approach have been reported in the past several decades. However, it is noted that one obvious difference between NCSs and the traditional sampled-data systems lies at the transmission of digital signals. The issue of digital signal transmission is usually neglected in the study of sampled-data systems, where it is assumed that all the signals can be transmitted reliably. While the study of control over unreliable networks is a significant research emphasis in NCSs. On the other hand, as is discussed in Section 1.1, all the network communication channels are with limited bandwidth and the increasing network traffic load can severely influence network transmission in the form of increased network-induced delays, packet dropouts, which can deteriorate control performances of the NCSs. In addition, it takes a certain amount of network resources to transmit any digital signal over a network channel while this kind of transmission cost is not considered in sampled-data control systems. Therefore, it is necessary to reevaluate the implementation of the time-triggered control approach in an NCS, where the network traffic flow follows the same rate with the time-triggered sampling rate.

It is worth noticing the use of the time-triggered control approach in NCSs with respect to the following aspects: i) *Practical implementation.* There are various time-triggered sensors with ADC available for different use, but a sensor is generally not made for a specific control application. The choice of sensors for an NCS is usually determined according to human experience. ii) *Analysis and design.* The analysis and synthesis of the time-triggered NCSs usually follow the methods developed for traditional sampled-data control systems. However, some essential properties of networks such as network resources, network traffic, are not be taken into account appropriately in pursuing design objectives. For example, a fast sampling rate can increase network load and transmission cost. While a maximum allowable sampling/transmission interval is usually obtained in the worst disturbance case, which may lead to unnecessary over-provisioning in an NCS and hence to a waste of the limited network resources. As is stated in [39], when the time-triggered control approach is employed in large-scale systems, the cost may become prohibitive. (iii) *Reliability and conservativeness.* With the time-triggered control approach in an NCS, the system can run in a reliable way in the sense of the predictable behaviors such as sampling, transmission, which is very favored in safety-critical applications. This property of predicability can make some limited packet dropouts and network-induced delays be easily considered in analysis and design of NCSs. In brief, the time-triggered control is not effective in utilization of network resources although this conservative approach can simplify the analysis and design.

1.3 Event-triggered control

Event-triggered control refers to control of systems, where a sensor transmits measurement when a measurement-dependent event threshold is violated, rather than when a certain time period is elapsed. The event-triggered control has a competitive advantage over its time-triggered counterpart in reducing the average transmission rate, which implies that network resources (network bandwidth and/or battery power) can be saved in NCSs by using an event-triggered transmission scheme. Several event-triggered sampling/transmission strategies have been developed in the literature to achieve different communication and/or control performances.

Event-triggered sampling, or Lebesgue sampling, has been studied as an alternative to the traditional periodic sampling in 1990s [43–46]. With event-triggered sampling, system output is sampled whenever the output exceeds some certain limits. It is shown that better performance can be achieved for first order stochastic systems by using a simple event-triggered sampling scheme than by the periodic sampling mode [45]. An event-triggered sampling technique is proposed for PID control [46], where simulations performed on a double tank process suggest that the CPU utilization can be effectively reduced with only little control performance degradation. It is also pointed out in the literature that a challenge to event-triggered control is the difficulty in system analysis and design, and there is a great need for the corresponding system theory on event-triggered control and scheduling.

In recent few years, event-triggered control receives a resurgence of interest due to the strong development of NCSs and related theory and hardware [40, 41, 48–54]. For real-time scheduling on embedded microprocessors, an event-triggered scheduler is proposed in [40], where a new control task is triggered to be executed when the norm of the state measurement error exceeds a state-dependent threshold. An event-triggered sampling generator is developed based on the error between the plant state and the estimated state in a way that a new sampling is generated whenever the norm of the error exceeds a given constant [49]. An event-triggered sampled-data feedback scheme is proposed in [54], where the sampling of system output is triggered by the crossings of the quantization levels. An event-triggered sampling/update scheme is proposed in [41] to study distributed networked control systems, where the current state of each subsystem is sampled and released only when the local error of the subsystem exceeds a specified state-dependent bound. Event-triggered sampling/transmission schemes have also been considered in output feedback systems [54, 55], decentralized control systems [56–58, 58], multi-agent systems [59, 60] due to their appealing advantages, which can be summarized as follows.

- The dynamical evolution of the system is taken into account to determine when to update the feedback data, which makes the data sampling/transmission more intelligent.

- Data sampling occurs when it is necessary (a data dependent threshold is violated), which implies that the sampling/transmission scheme is more effective in the sense that only the effective signals are selected to be transmitted. By this way, the average transmission interval can be increased and some amount of the limited network resources can be saved. In addition, the network and CPU occupying time may also be reduced.

- Robustness of the event-triggered control systems may be improved as the system output is continuously monitored.

However, there are two issues worth mentioning on the event-triggered sampling mechanism. (i) First, some real-time detection hardware is required for continuously monitoring system output so that the signal sampling can be executed at the moment when the event threshold is violated [51, 53]. This critical requirement on real-time hardware may restrict the implementation of event-triggered sampling schemes in practice. In addition, perhaps it would cost a large amount of energy to keep the radio on for detecting some possible event thresholds. (ii) In the event-triggered sampling implementations, the sampling times are determined by a state/output dependent condition that is checked online. Therefore, the sequence of sampling time instants may converge to a specific time instant, especially when the event-triggered sampling mechanism is applied in output feedback control systems [54, 55] and decentralized control systems [56–58]. Therefore, a lower bound of inter-sampling periods has to be evaluated to avoid the Zeno behaviour in the event-triggered control and it usually takes much attention to obtain and guarantee such a positive lower bound. Motivated by the observations, *several event-triggered transmission schemes will be considered for NCSs in this thesis to conquer the two challenging issues while retaining the appealing advantage of effectively saving the limited network resources.*

Substantial efforts have been made in the area of event-triggered control to deal with the challenging issues. To avoid the Zeno behaviour in decentralized event-triggered control, a separate term of a positive constant is involved in each subdecentralized event-triggered threshold in [55–57]. By this way, it generally only leads to a practical stability of the system [57], where the asymptotic stability is achieved only if the positive scalars could be appropriately adjusted online. A decentralized event-triggered sampling scheme is proposed in [61], where a minimum sampling interval should be pre-determined in the sampling logic rule to avoid the Zeno behaviour.

On the other hand, one way of avoiding the required real-time detection hardware is the self-triggered sampling, which is proposed for event-triggered control as an alternative to the event-triggered sampling. With a self-triggered sampling scheme, the next sampling time instant of the system is predicted at the current sampling instant based on the latest state measurement and system dynamics. Self-triggered sampling, or self-triggered task execution for real-time control systems is first introduced in [62], where the data sampling instance is dynamically predicted according to the controlled system performance and processor utilization. Further results on self-triggered control are presented for linear time-invariant systems with disturbances [51, 52] and nonlinear systems [53]. A self-triggered sampling scheme is proposed in [52] for real-time systems, where both of the next task release time and finishing time can be predicted based on knowledge of the sampled state and a nonzero inter-sample lower bound is obtained to avoid the Zeno behaviour. Compared with the event-triggered sampling mechanism, an advantage of self-triggered sampling is that some special real-time hardware is no longer needed as the current system state is not required for determining sampling times. However, it is difficult to predict the next sampling time when the construction detail of a system is not known *a priori* or the system is exposed to some unknown disturbances. Another issue is that the average sampling interval generated by a self-triggered sampling scheme is usually shorter than that generated by an event-triggered sampling scheme for the same plant, since the sampling intervals generated by the self-triggered sampling scheme is predicted under worst disturbance conditions even if these rarely occur. It is also noted that the existing study on the self-triggered control is restricted on state feedback control and it is difficult to be applied in other feedback control scenarios.

Another event-triggered control strategy is developed by integrating conventional periodic sampled-data control and the event-triggered mechanism. Existing results on this strategy is on event-triggered control for discrete-time systems [116–119]. Event-triggered and self-triggered strategies described previously are generalized to control of discrete-time systems in [117]. For reducing communication in discrete-time distributed control systems [116], identical estimators are employed at each node of the system to estimate system outputs at other nodes and the estimated values are used for feedback control at each node; the true output measurement is transmitted to the rest of the system only when the error between the output and its estimated value exceeds a pre-specified value. For practically stabilizing perturbed linear systems with a purpose of reducing the number of control calculations, a kind of event-triggered control scheme is proposed in [118], where the control input is calculated only when the system state is outside a given set and a certain time period is elapsed, then the event-triggered control scheme is analyzed in the discretized version of the original system. It can be seen that by combining periodic sampling and event-triggered mechanism in a system, the system output is no longer needed to be continuously monitored since the event-triggered threshold is only checked at some discrete time instants. Therefore, both of the two issues, special hardware and Zeno behaviour, have disappeared in this case, although the inter-sample behavior of the system is not considered. This kind of discrete event-triggered control strategy is recently further studied [119, 120] and formulated as the so-called periodic event-triggered control. The periodic event-triggered control is proposed for a discrete-time model-based linear system with a Luenberger observer and a predictor employed in the sensor [119], where the estimated state is transmitted whenever the error between it and the predicted state exceeds a threshold. With an impulsive system approach, the periodic event-triggered control is studied for a continuous-time linear system where the event-triggered condition is verified periodically instead of continuously [120]. It is noted that although the state measurement is transmitted over networks in [119, 120], none of network imperfections such as signal quantization, network-induced delays, packet dropouts are considered in the studies. One can see that the proposed event-triggered control strategies in [119, 120] will not be applied any longer when network-induced delays or packet dropouts occur during network transmissions.

For controller design in an event-triggered control system, most of the existing results follow a so-called emulation-based method, by which there are two steps to present the event-triggered control strategy. First, based on traditional control approaches, a controller is designed for the system such that a stability concept (e.g. input-to-state stability [40, 63, 117], $\mathcal{L}_2$ stability [41], exponential stability [120]) is guaranteed for the system. For nonlinear systems, it is usually assumed that there exists an appropriate Lyapunov function satisfying some stability conditions although the feasible controller may not always exist [115]. Second, an event-triggered threshold condition is proposed to preserve the chosen stability concept. For example, a pre-designed controller is employed in [40] to ensure that the closed-loop system is input-to-state stable (ISS) with respect to measurement errors, based on which an event-triggered sampling scheme that guarantees the monotonic decrease of the ISS Layapunov function is introduced. To relax the event condition presented in [40], a refined event-triggered sampling scheme is proposed [63], where the state is sampled when the Lyapunov function intersects an exponentially decreasing function. Based on a traditional LQR controller, a self-triggered sampling scheme and an event-triggered sampling scheme are proposed for distributed implementation over sansor/actuator networks [64]. The periodic event-triggered control strategy presented in [120] also follows the emulation-based approach, by which a standard periodic sampled-data controller is pre-designed by a traditional sampled-data control design method [66] such that the resulted closed-loop system is globally exponentially stable and with a prescribed $\mathcal{L}_2$ gain. It is shown that by the emulation-based method, a desired control performance of an event-triggered closed-loop system can be ensured while the average transmission interval can be increased. However, it is noted that the emulation-based approach may lead to some conservatism since the employed controller is usually pre-designed/assumed without considering the event-triggered sampling/transmission scheme. *Motivated by this observation, this thesis will consider the problem of controller design for event-triggered NCSs by studying the event-triggered transmission scheme and controller design in a unified framework.*

As stated previously, it has been shown in the literature that event-triggered control has an advantage of saving the limited network resources and reducing network traffic in NCSs. However, it is still challenging to apply event-triggered control in network control systems. It is noted that in most of the existing results on event-triggered control (e.g. [40, 64, 84, 119, 120]), there is an important assumption: the event-triggered data can be directly transmitted via a network reliably; while very few works consider network effects on signal transmissions such as signal quantization, network-induced delays, packet dropouts [68]. From a practical point of view, the effect of signal quantization should be studied in event-triggered NCSs since quantization is an inevitable procedure in NCSs, and the quantization error has a direct influence on measurement errors which are used in event-triggered thresholds. Therefore, event-triggered control schemes proposed in real-time systems with ideal information transmission should be reexamined when they are introduced in digital applications such as NCSs. Although the network traffic can be efficiently reduced in an NCS by using an event-triggered transmission scheme, network-induced delays and packet dropouts may still occur in the network transmissions, especially when a shared (contention-based) network is employed in the NCS. Both of the issues may deteriorate control performances of the event-triggered NCSs more severely than they do to those of the time-triggered NCSs. One major reason is that there usually exists a strong relationship between two consecutive broadcasted signals in an event-triggered NCS due to the error dependent event-triggering condition. *It is therefore of great importance to study these network related issues (signal quantization, network-induced delays, packet dropouts) in event-triggered NCSs to maintain some desired control performances while less information is required for network transmission.*

It is much more difficult to deal with signal quantization, network-induced delays and packet dropouts in the event-triggered NCSs than in the time-triggered NCSs since these issues can severely influence the design of the event threshold. As is shown in [70], the effects of quantized measurement are very critical in the selection of stabilizing thresholds. A more conservative event-triggered broadcasting scheme is required to tolerate some bounded network-induced delays and packet dropouts [41]. Based on an emulation-based approach, a quantization scheme design is proposed for distributed event-triggered control systems [67], where a dynamic sphere logarithmic quantizer with a zooming strategy is given to quantize signals in radius direction within the event-triggered framework, while the quantization of spherical angles is not taken into consideration. The issue of stabilizing bit rates for an event-triggered control system is studied in [115], where it is shown that the required bit rates are bounded by some relations among transmission delays, inter-sampling interval, and quantization error which is pre-assumed. Signal quantization and network-induced delays are considered in model-based event-triggered control systems [70], where several stabilizing thresholds are designed to preserve stability under the consideration of parameter uncertainties, logarithmic quantization error and/or time-varying delays. Transmission delays are considered in event-triggered NCSs [133, 134], where the delays are merged into sampling intervals in the system analysis. The independent and identically distributed packet dropouts are studied in event-triggered impulse control based on delta sampling [69], where the relationship among control performance, level thresholds and the packet loss probability is analyzed. Transmission delays and packet dropouts are considered in distributed event-triggered networked control systems [41], where one can find a tradeoff among event-triggered thresholds, allowable transmission delays and packet dropouts. By taking a smaller event-triggered sampling threshold than the one proposed in the delay free case, some limited packet dropouts and transmission delays can be tolerated [41], where the maximum allowable number of successive data dropouts (MANSD) and state-dependant delay upper bounds for each successful transmission to maintain some desired control performance are presented. However, as stated in [96], one critical issue of the method proposed in [41] is that the inter transmission time may become infinitely small as the system state approaches the origin. Under a static event-triggered condition, the distributed event-triggered control with packet dropouts is studied in [96], where two transmission protocols are proposed when a packet get lost: one is to retransmit the lost data after a waiting period while the other one is to transmit the current data after a waiting time. Both of the protocols, however, can only make the sate of each subsystem finally converge to a small region due to the static triggering condition.

It can be concluded from the above discussion that the study of event-triggered networked control systems under consideration of signal quantization, network-induced delays, packet dropouts is of significance and challenging. This thesis will consider controller design for the event-triggered NCSs with an easy implementable event-triggered transmission scheme, where signal quantization, network-induced delays, data packet dropouts will be dealt with.

1.4 Significance of this thesis

It is well known that networked control systems (NCSs) have an increasing number of applications in practice due to some appealing merits such as flexible installation, low-cost deployment and easy maintenance. On the other hand, diverse research efforts have been devoted to dealing with several non-ideal factors resulted from the introduced network channels in network-based control systems, such as network-induced delays, data packet dropouts, signal quantization, etc. It is noted that all the network channels are bandwidth limited, which may lead to degraded quality of service (QoS) as network traffic load becomes heavy. Therefore, it is of great importance to study how to reduce the required network bandwidth of an NCS by considering some fundamental properties of network transmission.

On the other hand, data update/transmission in NCSs is usually executed in a time-triggered manner as in traditional control systems, where there is no restriction on data transmission. One reason of adopting the periodic sampling/tranmsission scheme in NCSs is that system analysis and design are easy due to the well developed theory on sampled-data control systems. Another reason is that the periodic sampling/transmission is easy to be implemented in practice. However, since it takes a certain amount of network resources, for example, network bandwidth and battery power, to complete any data transmission via a network communication channel, here comes a significant question as to the time-triggered NCSs: is it always necessary to transmit all the sampled data in an NCS?

An observation is that when a system evolves steadily, the difference between two consecutive sampled data may be so small that the control inputs induced by them can be very similar. In this case, the latter sampled data is no longer needed to be transmitted for feedback control, which implies that the required network resources of an NCS can be decreased by reducing some unnecessary network transmissions. One important purpose of this thesis is to consider some easily implemented event-triggered transmission schemes for NCSs, by which only a small proportion of the sampled data are required for being transmitted via networks to maintain some desired control performance. This shows the significance of this research in practical NCSs applications in terms of saving the limited network resources, alleviating network traffic load, reducing CPU occupying time, etc.

In addition, two significant and challenging issues are remaining unresolved for the event-triggered NCSs. For example, how to design a feasible controller for NCSs under consideration of an event-triggered transmission scheme under non-ideal network environments? How to deal with the challenging issues of signal quantization, network-induced delays, packet dropouts in the event-triggered NCSs? The study of these research issues in this thesis will contribute to a system theory on event-triggered networked control systems.

1.5 Contributions of this thesis

The contributions of this thesis can be briefly summarized as follows

- For the purpose of saving the limited network resources and reducing network traffic, several event-triggered transmission schemes are proposed for NCSs to select which sampled data should be transmitted via networks for feedback control. More specifically, for a given NCS, an event-triggered transmission scheme is executed in a way that the current sampled data is triggered to be transmitted via a network channel whenever a sampled-data error dependent threshold is violated. It is shown in this thesis that by this way, the average transmission interval can be substantially increased while some desired control performance can still be maintained, which implies that a considerable amount of the limited network resources can be saved. In addition, the proposed event-triggered transmission schemes are much more easily to be implemented in theoretical analysis and practical applications compared with the event-triggered sampling schemes. On the one hand, a nonzero lower bound of inter-event intervals has to be evaluated when an event-triggered sampling scheme is implemented, while this difficult problem vanishes in this thesis since the processes of sampling and transmission are executed separately. On the other hand, the real time detection hardware required in event-triggered sampling schemes for continuously monitoring system outputs is no longer needed in this thesis.

- An interval time-varying delay is introduced to model the inter-transmission dynamics together with the sampled-data error bounds guaranteed by the adopted event-triggered transmission schemes. The sawtooth structure characteristic of the artificial delay can be considered in deriving system stability criteria, which makes that controller design and event-triggered transmission scheme can be studied in a unified framework. By the proposed method, a controller can be designed under consideration of an appropriate event-triggered transmission scheme, while most of the existing results on event-triggered control are obtained by a so-called emulation-based method, where a controller is usually pre-given without considering the event-triggered mechanism. Furthermore, the parameters of transmission scheme and controller can be obtained simultaneously in some cases considered in this thesis.

- Signal quantization is an important and inevitable procedure in information transmission over network channels. The number of quantization levels directly determines the data rate (bits per measurement) required for transmitting each measurement. An $\mathcal{L}_2$ controller design method is presented for an NCS with quantized measurement, where an event-triggered transmitter is proposed to choose which quantized measurement should be transmitted through a network for feedback. Moreover, an interactive design of finite-level quantizer and event-triggered communication scheme is proposed in a novel NCSs framework, where an output feedback controller is designed to ensure that the state of the closed-loop system is uniformly ultimately bounded.

- A decentralized event-triggered transmission scheme based on asynchronous sampling is proposed for NCSs, where network-induced delays are considered in each transmission channel. Input delays resulted from the transmission scheme and network-induced delays are appropriately considered by using a switching Lyapunov-Krasovskii functional. Both decentralized and central controllers are designed for the decentralized event-triggered NCSs to achieve a prescribed $\mathcal{L}_2$ control performance.

- Data packet dropouts occur occasionally in a general NCS and this issue has been well studied in the time-triggered NCSs. However, this problem becomes much more difficult when it is considered in an event-triggered NCS since there is a relationship between two consecutive event-triggered signals, which can be deteriorated by the random packet dropouts. To deal with this challenging issue, *a compensation scheme for packet dropouts* is proposed for an event-triggered NCS with random packet dropouts.

- The event-triggered transmission scheme is applied in a class of nonlinear systems described by a T-S fuzzy model, where the traditional parallel distribution compensation (PDC) can not apply since the fuzzy controller may not receive enough information about premise variables of the plant due to the event-triggered transmission scheme. A fuzzy dynamical output feedback controller is proposed to regularly generate the control input, which makes the controlled system stable with a certain H_∞ disturbance attenuation level.

1.6 Organisation of this thesis

This thesis is concerned with analysis and design of event-triggered networked control systems. Several event-triggered transmission schemes are presented for NCSs to save the limited network resources by reducing some unnecessary network transmissions. The organisation of this thesis is given as follows:

- Chapter 2 considers the problem of $\mathcal{L}_2$ controller design for an event-triggered NCS with quantized measurement. An event-triggered transmission scheme is proposed to select which quantized measurement should be transmitted via a network for feedback control. $\mathcal{L}_2$ control performance and the event-triggered transmission scheme are studied in a unified framework. It is shown that the average transmission interval could be substantially increased while a prescribed level of $\mathcal{L}_2$ control performance is maintained.

- Chapter 3 proposes an event-triggered quantized-data output feedback control scheme for linear systems. An event-triggered communication scheme is introduced to determine which sampled data should be quantized and transmitted to the controller. The event threshold is constructed by considering the error between the current sampled data and the latest quantized data. A finite-level dynamical quantizer is developed based on the event-triggered communication scheme and quantized data. A static output feedback controller is designed to ensure that the state of the closed-loop is uniformly ultimately bounded. A numerical example illustrates the effectiveness of the proposed approach.

- Chapter 4 introduces a decentralized event-triggered transmission scheme based on asynchronous sampling. The event-triggered transmission scheme does not depend on the full-order state of the system. Several spatially distributed sensor nodes are employed to collect the state measurement. Each node transmits the sampled data separately according to a sub-event threshold. Network-induced delays are taken into consideration for each transmission channel. A switching Lyapunov-Krasovskii functional method is employed to analyze the decentralized event-triggered control system. An $\mathcal{L}_2$ decentralized controller design is presented based on the analysis result. Several simulation examples are used to illustrates the effectiveness of the results.

- Chapter 5 considers the issue of packet dropouts in an event-triggered NCS by utilizing a compensation scheme. The current sampled data is transmitted via a network to the controller when a state dependent threshold is violated. The sampled data is also pushed in an FIFO queue implemented in a buffer. When an event-triggered data gets lost during the network transmission, the transmitter will retrieve the lost data and retransmit it to the controller. A corresponding state feedback controller is designed to guarantee the finite-gain $\mathcal{L}_2$ stability of the resulted control system. It is shown that the number of data packets required to be transmitted is significantly reduced while maintaining the desired control performance.

- Chapter 6 studies the H_∞ control problem for discrete-time Takagi-Sugeno (T-S) model fuzzy systems with event-triggered output feedback. The measurement output is transmitted to a fuzzy controller when the output error exceeds a pre-given threshold. The parallel distribution compensation (PD-C) can not be used for controller design since the controller may not receive enough information about premise variables of the plant due to the event-triggered transmission scheme. A fuzzy dynamical output feedback controller is proposed to regularly generate the control input, which makes the controlled system stable with a certain H_∞ disturbance attenuation level.

- Chapter 7 gives a conclusion to the thesis and presents some related research issues for future study.

Chapter 2

Event-triggered $\mathcal{L}_2$ controller design of networked control systems with quantized measurement

2.1 Introduction

Generally, in a networked control system(NCS), the system output is firstly sampled and quantized; then the quantized data is encoded into a digital form before being transmitted via a network to a controller or a filter. Signal quantization is usually an inevitable procedure in an NCS since analog signals have to be quantized before being transmitted through a digital communication channel with limited bandwidth. It is acknowledged that the processes of sampling, quantization and transmission are time-triggered, and they are executed at the same rate. In this way, all the sampled/quantized signals in the control system are transmitted for feedback. However, since it takes some amount of network resources (e.g. network bandwidth and/or battery power) to transmit any signal through the network, it is worthwhile to study if it is necessary to transmit *all* the measurement to achieve and maintain a desired control performance.

As stated in Chapter 1, event-triggered sampling has received increasing attention recently in the system control area due to some experimental results suggesting that the average sampling interval could be increased by using an event-triggered sampling scheme [40, 41, 115]. This is a very attractive advantage in NCSs since it provides a way of reducing the transmission frequency, which implies that the limited network resources could be saved. Several event-triggered sampling schemes and control strategies have been proposed in the literature to achieve different control performances. For example, an event-triggered sampling scheme is developed for real-time scheduling of stabilizing control tasks on an embedded processor [40], where the current state is sampled only when the measurement error exceeds a specified state-dependent threshold. It is noted that in most of the existing results on event-triggered control: i) an event-triggered control strategy is usually developed based on the so-called "emulation based" method, by which a controller is first designed for the system *without* considering the event-triggered mechanism, then the event-triggered sampling/tranmsission technique is put forward based on the conditions that preserve the chosen stability concept such as input-to-state stability [41,57,115], $\mathcal{L}_2$ stability [41], exponential stability [120]; ii) some special real-time detection hardware is required to *continuously* measure system output and the sampling/tranmsission is executed at the moment when the event threshold is violated; in this way, the measurement error could always be assumed to be within the event threshold; iii) a nonzero lower bond of inter-event intervals has to be evaluated, that is, the Zeno phenomenon must be avoided. These observations motivate this chapter.

This chapter will consider an event-triggered NCS framework, where an event-triggered transmitter is employed to determine which quantized measurement is required for network transmission. A quantized signal is released for transmission only when a pre-designed threshold is violated. With the quantization error taken into account, an $\mathcal{L}_2$ stability criterion is developed for the event-triggered NCS. Based on the stability criterion, a controller design method is given to obtain a desired controller such that the event-triggered closed-loop system is finite-gain $\mathcal{L}_2$ stable.

There are two aspects worth mentioning about the event-triggered NCS framework in this chapter. On the one hand, the sampling and event-triggered transmission processes are executed separately. Therefore, the real-time detection hardware is no longer needed compared with the continuous event-triggered sampling technique. On the other hand, the event-triggered transmitter is introduced to trigger the release of the *quantized data*. This structure is expected to be more effective than the structure where the event-triggered mechanism is implemented prior to the quantization process in the sense of saving the limited network resources. In the latter case, the average error of the event-triggered sampled data may become small due to the effect of quantization, which can reduce the effect of the event-triggered mechanism.

The organisation of this chapter is as follows. An event-triggered transmission scheme is proposed in Section 2.2. System modeling analysis is given in Section 2.3. The $\mathcal{L}_2$ stability analysis and controller design results are presented in Section 2.4. Two numerical examples are given in Section 2.5 to show the effectiveness of the proposed method. This chapter is concluded in Section 2.6.

2.2 An event-triggered transmission scheme

In this section, we present an event-triggered transmission scheme for a network-based control system, where some sensor and/or control signals are transmitted through network channels. Since network resources such as network bandwidth and battery power are limited, the event-triggered transmission scheme is proposed to save the limited resources and alleviate the network traffic. Figure 2.1 gives a conceptual framework of such a networked control system.

In the framework given above, it is noted that

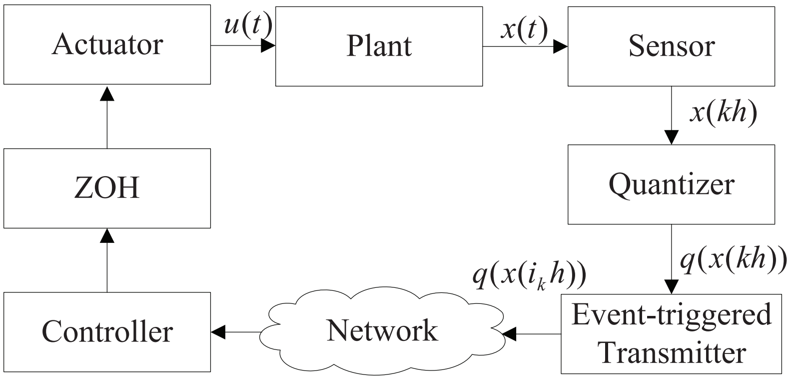

Figure 2.1: An NCS with an event-triggered transmitter

(i) The system state is assumed to be fully measurable and it is sampled by a time-driven sensor; the sampling rate could be fixed or time varying although a constant sampling rate (with sampling period h) is taken in the following sections for simplicity;

(ii) The sampled data $x(kh)$ need to be quantized by a quantizer $q(\cdot)$ before being transmitted through the network;

(iii) For the efficient use of the network resources, an event-triggered transmitter is proposed to determine whether or not the current quantized measurement $q(x(kh))$ should be transmitted to the controller.

It can be seen that the sampling time sequence is $\{kh\}_{k=1}^{\infty}$. For notational convenience, we denote the transmitter broadcast time sequence as $\{i_k h\}_{k=1}^{\infty}$. Notice that $\{i_k h\}_{k=1}^{\infty}$ is a subsequence of $\{kh\}_{k=1}^{\infty}$. The broadcast release time instants $\{i_k h\}_{k=1}^{\infty}$ are generated by the event-triggered transmitter according to the following transmission logic

$$i_{k+1}h = i_k h + \min_{l \in \mathbb{Z}^+} \{lh | e^T(r_{k,l}h)\Phi e(r_{k,l}h) \geq \delta q^T(x(i_k h))\Phi q(x(i_k h))\} \tag{2.1}$$

where the positive matrix Φ is a weighting matrix; δ satisfying $0 < \delta < 1$ is the threshold of the event-triggered transmission scheme; $\mathbb{Z}^+$ is the set of positive integers; and

$$e(r_{k,l}h) = q(x(r_{k,l}h)) - q(x(i_k h)), \tag{2.2}$$

$$r_{k,l}h = i_k h + lh, \quad l = 1, 2, 3, \ldots. \tag{2.3}$$

It can be seen from (2.1) that the event-triggered transmission scheme is executed based on the error $e(r_{k,l}h)$ between the current quantized data $q(x(kh))$ and the latest transmitted one $q(x(i_k h))$. Once the transmitter receives quantized data $q(x(r_{k,l}h))$, it will check the error $e(r_{k,l}h)$ according to the threshold condition

$$e^T(r_{k,l}h)\Phi e(r_{k,l}h) < \delta q^T(x(i_k h))\Phi q(x(i_k h)). \tag{2.4}$$

If the inequality (2.4) holds, which implies that it may be not necessary to update $q(x(i_k h))$ with $q(x(r_{k,l}h))$ by transmitting $q(x(r_{k,l}h))$ through the network to the controller, the transmitter will discard $q(x(r_{k,l}h))$ instead of transmitting it. On the contrary, if the error bound in (2.4) is exceeded, the transmitter will transmit $q(r_{k,l}h)$ instantly; and $q(x(i_{k+1}h)) := q(r_{k,l}h)$ will be stored in the transmitter.

Remark 2.1. *It can be seen that the number of data packets required to be transmitted can be reduced by introducing the event-triggered transmitter since some unnecessary data may be discarded by the transmitter. One can see from (2.1) that as the value of δ increases, the number of data packets triggered for transmission will be reduced. On the other hand, if $\delta \to 0+$, the event-triggered transmission scheme gradually degenerates towards a time-triggered transmission scheme.*

Remark 2.2. *It is clear that one can not reduce arbitrary many quantized data packets without considering the control performance. Therefore, the value of parameters Φ and δ are chosen under consideration of the desired control performance, which will be shown in the following sections. The purpose of introducing the event-triggered transmission scheme (2.1) is to save the limited network resources while guaranteeing a certain level of control performance for a given network-based control system.*

Remark 2.3. *Different from the event-triggered sampling scheme as in [40, 41, 55], the proposed event-triggered transmission and the sampling process are executed separately. Therefore, the real-time detection hardware is no longer needed while the network traffic load can still be reduced.*

2.3 System modeling

In this section, we will apply the event-triggered transmission scheme (2.1) to a class of linear systems. The continuous system dynamics and discrete transmission events will be studied in a unified framework.

Consider the following linear time-invariant system

$$\dot{x}(t) = Ax(t) + Bu(t) + B_\omega \omega(t), \ t \geq t_0 \tag{2.5}$$

where $x(t) \in \mathbb{R}^n$, $u(t) \in \mathbb{R}^m$ and $\omega(t) \in \mathcal{L}_2[0, \infty)$ are the system state, control input and the exogenous disturbance respectively; A, B and B_ω are constant matrices with appropriate dimensions; and the initial condition of the system (2.5) is given by $x(t_0) = x_0$.

To better convey the idea of the event-triggered transmission scheme in this chapter, the network transmission delay and data processing delay are not taken into consideration. We are interested in designing the following controller

$$u(t) = Kq(x(i_k h)), \qquad t \in [i_k h, i_{k+1} h) \tag{2.6}$$

where $K \in \mathbb{R}^{m \times n}$ is to be determined.

The quantizer $q(\cdot)$ is assumed to be in the form of $q(\cdot) = [q_1(\cdot) \ \ldots \ q_n(\cdot)]^T$. Each scalar quantizer $q_j(\cdot)$, $j \in \{1, 2, \ldots, n\}$, is a logarithmic quantizer with the quantization levels set

$$\mathcal{U}_j := \{\pm u_i^{(j)}, \ u_i^{(j)} = \rho_j^i u_0^{(j)}, \ i = 0, \pm 1, \pm 2, \ldots\} \cup \{0\}, \quad 0 < \rho_j < 1, \ u_0^{(j)} > 0.$$

The parameter i is the exponent of the exponential function and it can also be used as the number index of the countable quantization levels. Each quantization level

$u_i^{(j)}$ corresponds to a segment such that $q_j(\cdot)$ can map the whole segment to this quantization level and all the segments form a partition of the real set $\mathbb{R}$. The associated quantizer $q_j(\cdot)$ is defined as [104, 112]:

$$q_j(v) = \begin{cases} u_i^{(j)}, & \text{if } \ 0 < \frac{1}{1+\sigma_j} u_i^{(j)} < v \le \frac{1}{1-\sigma_j} u_i^{(j)} \\ 0, & \text{if } \qquad\qquad v = 0 \\ -q_j(-v), & \text{if } \qquad\qquad v < 0 \end{cases}$$

where

$$\sigma_j = \frac{1-\rho_j}{1+\rho_j} \tag{2.7}$$

and ρ_j satisfying $0 < \rho_j < 1$ is referred to as the quantization density. Let $\Lambda = diag\{\sigma_1, \sigma_2, \ldots, \sigma_n\}$.

From the definition of the quantizer $q(\cdot)$ described above, one can see

$$|q_j(v) - v| \le \sigma_j v. \tag{2.8}$$

Then $q(x(i_k h))$ can be written as

$$q(x(i_k h)) = (I + \Delta)x(i_k h) \tag{2.9}$$

where

$$\Delta = diag\{\Delta_1, \Delta_2, \ldots, \Delta_n\}, \ |\Delta_j| \le \sigma_j, j = 1, 2, \ldots, n. \tag{2.10}$$

For the convenience of data analysis, let

$$\tilde{r}_{k,l} h = \begin{cases} i_k h, & l = 0 \\ r_{k,l} h, & 1 \le l < i_{k+1} - i_k. \end{cases} \tag{2.11}$$

One can see that there is a one-to-one correspondence between the sampling time instants sequences $\{kh\}_{k=i_1}^{\infty}$ and $\{\tilde{r}_{k,l} h\}_{k=1}^{\infty}$ as the system evolves. The interval $[i_k h, i_{k+1} h)$ can be written as

$$[i_k h, i_{k+1} h) = \bigcup_{s=1}^{l_k} I_{k,s} \tag{2.12}$$

where

$$I_{k,s} = [i_k h + (s-1)h, i_k h + sh), \quad s = 1, 2, \ldots, l_k \tag{2.13}$$

$$l_k = i_{k+1} - i_k, \quad k = 1, 2, \ldots. \tag{2.14}$$

Now, we define two functions $\eta(t)$ and $e_k(t)$ on $[i_k h, i_{k+1}h)$ as

$$\eta(t) := \begin{cases} t - i_k h, & t \in I_{k,1} \\ t - i_k h - h, & t \in I_{k,2} \\ \vdots, & \vdots \\ t - i_k h - l_k h + h, & t \in I_{k,l_k} \end{cases} \tag{2.15}$$

$$e_k(t) := \begin{cases} 0, & t \in I_{k,1} \\ q(x(\tilde{r}_{k,1}h)) - q(x(i_k h)), & t \in I_{k,2} \\ \vdots, & \vdots \\ q(x(\tilde{r}_{k,l_k-1}h)) - q(x(i_k h)), & t \in I_{k,l_k} \end{cases} \tag{2.16}$$

Then one can obtain that

$$q(x(i_k h)) = q(x(t - \eta(t))) - e_k(t), \quad t \in [i_k h, i_{k+1}h). \tag{2.17}$$

The error-dependent closed-loop system can be derived from (2.5), (2.6), (2.9) and (2.17) as

$$\dot{x}(t) = Ax(t) + BK(I + \Delta)x(t - \eta(t)) - BKe_k(t) + B_\omega \omega(t), \; t \in [i_k h, i_{k+1}h). \tag{2.18}$$

We supplement the initial condition of the state on $[t_0 - h, t_0]$ as $x(t) = \varphi(t), t \in [t_0 - h, t_0]$, where $\varphi(t)$ is a continuous function on $[t_0 - h, t_0]$ and $\varphi(t_0) = x_0$.

Remark 2.4. *Since only the event-triggered data $q(x(i_k h))$ is transmitted to the controller, an interval time-varying variable $\eta(t)$ is introduced to model the system dynamics in transmission intervals together with the event-triggered threshold condition (2.4).*

The purpose of this chapter is to design a controller in the form of (2.6) such that

(i) The closed-loop system (2.18) with $\omega(t) = 0$ is asymptotically stable;

(ii) The $\mathcal{L}_2$ gain from ω to x is less than a given scalar $\gamma > 0$, that is, under zero initial condition, $\|x(t)\|_2 < \gamma \|\omega(t)\|_2$ for any nonzero $\omega(t) \in \mathcal{L}_2[0, \infty)$.

2.4 $\mathcal{L}_2$ stability analysis and controller design

In this section, we will first analyze the $\mathcal{L}_2$ stability of the event-triggered networked control system by employing the Lyapunov-Krasovskii functional method. Then a controller will be designed to ensure the $\mathcal{L}_2$ stability of the error-dependent closed-loop control system.

Proposition 2.1. *Given a scalar $\gamma > 0$, under event-triggered transmission scheme (2.1), the system (2.18) is finite-gain $\mathcal{L}_2$ stable from ω to x with a gain less than γ, if there exist real matrices $P > 0, Q > 0, R_1 = R_1^T, R_2, Y_1, Y_2, Y_3, Z_1,\ Z_2$ with appropriate dimensions and a scalar $\varepsilon > 0$ such that*

$$\begin{bmatrix} P + hR_1 & hR_2 - hR_1 \\ * & hR_1 - hR_2 - hR_2^T \end{bmatrix} > 0 \tag{2.19}$$

$$\begin{bmatrix} \Gamma_{11} & -Z_1^T + A^T Z_2 - Y_2 + P & \Gamma_{13} & hY_1^T & \Gamma_{15} \\ * & -Z_2 - Z_2^T & Y_2^T & hY_2^T & \Gamma_{25} \\ * & * & \Gamma_{33} & hY_3^T & \Gamma_{35} \\ * & * & * & -hQ & 0 \\ * & * & * & * & \Gamma_{55} \end{bmatrix} < 0 \tag{2.20}$$

$$\begin{bmatrix} \Gamma_{11} & -Z_1^T + A^T Z_2 - Y_2 + P + hR_1 & \Gamma_{13} & \Gamma_{15} \\ * & -Z_2 - Z_2^T + hQ & \Gamma_{23} & \Gamma_{25} \\ * & * & \Gamma_{33} & \Gamma_{35} \\ * & * & * & \Gamma_{55} \end{bmatrix} < 0 \tag{2.21}$$

where

$$\begin{aligned}
\Gamma_{11} &= Z_1^T A + A^T Z_1 - R_1 + I - Y_1 - Y_1^T \\
\Gamma_{13} &= Y_1^T - Y_3 + R_1 - R_2 \\
\Gamma_{15} &= \begin{bmatrix} 0 & Z_1^T B_\omega & 0 & Z_1^T BK \end{bmatrix} \\
\Gamma_{23} &= Y_2^T + h(R_2 - R_1) \\
\Gamma_{25} &= \begin{bmatrix} 0 & Z_2^T B_\omega & 0 & Z_2^T BK \end{bmatrix} \\
\Gamma_{33} &= -R_1 + R_2 + R_2^T - \Phi + Y_3 + Y_3^T + \varepsilon\Lambda^2 \\
\Gamma_{35} &= \begin{bmatrix} -\Phi & 0 & 0 & 0 \end{bmatrix}
\end{aligned}$$

$$\Gamma_{55} = \begin{bmatrix} -\Phi-\varepsilon I & 0 & 0 & -\varepsilon I \\ 0 & -\gamma^2 I & 0 & 0 \\ 0 & 0 & -\delta\Phi & \delta\Phi \\ -\varepsilon I & 0 & \delta\Phi & -\varepsilon I \end{bmatrix}.$$

Proof. It follows from the transmission scheme (2.1) that for $\forall t \in [i_k h, i_{k+1}h)$, the following error condition holds:

$$\begin{aligned} e_k^T(t)\Phi e_k(t) &\leq \delta q^T(x(i_k h))\Phi q(x(i_k h)) \\ &= \delta[q(x(t-\eta(t))) - e_k(t)]^T\Phi[q(x(t-\eta(t))) - e_k(t)]. \end{aligned} \tag{2.22}$$

Choose the following Lyapunov-Krasovskii functional candidate

$$\begin{aligned} V(t,x(t)) &= x^T(t)Px(t) + (h-\eta(t))\int_{t-\eta(t)}^{t} \dot{x}^T(s)Q\dot{x}(s)ds \\ &+(h-\eta(t))\begin{bmatrix} x(t) \\ x(t-\eta(t)) \end{bmatrix}^T \begin{bmatrix} R_1 & R_2-R_1 \\ * & R_1-R_2-R_2^T \end{bmatrix}\begin{bmatrix} x(t) \\ x(t-\eta(t)) \end{bmatrix}. \end{aligned} \tag{2.23}$$

One can get from (2.19) that $V(t,x(t))$ is non-negative definite by considering

$$\begin{aligned} V(t,x(t)) &= \frac{\eta(t)}{h}x^T(t)Px(t) + (h-\eta(t))\int_{t-\eta(t)}^{t} \dot{x}^T(s)Q\dot{x}(s)ds \\ &+\frac{h-\eta(t)}{h}\begin{bmatrix} x(t) \\ x(t-\eta(t)) \end{bmatrix}^T \begin{bmatrix} P+hR_1 & hR_2-hR_1 \\ * & hR_1-hR_2-hR_2^T \end{bmatrix}\begin{bmatrix} x(t) \\ x(t-\eta(t)) \end{bmatrix}. \end{aligned}$$

It is verified that $V(t,x(t))$ is continuous over time. However, it is not differentiable at the sampling time instants $\{kh|k=1,2,3,\ldots\}$. As a result, we consider the right derivative of $V(t,x(t))$ with respect to t.

Notice that $\dot{\eta}(t) = 1$, $0 \leq \eta(t) < h$. Taking the right derivative of $V(t,x(t))$ with respect to t along the trajectory of (2.18) yields

$$\begin{aligned} \dot{V}(t,x(t)) &= 2x^T(t)P\dot{x}(t) - \int_{t-\eta(t)}^{t} \dot{x}^T(s)Q\dot{x}(s)ds + 2(h-\eta(t))[x^T(t)R_1 \\ &+x^T(t-\eta(t))(R_2^T-R_1)]\dot{x}(t) + (h-\eta(t))\dot{x}^T(t)Q\dot{x}(t) \\ &-\begin{bmatrix} x^T(t) & x^T(t-\eta(t)) \end{bmatrix}\begin{bmatrix} R_1 & R_2-R_1 \\ * & R_1-R_2-R_2^T \end{bmatrix}\begin{bmatrix} x(t) \\ x(t-\eta(t)) \end{bmatrix} \end{aligned} \tag{2.24}$$

where $\dot{V}(t,x(t)) = \lim_{\triangle t\to 0+}\frac{V(t+\triangle t, x(t+\triangle t)) - V(t,x(t))}{\triangle t}$. Using the Jensen's inequality [47], one can get

$$-\int_{t-\eta(t)}^{t} \dot{x}^T(s)Q\dot{x}(s)ds \leq -\eta(t)v^TQv \tag{2.25}$$

with $v = \frac{\int_{t-\eta(t)}^{t} \dot{x}(s)ds}{\eta(t)}$. And $v|_{\eta(t)=0} := \lim_{\eta(t)\to 0^+} v = \dot{x}(t)$.

Notice that for matrices Y_1, Y_2, Y_3, Z_1 and Z_2 with appropriate dimensions, the following equalities hold:

$$2(x^T(t)Y_1^T + \dot{x}^T(t)Y_2^T + x^T(t-\eta(t))Y_3^T)(-x(t) + x(t-\eta(t)) + \eta(t)v) = 0, \qquad (2.26)$$

$$\begin{aligned} &2(x^T(t)Z_1^T + \dot{x}^T(t)Z_2^T)(Ax(t) + BK(I+\Delta)x(t-\eta(t)) - BKe_k(t) \\ &\qquad + B_\omega \omega(t) - \dot{x}(t)) = 0. \end{aligned} \qquad (2.27)$$

Considering (2.22), (2.24)-(2.27) together, one can get

$$\dot{V}(t, x(t)) \leq \xi^T(t)\Xi\xi(t) - x^T(t)x(t) + \gamma^2 w^T(t)w(t), \ t \in [i_k h, i_{k+1} h) \qquad (2.28)$$

where

$$\xi(t) = \begin{bmatrix} x^T(t) & \dot{x}^T(t) & x^T(t-\eta(t)) & v^T & e_k^T(t) & \omega^T(t) \end{bmatrix}^T$$

$$\Xi = \begin{bmatrix} \Gamma_{11} & \Xi_{12} & \Xi_{13} & \eta(t)Y_1^T & -Z_1^T BK & Z_1^T B_\omega \\ * & \Xi_{22} & \Xi_{23} + Z_2^T BK(I+\Delta) & \eta(t)Y_2^T & -Z_2^T BK & Z_2^T B_\omega \\ * & * & \Xi_{33} & \eta(t)Y_3^T & -\delta(I+\Delta)\Phi & 0 \\ * & * & * & -\eta(t)Q & 0 & 0 \\ * & * & * & * & -(1-\delta)\Phi & 0 \\ * & * & * & * & * & -\gamma^2 I \end{bmatrix}$$

with

$$\Xi_{12} = -Z_1^T + A^T Z_2 - Y_2 + P + (h-\eta(t))R_1$$

$$\Xi_{13} = \Gamma_{13} + Z_1^T BK(I+\Delta), \quad \Xi_{22} = -Z_2 - Z_2^T + (h-\eta(t))Q$$

$$\Xi_{23} = Y_2^T + (h-\eta(t))(R_2 - R_1)$$

$$\Xi_{33} = -R_1 + R_2 + R_2^T + Y_3 + Y_3^T + \delta(I+\Delta)\Phi(I+\Delta).$$

Noticing that $\dot{\eta}(t) = 1$, $0 \leq \eta(t) < h$, one can get from (2.20) and (2.21) that for $\forall \eta(t) \in [0, h)$,

$$\begin{bmatrix} \Gamma_{11} & \Xi_{12} & \Gamma_{13} & \eta(t)Y_1^T & \Gamma_{15} \\ * & \Xi_{22} & \Xi_{23} & \eta(t)Y_2^T & \Gamma_{15} \\ * & * & \Gamma_{33} & \eta(t)Y_3^T & \Gamma_{35} \\ * & * & * & -\eta(t)Q & 0 \\ * & * & * & * & \Gamma_{55} \end{bmatrix} < 0. \qquad (2.29)$$

By Schur complement and some congruence transformations, one can find that (2.29) is equivalent to

$$\begin{bmatrix} \Gamma_{11} & \Xi_{12} & \Gamma_{13} & \eta(t)Y_1^T & -Z_1^TBK & Z_1^TB_\omega & 0 \\ * & \Xi_{22} & \Xi_{23} & \eta(t)Y_2^T & -Z_2^TBK & Z_2^TB_\omega & 0 \\ * & * & \Gamma_{33}-\varepsilon\Lambda^2 & \eta(t)Y_3^T & -\Phi & 0 & 0 \\ * & * & * & -\eta(t)Q & 0 & 0 & 0 \\ * & * & * & * & -\Phi & 0 & -\delta\Phi \\ * & * & * & * & * & -\gamma^2 I & 0 \\ * & * & * & * & * & * & -\delta\Phi \end{bmatrix} + \varepsilon\mathcal{L}_1^T\Delta\Lambda^{-1}\mathcal{L}_1 + \varepsilon^{-1}\mathcal{L}_2\Lambda^{-1}\Delta\mathcal{L}_2^T < 0 \quad (2.30)$$

where

$$\mathcal{L}_1 = \begin{bmatrix} 0 & 0 & \Lambda & 0 & 0 & 0 & 0 \end{bmatrix}$$
$$\mathcal{L}_2 = \begin{bmatrix} K^TB^TZ_1 & K^TB^TZ_2 & 0 & 0 & 0 & 0 & \delta\Phi \end{bmatrix}^T.$$

Notice that $\Delta\Lambda^{-1}\Lambda^{-1}\Delta \le I$. Then it follows from (2.30) that

$$\begin{bmatrix} \Gamma_{11} & \Xi_{12} & \Gamma_{13}+Z_1^TBK\Delta & \eta(t)Y_1^T & -Z_1^TBK & Z_1^TB_\omega & 0 \\ * & \Xi_{22} & \Xi_{23}+Z_2^TBK\Delta & \eta(t)Y_2^T & -Z_2^TBK & Z_2^TB_\omega & 0 \\ * & * & \Gamma_{33}-\varepsilon\Lambda^2 & \eta(t)Y_3^T & -\Phi & 0 & \delta\Delta\Phi \\ * & * & * & -\eta(t)Q & 0 & 0 & 0 \\ * & * & * & * & -\Phi & 0 & -\delta\Phi \\ * & * & * & * & * & -\gamma^2 I & 0 \\ * & * & * & * & * & * & -\delta\Phi \end{bmatrix} < 0. \quad (2.31)$$

By Schur complement and some congruence transformations, one can get from (2.31) that $\Xi < 0$ holds. It then follows from (2.28) that, for $\forall t \in [i_k h, i_{k+1}h)$,

$$\dot{V}(t, x(t)) \le -x^T(t)x(t) + \gamma^2\omega^T(t)\omega(t). \quad (2.32)$$

For any integer $N > 0$, we have

$$\int_0^{i_N h} \dot{V}(s, x(s))ds \le \int_0^{i_N h} -x^T(s)x(s) + \gamma^2\omega^T(s)\omega(s)ds.$$

On the other hand,

$$\int_0^{i_N h} \dot{V}(s, x(s))ds = V(i_N h, x(i_N h)) - V(0, x(0)).$$

Let $N \to \infty$. It is clear that, under zero initial condition,

$$\int_0^\infty x^T(s)x(s)ds \le \gamma^2 \int_0^\infty \omega^T(s)\omega(s)ds \tag{2.33}$$

holds for any nonzero $\omega(t) \in \mathcal{L}_2[0,\infty)$.

When $\omega(t) = 0$, (2.32) becomes

$$\dot{V}(t, x(t)) \le -x^T(t)x(t), \quad t \in [i_k h, i_{k+1}h) \tag{2.34}$$

from which one can conclude the asymptotic stability of the error-dependent closed-loop system (2.18). The proof is completed. □

Based on the stability analysis result, we are now in a position to design a controller for system (2.5) such that the closed-loop system (2.18) is finite-gain $\mathcal{L}_2$ stable from ω to x with a gain less than γ.

Proposition 2.2. *For given real constants $\delta > 0$ and $\gamma > 0$, under the event-triggered transmission scheme (2.1) with $\Phi = \tilde{Z}^{-T}\tilde{\Phi}\tilde{Z}^{-1}$, the system (2.18) is finite-gain $\mathcal{L}_2$ stable from ω to x with a gain less than γ, if there exist real matrices $\tilde{P} > 0, \tilde{Q} > 0, \tilde{\Phi} > 0, \tilde{R}_1 = \tilde{R}_1^T, \tilde{R}_2, \tilde{Y}_1, \tilde{Y}_2, \tilde{Y}_3, \tilde{Z}$, $\tilde{K}$ with appropriate dimensions and real constants κ and $\varepsilon > 0$ such that*

$$\begin{bmatrix} \tilde{P} + h\tilde{R}_1 & h\tilde{R}_2 - h\tilde{R}_1 \\ * & h\tilde{R}_1 - h\tilde{R}_2 - h\tilde{R}_2^T \end{bmatrix} > 0 \tag{2.35}$$

$$\begin{bmatrix} \Upsilon_{11} & -\tilde{Z} + \tilde{Z}^T A^T - \tilde{Y}_2 + \tilde{P} & \Upsilon_{13} & h\tilde{Y}_1^T & \Upsilon_{15} \\ * & -\tilde{Z} - \tilde{Z}^T & \tilde{Y}_2^T & h\tilde{Y}_2^T & \Upsilon_{25} \\ * & * & \Upsilon_{33} & h\tilde{Y}_3^T & \Upsilon_{35} \\ * & * & * & -h\tilde{Q} & 0 \\ * & * & * & * & \Upsilon_{55} \end{bmatrix} < 0 \tag{2.36}$$

$$\begin{bmatrix} \Upsilon_{11} & -\tilde{Z} + \tilde{Z}^T A^T - \tilde{Y}_2 + \tilde{P} + h\tilde{R}_1 & \Upsilon_{13} & \Upsilon_{15} \\ * & -\tilde{Z} - \tilde{Z}^T + h\tilde{Q} & \Upsilon_{23} & \Upsilon_{25} \\ * & * & \Upsilon_{33} & \Upsilon_{35} \\ * & * & * & \Upsilon_{55} \end{bmatrix} < 0 \tag{2.37}$$

where

$$\Upsilon_{11} = A\tilde{Z} + \tilde{Z}^T A^T - \tilde{R}_1 - \tilde{Y}_1 - \tilde{Y}_1^T, \quad \Upsilon_{13} = \tilde{Y}_1^T - \tilde{Y}_3 + \tilde{R}_1 - \tilde{R}_2$$

$$\Upsilon_{15} = \begin{bmatrix} -B\tilde{K} & B_\omega & 0 & \varepsilon B\tilde{K} & 0 & \tilde{Z}^T \end{bmatrix}$$

$$\Upsilon_{23} = \tilde{Y}_2^T + h\tilde{R}_2 - h\tilde{R}_1, \quad \Upsilon_{25} = \begin{bmatrix} -B\tilde{K} & B_\omega & 0 & \varepsilon B\tilde{K} & 0 & 0 \end{bmatrix}$$

$$\Upsilon_{33} = -\tilde{R}_1 + \tilde{R}_2 + \tilde{R}_2^T - \tilde{\Phi} + \tilde{Y}_3 + \tilde{Y}_3^T, \quad \Upsilon_{35} = \begin{bmatrix} -\tilde{\Phi} & 0 & 0 & 0 & \tilde{Z}^T\Lambda & 0 \end{bmatrix}$$

$$\Upsilon_{55} = \begin{bmatrix} -\tilde{\Phi} & 0 & -\delta\tilde{\Phi} & 0 & 0 & 0 \\ 0 & -\gamma^2 I & 0 & 0 & 0 & 0 \\ -\delta\tilde{\Phi} & 0 & -\delta\tilde{\Phi} & \varepsilon\delta\tilde{\Phi} & 0 & 0 \\ 0 & 0 & \varepsilon\delta\tilde{\Phi} & \Upsilon_{88} & 0 & 0 \\ 0 & 0 & 0 & 0 & -\varepsilon I & 0 \\ 0 & 0 & 0 & 0 & 0 & -I \end{bmatrix}$$

$$\Upsilon_{88} = \varepsilon\kappa^2 I - \varepsilon\kappa\tilde{Z} - \varepsilon\kappa\tilde{Z}^T.$$

Furthermore, the controller parameter matrix K can be obtained by $K = \tilde{K}\tilde{Z}^{-1}$.

Proof. It follows from (2.36) that $\tilde{Z} + \tilde{Z}^T > 0$, which implies that matrix $\tilde{Z}$ is nonsingular. Perform congruence transformations to (2.35), (2.36) and (2.37) by $\mathrm{diag}(\tilde{Z}^{-1}, \tilde{Z}^{-1})$, $\mathrm{diag}(\tilde{Z}^{-1}, \tilde{Z}^{-1}, \tilde{Z}^{-1}, \tilde{Z}^{-1}, \tilde{Z}^{-1}, I, \tilde{Z}^{-1}, \tilde{Z}^{-1}, I, I)$ and $\mathrm{diag}(\tilde{Z}^{-1}, \tilde{Z}^{-1}, \tilde{Z}^{-1}, \tilde{Z}^{-1}, I, \tilde{Z}^{-1}, \tilde{Z}^{-1}, I, I)$, respectively. Let

$$P = \tilde{Z}^{-T}\tilde{P}\tilde{Z}^{-1}, \quad Q = \tilde{Z}^{-T}\tilde{Q}\tilde{Z}^{-1}, \quad \Phi = \tilde{Z}^{-T}\tilde{\Phi}\tilde{Z}^{-1}, \quad R_1 = \tilde{Z}^{-T}\tilde{R}_1\tilde{Z}^{-1}$$

$$R_2 = \tilde{Z}^{-T}\tilde{R}_2\tilde{Z}^{-1}, \quad Y_1 = \tilde{Z}^{-T}\tilde{Y}_1\tilde{Z}^{-1}, \quad Y_2 = \tilde{Z}^{-T}\tilde{Y}_2\tilde{Z}^{-1}, \quad Y_3 = \tilde{Z}^{-T}\tilde{Y}_3\tilde{Z}^{-1}$$

$$Z_1 = \tilde{Z}^{-1}, \quad Z_2 = \tilde{Z}^{-1}, \quad K = \tilde{K}\tilde{Z}^{-1}.$$

Notice that $-\tilde{Z}^T\tilde{Z} \leq -\kappa\tilde{Z} - \kappa\tilde{Z}^T + \kappa^2 I$ holds for any $\kappa \in R$. Then it can be verified that (2.19), (2.20) and (2.21) are satisfied by Schur complement. The result follows from Proposition 2.1. □

Remark 2.5. *One can see that the weighting matrix Φ in event-triggered transmission scheme (2.1) can be given by Proposition 2.2. Therefore, for the event-triggered transmitter, only the parameter $\delta \in (0, 1)$ is to be determined according to a tradeoff between available resources and desired control performance.*

In the remainder of this chapter, we consider the case where the event-triggered mechanism is executed prior to the signal quantization, as is shown in Figure 2.2.

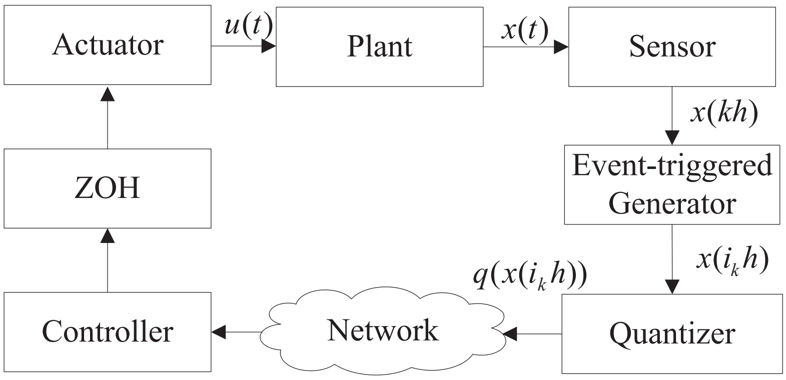

Figure 2.2: An NCS with an event-triggered generator

In Figure 2.1, an event-triggered transmitter is used to reduce the unnecessary transmissions by checking the quantized measurement according to the transmission scheme 2.1. Now we employ an event-triggered generator to determined which sampled measurement should be quantized and all the quantized signals are transmitted for feedback. The corresponding results can be obtained. In this case, the transmission scheme (2.1) becomes

$$i_{k+1}h = i_k h + \min_{l \in \mathbb{Z}^+} \{lh | \tilde{e}^T(r_{k,l}h)\Phi\tilde{e}(r_{k,l}h) \geq \delta x^T(i_k h)\Phi x(i_k h)\} \tag{2.38}$$

with $\tilde{e}(r_{k,l}h) = x(r_{k,l}h) - x(i_k h)$. The error-dependent closed-loop system can be obtained as

$$\dot{x}(t) = Ax(t) + BK(I + \Delta)x(t - \eta(t)) - BK(I + \Delta)\tilde{e}(t) + B_\omega \omega(t) \tag{2.39}$$

with $x(t) = \varphi(t), t \in [t_0 - h, t_0]$.

Following the similar line as in Proposition 2.1, we have the following $\mathcal{L}_2$ stability result.

Proposition 2.3. *Given a scalar $\gamma > 0$, with communication scheme (2.38), the system (2.39) is finite-gain $\mathcal{L}_2$ stable from ω to x with a gain less than γ, if there* *exist real matrices* $P > 0, Q > 0, R_1 = R_1^T, R_2, Y_1, Y_2, Y_3, Z_1,\ Z_2$ *with appropriate dimensions and a scalar* $\varepsilon > 0$ *such that inequalities (2.19)-(2.21) hold, where* $\Gamma_{33}, \Gamma_{15}, \Gamma_{25}, \Gamma_{35}, \Gamma_{55}$ *are replaced by* $\tilde{\Gamma}_{33}, \tilde{\Gamma}_{15}, \tilde{\Gamma}_{25}, \tilde{\Gamma}_{35}, \tilde{\Gamma}_{55}$*, respectively, and*

$$\begin{aligned}
\tilde{\Gamma}_{33} &= -R_1 + R_2 + R_2^T - \Phi + Y_3 + Y_3^T \\
\tilde{\Gamma}_{15} &= \begin{bmatrix} -Z_1^T BK & Z_1^T B_\omega & Z_1^T BK \end{bmatrix} \\
\tilde{\Gamma}_{25} &= \begin{bmatrix} -Z_2^T BK & Z_2^T B_\omega & Z_2^T BK \end{bmatrix} \\
\tilde{\Gamma}_{35} &= \begin{bmatrix} -\Phi & 0 & 0 \end{bmatrix} \\
\tilde{\Gamma}_{55} &= \begin{bmatrix} (\delta-1)\Phi + \varepsilon\Lambda^2 & 0 & 0 \\ 0 & -\gamma^2 I & 0 \\ 0 & 0 & -\varepsilon I \end{bmatrix}.
\end{aligned}$$

2.5 Illustrative examples

In this section, two examples are given to demonstrate the effectiveness of the proposed approach.

Example 1: Consider the attitude control of a satellite in orbit, which is studied in [73–75]. The model is established by two rigid bodies connected by a flexible boom with torque constant k and viscous damping constant f. The diagram of the satellite and the model can be found in [73, 74]. The equations of motion are

$$J_1\ddot{\theta}_1 + f(\dot{\theta}_1 - \dot{\theta}_2) + k(\theta_1 - \theta_2) = u \tag{2.40}$$

$$J_2\ddot{\theta}_2 + f(\dot{\theta}_2 - \dot{\theta}_1) + k(\theta_2 - \theta_1) = \omega \tag{2.41}$$

where θ_1 denotes the angle of the main satellite with respect to the star and θ_2 is the angle between the star sensor and the instrument module; J_1 and J_2 are inertias; u and ω are the control torque and disturbance torque respectively. Let the state vector of the system be

$$x = \begin{bmatrix} \theta_1 & \theta_2 & \dot{\theta}_1 & \dot{\theta}_2 \end{bmatrix}^T.$$

Then one can get the state equation

$$\dot{x}=\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\frac{k}{J_1} & \frac{k}{J_1} & -\frac{f}{J_1} & \frac{f}{J_1} \\ \frac{k}{J_2} & -\frac{k}{J_2} & \frac{f}{J_2} & -\frac{f}{J_2} \end{bmatrix} x+\begin{bmatrix} 0 \\ 0 \\ \frac{1}{J_1} \\ 0 \end{bmatrix} u+\begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{1}{J_2} \end{bmatrix} \omega. \tag{2.42}$$

Based on physical analysis of the boom, it is assumed that the parameters k and f may vary with the temperature but are bounded by [73]:

$$0.09 \leq k \leq 0.4, \quad 0.038\sqrt{\frac{k}{10}} \leq f \leq 0.2\sqrt{\frac{k}{10}}. \tag{2.43}$$

Here we choose $k = 0.3$, $f = 0.1\sqrt{\frac{k}{10}}$ and $J_1 = J_2 = 1$. It is assumed that the sampling period $h = 10$ms, the event-triggered transmission scheme parameter $\delta = 0.1$ and the quantization density $\rho_i = 0.85$, $i = 1, 2, 3, 4$.

By using Proposition 2.2, we can find that under event-triggered transmission scheme (2.1), the system (2.42) with controller (2.6) is finite-gain $\mathcal{L}_2$ stable with a gain $\gamma = 200$ and parameters $\varepsilon = 0.001, \kappa = 0.01$. A feasible solution to (2.35)-(2.37) leads to the following controller parameter matrix and transmission scheme weighting matrix

$$K = \begin{bmatrix} -3.7929 & 0.3849 & -3.2782 & -5.5872 \end{bmatrix}$$
$$\Phi = \begin{bmatrix} 177.5318 & -11.8767 & 127.7834 & 266.7983 \\ -11.8767 & 23.6276 & -13.8904 & -3.7243 \\ 127.7834 & -13.8904 & 109.4995 & 190.5661 \\ 266.7983 & -3.7243 & 190.5661 & 462.9764 \end{bmatrix}.$$

For the purpose of simulation, the initial condition and the disturbance torque are assumed to be $x(0) = [-0.5\ 1.3\ 0.5\ \ -0.5]^T$, $\omega(t) = 1/(1+t)$, respectively. The parameters of the quantizer are chosen as $u_0^{(j)} = 1, j = 1, 2, 3, 4$. With the controller (2.6) and the transmission scheme (2.1), the state response of the closed-loop system is shown in Figure 2.3. Figure 2.4 demonstrates the event-triggered broadcast release time instants distribution in the simulation.

Within the simulation period $T_s = 25s$, it is shown that the system state is sampled and quantized 2500 times, respectively, while only 112 quantized signals

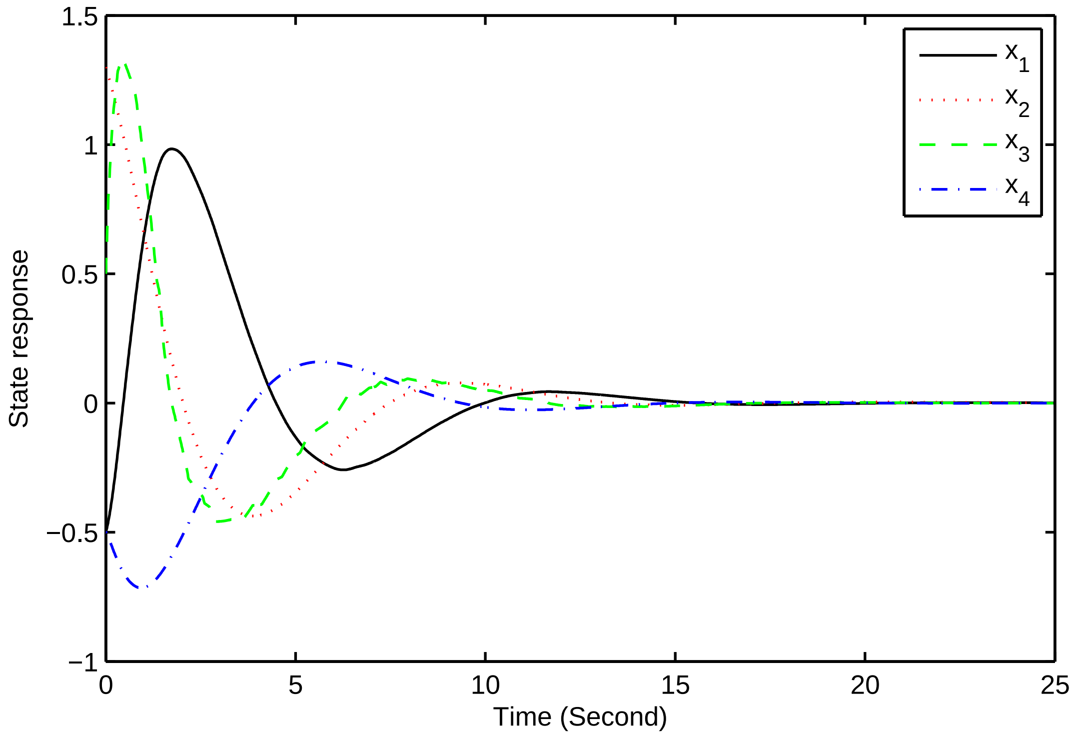

Figure 2.3: State response of the event-triggered closed-loop system.

Table 2.1: Number of transmitted packets N_t for different δ

δ	0	0.1	0.2	0.3	0.4	0.5
N_t	2500	112	76	56	32	29

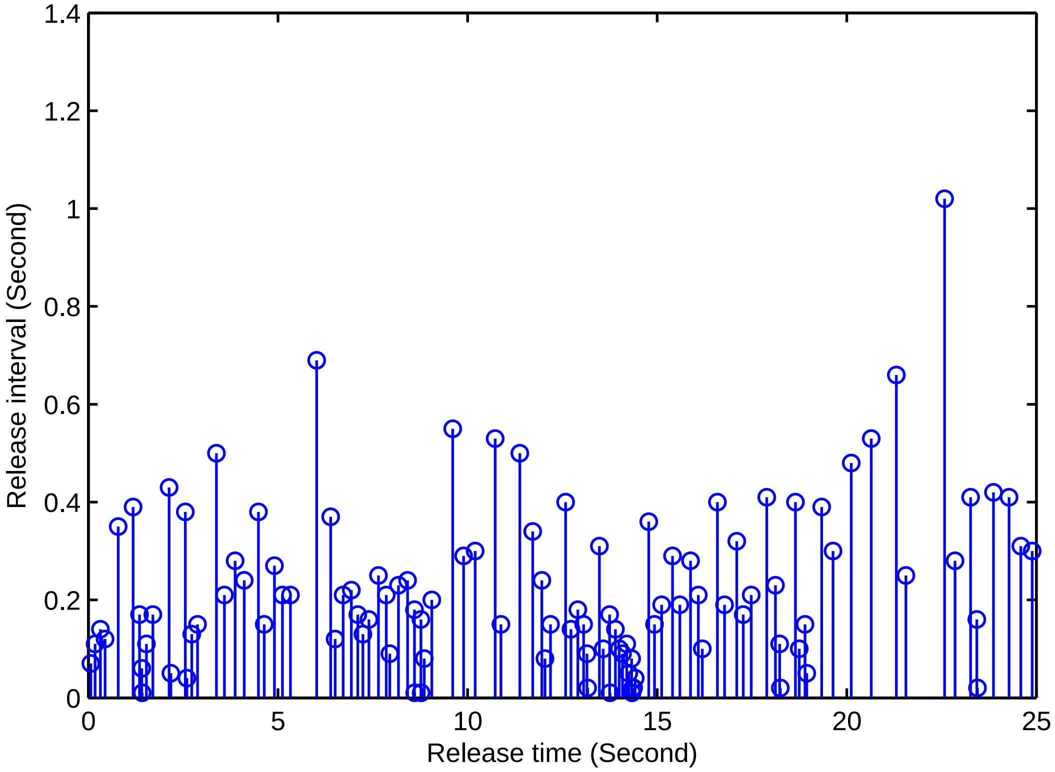

Figure 2.4: Broadcast release time distribution in the simulation.

are triggered to be transmitted to the controller through the network. The average broadcast release time interval is 0.2232s, which is much longer than the sampling period 0.01s. Compared with the periodic transmission scheme (when $\delta \to 0+$), it is clear that a large proportion of the required network resources may be saved, which shows the effectiveness of the approach. Table 2.1 shows the corresponding transmitted packets number N_t within the simulation time $T = 25s$ when different value of δ is used in the event-triggered transmitter. One can see that as the value of δ increases, the transmitted packets number N_t decreases.

Example 2: Consider the following system [31, 76]

$$\dot{x}(t) = \begin{bmatrix} 0 & 1 \\ 0 & -0.1 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 0.1 \end{bmatrix} u(t). \tag{2.44}$$

A non-networked controller $u = [-3.75 \quad -11.5]x(t)$ is used to compute the maximum allowable sampling period h_{max} as in [31, 76]. The quantization density is chosen as $\rho_1 = \rho_2 = 0.85$. We consider the system first in the framework where the event-triggered mechanism is executed after the signal quantization, then in the framework where the event-triggered mechanism is executed before the signal quantization. Table 2.2 shows the value of h_{max} when the system is studied in the two different frameworks, respectively. One can see that, in the first framework, the value of h_{max} is larger than the one in the other framework by around 10% with the same value of the event-triggering parameter δ. It can also be seen that the range of δ in the two frameworks are different. And as the value of δ increases, the state of the system is required to be sampled more frequently, although not all the measurement is to be transmitted.

2.6 Conclusion

This chapter has considered an event-triggered networked control system framework where the quantized measurement is event-triggered to be released for transmission through a network channel to a controller. It has been shown that by this way,

Table 2.2: Maximum allowable sampling period h_{max} in Example 2

δ	h_{max} with Proposition 2.1	h_{max} with Proposition 2.3
0.1	1.0562	1.0549
0.2	0.8487	0.8453
0.3	0.6874	0.6800
0.4	0.5496	0.5430
0.5	0.4269	/
0.6	0.3135	/

the average transmission interval can be increased substantially, which implies the required network resources for the NCS can be reduced while a prescribed level of $\mathcal{L}_2$ control performance is still maintained. In addition, the event-triggered threshold is checked for each discrete-time quantized measurement and the real-time detection hardware is no longer needed compared with the general event-triggered sampling scheme.

Chapter 3

Finite-level quantized event-triggered output feedback control for linear systems

3.1 Introduction

In the previous chapter, we considered an event-triggered networked control system (NCS) with a static quantizer, which is easy to be implemented. The processes of static quantization and event-triggered transmission are executed separately. While in this chapter, a finite-level dynamical quantizer and an event-triggered communication scheme are proposed in an integrated way. An event-triggered communication scheme is proposed to determine whether or not the current sampled data should be quantized and used for feedback. The current sampled data is released to be quantized and transmitted to the controller when the error between the current sampled data and the latest quantized data exceeds a specific threshold. We construct a dynamical finite-level quantizer based on the event-triggered communication scheme and the latest quantized data. A static output feedback controller is given to ensure that the state of the resulted system is uniformly ultimately bounded.

The contribution of this chapter is an integrated design method for an event-triggered quantized feedback control scheme. The finite-level quantizer and the event-triggered communication scheme share the same parameters and they are closely correlated. Compared with the continuous event-triggered sampling scheme, the real-time detection hardware is no longer needed. A static output feedback controller is also designed in this framework to guarantee the uniform ultimate boundedness of the system state.

The organisation of this chapter is as follows. Section 3.2 presents an event-triggered communication scheme and formulates the quantized output feedback control problem. The uniform ultimate boundedness analysis is addressed for the event-triggered system with quantized data in Section 3.3. A static output feedback controller design method is presented in Section 3.4. The finite-level dynamic quantizer is given in Section 3.5. A simulation example is given to demonstrate the effectiveness of the proposed approach in Section 3.6. This chapter is concluded in Section 3.7.

3.2 Problem statement

Consider the following linear time-invariant system

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) \\ y(t) = Cx(t) \end{cases} \tag{3.1}$$

where $x(t) \in \mathbb{R}^n$, $u(t) \in \mathbb{R}$ and $y(t) \in \mathbb{R}$ are the system state, control input and the measurement output, respectively; A, B and C are constant matrices with appropriate dimensions; and the initial condition of the system is given by $x(0) = x_0$.

In this chapter, only limited information about the output $y(t)$ is available for feedback due to the following constraints:

- $y(t)$ is periodically sampled with a sampling period $h > 0$;
- For the efficient use of communication resources, an event-triggered transmitter is employed to select which sampled data should be quantized and used for feedback;

- The selected sampled data is quantized, and represented by a finite number of bits before being transmitted to a controller through a digital communication channel.

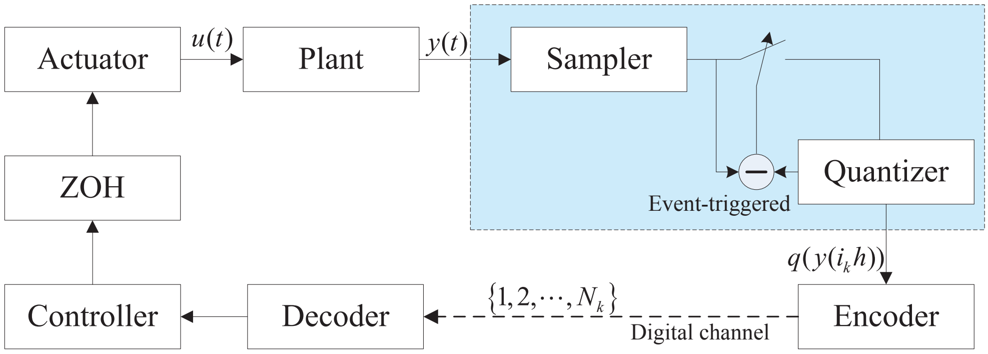

Figure 3.1: A framework of quantized event-triggered control

A conceptual framework of the event-triggered feedback control system with quantized data is shown in Figure 3.1. Once the sampled data $y(kh)$ is obtained in the sensor, whether it is used for feedback or not is determined by the smart sensor according to an event-triggered communication scheme in the form of

$$i_{k+1}h = i_k h + \min_{l\in\mathbb{Z}^+}\{lh \mid |y(i_k h + lh) - q(y(i_k h))|^2 \geq \delta |q(y(i_k h))|^2 + \epsilon^3\} \qquad (3.2)$$

where $q(\cdot)$ is a finite-level dynamical quantizer to be designed later; δ and ϵ, which satisfies $0 < \delta < 1$ and $\epsilon > 0$, are the threshold parameters of the communication scheme; $\mathbb{Z}^+$ is the set of positive integers; the sampling time sequence is $\{kh\}_{k=1}^{\infty}$; $\{y(i_k h)\}_{k=1}^{\infty}$ is used to represent the selected sampled measurement. $\{i_k h\}_{k=1}^{\infty}$ is a subsequence of $\{kh\}_{k=1}^{\infty}$ since the time delays during the process of signal sampling, quantization and transmission are not considered in this chapter. As is shown in Figure 3.1, the quantizer $q(\cdot)$ is finite-level and the number of quantization levels N_k may be time-varying. This implies that $q(y(i_k h))$ can be encoded into $\lceil \log_2 N_k \rceil$ bits of information to be transmitted through the digital channel to a decoder collated with the controller. It is known that there is usually no information loss in the process of information encoding and decoding. Therefore, the structure of the employed encoder and decoder is not elaborated in this chapter.

Remark 3.1. *It can be seen that the event-triggered communication scheme (3.2) is executed separately with the sampling process. In this way, the real-time detection hardware is no longer needed compared with the continuous event-triggered sampling/transmission scheme. Furthermore, the Zeno behavior is automatically avoided since the inter-event intervals are clearly lower bounded by a sampling interval h.*

Remark 3.2. *The separate terms of the positive scalars involved in the event-triggered mechanism conditions in [55, 56] are used to guarantee a nonzero minimum inter-event time. While in this chapter, the main purpose of introducing $\epsilon > 0$ in (3.2) is to design a finite-level quantizer in Section 3.5.*

In this chapter, we are interested in designing the following controller

$$u(t) = Kq(y(i_k h)), \qquad t \in [i_k h, i_{k+1} h) \tag{3.3}$$

where K is the feedback gain to be determined.

We partition the interval $[i_k h, i_{k+1} h)$ as

$$[i_k h, i_{k+1} h) = \bigcup_{j=1}^{l_k} I_{k,j} \tag{3.4}$$

where

$$I_{k,j} = [i_k h + (j-1)h, i_k h + jh), \quad j = 1, 2, \ldots, l_k \tag{3.5}$$

$$l_k = i_{k+1} - i_k, \quad k = 1, 2, \ldots. \tag{3.6}$$

For $t \in [i_k h, i_{k+1} h)$, let

$$\eta(t) := \begin{cases} t - i_k h, & t \in I_{k,1} \\ t - i_k h - h, & t \in I_{k,2} \\ \vdots, & \vdots \\ t - i_k h - l_k h + h, & t \in I_{k,l_k} \end{cases} \tag{3.7}$$

$$e_k(t) := \begin{cases} y(i_k h) - q(y(i_k h)), & t \in I_{k,1} \\ y(i_k h + h) - q(y(i_k h)), & t \in I_{k,2} \\ \vdots, & \vdots \\ y(i_k h + l_k h - h) - q(y(i_k h)), & t \in I_{k,l_k} \end{cases} \tag{3.8}$$

Then one can see that

$$q(y(i_k h)) = y(t - \eta(t)) - e_k(t), \quad t \in [i_k h, i_{k+1} h) \tag{3.9}$$

The error-dependent closed-loop system can be obtained as

$$\dot{x}(t) = Ax(t) + BKCx(t - \eta(t)) - BKe_k(t), \quad t \in [i_k h, i_{k+1} h). \tag{3.10}$$

It is noted that $\eta(t) \in [0, h)$. We supplement the initial condition of the system on $[-h, 0]$ as [72]

$$x(t_0 + \theta) = \varphi(\theta), \quad \theta \in [-h, 0] \tag{3.11}$$

where $\varphi \in \mathbb{W}$ with $\mathbb{W}$ denoting the Banach space of absolutely continuous functions $[-h, 0] \to R^n$ with square integrable derivative and with the norm

$$\|\varphi\|_W^2 = \|\varphi(0)\|^2 + \int_{-h}^{0} \|\varphi(s)\|^2 ds + \int_{-h}^{0} \|\dot{\varphi}(s)\|^2 ds \tag{3.12}$$

where the vector norm $\|\cdot\|$ represents the Euclidean norm.

We introduce the following definition of uniform ultimate boundedness as in [47, 77] and Lemma 1 as a analysis tool of the uniform ultimate boundedness. The proof of Lemma 1 is similar to that in [77], and thus is omitted here.

Definition 3.1. *The state of the system (3.10) is said to be uniformly ultimately bounded, if there exists a compact set $U \in \mathbb{W}$ such that for all $x(t_0 + \theta) = x_{t_0} \in U, \theta \in [-h, 0]$, there exists a scalar $\varepsilon > 0$ and a positive number $T(\varepsilon, x_{t_0}) > 0$ such that $\|x(t)\| < \varepsilon, \forall t \geq t_0 + T$.*

Lemma 3.1. *Let $V(t, x_t)$ with $x_t = x(t+\theta), \theta \in [-h, 0]$, be a Lyapunov functional of the system (3.10) which satisfies*

$$\begin{cases} \zeta_1(\|x(t)\|) \leq V(t, x_t) \leq \zeta_2(\|x_t\|_W) \\ \dot{V}(t, x_t) \leq -\zeta_3(\|x(t)\|) + \zeta_3(\varepsilon) \end{cases} \tag{3.13}$$

where $\varepsilon > 0$ is a positive constant, $\zeta_1(\cdot)$ and $\zeta_2(\cdot)$ are continuous, strictly increasing functions, and $\zeta_3(\cdot)$ is a continuous, nondecreasing function. If

$$\dot{V}(t, x_t) < 0, \text{ when } \|x(t)\| \geq \varepsilon \tag{3.14}$$

then the state of the system (3.10) is uniformly ultimately bounded.

The purpose of this chapter is to design a controller in the form of (3.3) and a finite-level quantizer $q(\cdot)$ such that the state of system (3.10) is uniformly ultimately bounded under the proposed event-triggered communication scheme (3.2).

3.3 Uniform ultimate boundedness analysis

In this section, we will derive a stability criterion for system (3.10) with event-triggered communication scheme (3.2).

Proposition 3.1. *Under the communication scheme (3.2) and with controller (3.3), the state of system (3.10) is uniformly ultimately bounded, if there exist real matrices $P > 0, Q > 0, R_1 = R_1^T, R_2, Y_1, Y_2, Y_3, Z_1$, Z_2 with appropriate dimensions such that*

$$\begin{bmatrix} P + hR_1 & hR_2 - hR_1 \\ * & hR_1 - hR_2 - hR_2^T \end{bmatrix} > 0 \tag{3.15}$$

$$\begin{bmatrix} \Gamma_{11} & -Z_1^T + A^T Z_2 - Y_2 + P & \Gamma_{13} & hY_1^T & -Z_1^T BK \\ * & -Z_2 - Z_2^T & \Gamma_{23} & hY_2^T & -Z_2^T BK \\ * & * & \Gamma_{33} & hY_3^T & -\delta C^T \\ * & * & * & -hQ & 0 \\ * & * & * & * & \delta - 1 \end{bmatrix} < 0 \tag{3.16}$$

$$\begin{bmatrix} \Gamma_{11} & -Z_1^T + A^T Z_2 - Y_2 + P + hR_1 & \Gamma_{13} & -Z_1^T BK \\ * & -Z_2 - Z_2^T + hQ & \Gamma_{23} + h(R_2 - R_1) & -Z_2^T BK \\ * & * & \Gamma_{33} & -\delta C^T \\ * & * & * & \delta - 1 \end{bmatrix} < 0 \tag{3.17}$$

where

$$\begin{aligned}\Gamma_{11} &= Z_1^T A + A^T Z_1 - R_1 + \epsilon I - Y_1 - Y_1^T \\ \Gamma_{13} &= Y_1^T - Y_3 + R_1 - R_2 + Z_1^T BKC, \quad \Gamma_{23} = Y_2^T + Z_2^T BKC \\ \Gamma_{33} &= -R_1 + R_2 + R_2^T + Y_3 + Y_3^T + \delta C^T C. \end{aligned} \tag{3.18}$$

Proof. Let

$$\begin{aligned} r_{k,j}h &= i_k h + jh, \ j = 0, 1, \ldots, i_{k+1} - i_k - 1, \\ e(r_{k,j}h) &= y(r_{k,j}h) - q(y(i_k h)). \end{aligned}$$

When $j \neq 0$, it follows from the communication scheme (3.2) that

$$|e(r_{k,j}h)|^2 < \delta |q(y(i_k h))|^2 + \epsilon^3. \tag{3.19}$$

When $j = 0$, $e(r_{k,j}h) = y(i_k h) - q(y(i_k h))$, which denotes the quantization error. The quantizer $q(\cdot)$ we employed will be designed in Section 3.5 such that

$$|y(i_k h) - q(y(i_k h))|^2 < \delta |q(y(i_k h))|^2 + \epsilon^3. \tag{3.20}$$

Then for all $e_k(t)$ in (3.8), we have that for $\forall t \in [i_k h, i_{k+1} h)$

$$\begin{aligned} |e_k(t)|^2 &< \delta |q(y(i_k h))|^2 + \epsilon^3 \\ &= \delta |y(t - \eta(t)) - e_k(t)|^2 + \epsilon^3. \end{aligned} \tag{3.21}$$

Choose the following Lyapunov-Krasovskii functional

$$\begin{aligned} V(t, x(t)) &= x^T(t) P x(t) + (h - \eta(t)) \int_{t-\eta(t)}^{t} \dot{x}^T(s) Q \dot{x}(s) ds \\ &+ (h - \eta(t)) \begin{bmatrix} x^T(t) & x^T(t - \eta(t)) \end{bmatrix} \begin{bmatrix} R_1 & R_2 - R_1 \\ * & R_1 - R_2 - R_2^T \end{bmatrix} \begin{bmatrix} x(t) \\ x(t - \eta(t)) \end{bmatrix}. \end{aligned}$$

One can get from (3.15) that $V(t, x(t))$ is non-negative definite by considering

$$\begin{aligned} V(t, x(t)) &= \frac{\eta(t)}{h} x^T(t) P x(t) + (h - \eta(t)) \int_{t-\eta(t)}^{t} \dot{x}^T(s) Q \dot{x}(s) ds \\ &+ \frac{h - \eta(t)}{h} \begin{bmatrix} x(t) \\ x(t - \eta(t)) \end{bmatrix}^T \begin{bmatrix} P + hR_1 & hR_2 - hR_1 \\ * & hR_1 - hR_2 - hR_2^T \end{bmatrix} \begin{bmatrix} x(t) \\ x(t - \eta(t)) \end{bmatrix}. \end{aligned}$$

Notice that $\dot{\eta}(t) = 1$. Taking the right derivative of $V(t, x(t))$ with respect to t along the trajectory of (3.10) yields

$$\begin{aligned}
\dot{V}(t, x(t)) = {}& 2x^T(t)P\dot{x}(t) - \int_{t-\eta(t)}^{t} \dot{x}^T(s)Q\dot{x}(s)ds + 2(h - \eta(t))[x^T(t)R_1 \\
& + x^T(t - \eta(t))(R_2^T - R_1)]\dot{x}(t) + (h - \eta(t))\dot{x}^T(t)Q\dot{x}(t) \\
& - \begin{bmatrix} x^T(t) & x^T(t - \eta(t)) \end{bmatrix} \begin{bmatrix} R_1 & R_2 - R_1 \\ * & R_1 - R_2 - R_2^T \end{bmatrix} \begin{bmatrix} x(t) \\ x(t - \eta(t)) \end{bmatrix} \quad (3.22)
\end{aligned}$$

where $\dot{V}(t, x(t)) = \lim_{\triangle t \to 0+} \frac{V(t+\triangle t, x(t+\triangle t)) - V(t, x(t))}{\triangle t}$.

Using the Jensen's inequality [47], one can get

$$-\int_{t-\eta(t)}^{t} \dot{x}^T(s)Q\dot{x}(s)ds \le -\eta(t)v^T Q v \quad (3.23)$$

with $v = \frac{\int_{t-\eta(t)}^{t} \dot{x}(s)ds}{\eta(t)}$. And $v|_{\eta(t)=0} := \lim_{\eta(t)\to 0^+} v = \dot{x}(t)$.

Notice that for matrices Y_1, Y_2, Y_3, Z_1 and Z_2 with appropriate dimensions, the following equalities hold.

$$2(x^T(t)Y_1^T + \dot{x}^T(t)Y_2^T + x^T(t - \eta(t))Y_3^T)(-x(t) + x(t - \eta(t)) + \eta(t)v) = 0 \quad (3.24)$$

$$2(x^T(t)Z_1^T + \dot{x}^T(t)Z_2^T)(Ax(t) + BKCx(t - \eta(t)) - BKe_k(t) - \dot{x}(t)) = 0. \quad (3.25)$$

Considering (3.21), (3.22)-(3.25) together, one can get

$$\dot{V}(t, x(t)) \le \xi^T(t)\Xi\xi(t) - \epsilon x^T(t)x(t) + \epsilon^3 \quad (3.26)$$

where

$$\xi(t) = \mathrm{col}\{x(t), \dot{x}(t), x(t - \eta(t)), v, e_k(t)\} \quad (3.27)$$

$$\Xi = \begin{bmatrix} \Gamma_{11} & -Z_1^T + A^T Z_2 - Y_2 + P + (h - \eta(t))R_1 & \Gamma_{13} & \eta(t)Y_1^T & -Z_1^T BK \\ * & -Z_2 - Z_2^T + (h - \eta(t))Q & \Xi_{23} & \eta(t)Y_2^T & -Z_2^T BK \\ * & * & \Gamma_{33} & \eta(t)Y_3^T & -\delta C^T \\ * & * & * & -\eta(t)Q & 0 \\ * & * & * & * & \delta - 1 \end{bmatrix}$$

with $\Gamma_{11}, \Gamma_{13}, \Gamma_{33}$ given by (3.18), and

$$\Xi_{23} = Z_2^T BKC + Y_2^T + (h - \eta(t))(R_2 - R_1).$$

One can get from (3.16) and (3.17) that $\Xi < 0$ holds. When $\|x(t)\| \geq \epsilon$, we have $-\epsilon x^T(t)x(t) + \epsilon^3 \leq 0$, which implies $\dot{V}(t, x(t)) < 0$ via (3.26). By Lemma 1, one can find that the state of system (3.10) is uniformly ultimately bounded. This completes the proof. □

3.4 Controller synthesis

Based on the stability analysis result obtained in the previous section, we are now in a position to give a method of designing an output feedback gain K which can guarantee the uniform ultimate boundedness of the state of system (3.10).

Proposition 3.2. *Under event-triggered communication scheme (3.2), the state of system (3.10) with an output feedback gain $K = \tilde{K}$ is uniformly ultimately bounded, if there exist real matrices $\tilde{P} > 0, \tilde{Q} > 0, \tilde{R}_1 = \tilde{R}_1^T, \tilde{R}_2, \tilde{Y}_1, \tilde{Y}_2, \tilde{Y}_3, \tilde{Z}$, $\tilde{K}$ and U with appropriate dimensions such that*

$$\begin{bmatrix} \tilde{P} + h\tilde{R}_1 & h\tilde{R}_2 - h\tilde{R}_1 \\ * & h\tilde{R}_1 - h\tilde{R}_2 - h\tilde{R}_2^T \end{bmatrix} > 0 \tag{3.28}$$

$$\begin{bmatrix} \Upsilon_{11} & -U^T\tilde{Z} + \tilde{Z}^T A^T - \tilde{Y}_2 + \tilde{P} & \Upsilon_{13} & h\tilde{Y}_1^T & \Upsilon_{15} \\ * & -\tilde{Z} - \tilde{Z}^T & \tilde{Y}_2^T & h\tilde{Y}_2^T & \Upsilon_{25} \\ * & * & \Upsilon_{33} & h\tilde{Y}_3^T & \Upsilon_{35} \\ * & * & * & -h\tilde{Q} & 0 \\ * & * & * & * & \Upsilon_{55} \end{bmatrix} < 0 \tag{3.29}$$

$$\begin{bmatrix} \Upsilon_{11} & -U^T\tilde{Z} + \tilde{Z}^T A^T - \tilde{Y}_2 + \tilde{P} + h\tilde{R}_1 & \Upsilon_{13} & \Upsilon_{15} \\ * & -\tilde{Z} - \tilde{Z}^T + h\tilde{Q} & \Upsilon_{23} & \Upsilon_{25} \\ * & * & \Upsilon_{33} & \Upsilon_{35} \\ * & * & * & \Upsilon_{55} \end{bmatrix} < 0 \tag{3.30}$$

where

$$\Upsilon_{11} = U^T A\tilde{Z} + \tilde{Z}^T A^T U - \tilde{R}_1 - \tilde{Y}_1 - \tilde{Y}_1^T, \quad \Upsilon_{13} = \tilde{Y}_1^T - \tilde{Y}_3 + \tilde{R}_1 - \tilde{R}_2$$

$$\Upsilon_{15} = \begin{bmatrix} -U^T B\tilde{K} & \epsilon\tilde{Z}^T \end{bmatrix}, \quad \Upsilon_{23} = \tilde{Y}_2^T + h\tilde{R}_2 - h\tilde{R}_1 + BX$$

$$\Upsilon_{25} = \begin{bmatrix} -B\tilde{K} & 0 \end{bmatrix}, \quad \Upsilon_{33} = -\tilde{R}_1 + \tilde{R}_2 + \tilde{R}_2^T + \tilde{Y}_3 + \tilde{Y}_3^T - \tilde{Z}^T C^T C\tilde{Z}$$

$$\Upsilon_{35} = \begin{bmatrix} -\tilde{Z}^T C^T & 0 \end{bmatrix}, \quad \Upsilon_{55} = diag\{(\delta - 1)I, -\epsilon I\}.$$

Proof. It can be seen from (3.29) that $-\tilde{Z} - \tilde{Z}^T < 0$, which implies that $\tilde{Z}$ is nonsingular. Make congruence transformations to (3.28), (3.29), (3.30) by $diag\{\tilde{Z}^{-1}, \tilde{Z}^{-1}\}$, $diag\{\tilde{Z}^{-1}, \tilde{Z}^{-1}, \tilde{Z}^{-1}, \tilde{Z}^{-1}, I, I\}$, $diag\{\tilde{Z}^{-1}, \tilde{Z}^{-1}, \tilde{Z}^{-1}, I, I\}$, respectively. Let

$$\begin{aligned}
&Z_2 = \tilde{Z}^{-1}, \quad P = \tilde{Z}^{-T}\tilde{P}\tilde{Z}^{-1}, \quad Q = \tilde{Z}^{-T}\tilde{Q}\tilde{Z}^{-1}, \quad R_1 = \tilde{Z}^{-T}\tilde{R}_1\tilde{Z}^{-1}, \quad R_2 = \tilde{Z}^{-T}\tilde{R}_2\tilde{Z}^{-1} \\
&Y_1 = \tilde{Z}^{-T}\tilde{Y}_1\tilde{Z}^{-1}, \quad ,Y_2 = \tilde{Z}^{-T}\tilde{Y}_2\tilde{Z}^{-1}, \quad Y_3 = \tilde{Z}^{-T}\tilde{Y}_3\tilde{Z}^{-1}, \quad Z_1 = U\tilde{Z}^{-1}, \quad K = \tilde{K} \\
&\mathcal{C}_1 = I + \begin{bmatrix} 0 & 0 & 0 & 0 & I & 0 \end{bmatrix}^T \begin{bmatrix} 0 & 0 & -C & 0 & 0 & 0 \end{bmatrix} \\
&\mathcal{C}_2 = I + \begin{bmatrix} 0 & 0 & 0 & I & 0 \end{bmatrix}^T \begin{bmatrix} 0 & 0 & -C & 0 & 0 \end{bmatrix}
\end{aligned}$$

Make congruence transformations to (3.29) and (3.30) by $\mathcal{C}_1$ and $\mathcal{C}_2$, respectively. Then by Schur complement, one can find that the matrix inequalities (3.15)-(3.17) in Proposition 3.1 hold. The result then follows from from Proposition 3.1. □

3.5 Quantizer design

Now we consider the problem of how to quantize $y(i_k h)$ by using a finite-level quantizer $q(\cdot)$. As mentioned earlier, a requirement of the quantizer $q(\cdot)$ is that its quantization error should be bounded as in (3.20), i.e.

$$|y(i_k h) - q(y(i_k h))|^2 < \delta |q(y(i_k h))|^2 + \epsilon^3.$$

For this purpose, we take the following quantizer $q(\cdot)$:

$$q(y(i_{k+1}h)) = \begin{cases} \frac{1}{2}\epsilon^{\frac{3}{2}}, & 0 \le y(i_{k+1}h) \le \epsilon^{\frac{3}{2}} \\ a_k\rho^j, & \epsilon^{\frac{3}{2}} \le (1-\sqrt{\delta})a_k\rho^j < y(i_{k+1}h) \le (1+\sqrt{\delta})a_k\rho^j, j \in \mathbb{Z} \\ -q(-y(i_{k+1}h)), & y(i_{k+1}h) < 0 \end{cases} \quad (3.31)$$

where δ and ϵ are the parameters of communication scheme (3.2); and

$$a_k = |q(y(i_k h))|, \quad \rho = \frac{1+\sqrt{\delta}}{1-\sqrt{\delta}}.$$

It is assumed that the initial quantized value $q(y(0)) := y(0) = Cx_0$ is known to the quantizer and the decoder, which is co-located with the controller.

Remark 3.3. *One can find that quantizer (3.31) and communication scheme (3.2) share the same parameters δ and ϵ. In fact, they are designed in an integrated way. If the term of ϵ^3 is not included in the event-triggered communication scheme (3.2), in this case the communication scheme (3.2) becomes*

$$i_{k+1}h = i_k h + \min_{l \in \mathbb{Z}^+}\{lh \mid |y(i_k h + lh) - q(y(i_k h))|^2 \geq \delta |q(y(i_k h))|^2\}.$$

Then one can find that as $y(i_k h) \rightarrow 0$, the number of quantization levels will tend to infinity to guarantee the corresponding quantization error bound: $|y(i_k h) - q(y(i_k h))| < \sqrt{\delta}|q(y(i_k h))|$.

Remark 3.4. *A finite-level quantizer in control systems usually has an adjustable parameter to dynamically scale the quantization region as in [78–80]. In this case, the parameter should also be represented in a certain number of bits and transmitted to the decoder; or the quantizer should be designed in a way such that the decoder could compute the current value of the parameter based on the received quantized data, as in [79]. As to our proposed quantizer (3.31), the adjustable parameter is no longer needed. The decoder can obtain the current quantized-data $q(y(i_{k+1}h))$ by using the quantization rule in (3.31) and the received data $q(y(i_k h))$.*

It can be verified from the definition of $q(\cdot)$ that

$$|y(i_k h) - q(y(i_k h))| \leq \frac{1}{2}\epsilon^{\frac{3}{2}}, \text{ if } |y(i_k h)| \leq \epsilon^{\frac{3}{2}}$$

and

$$|y(i_k h) - q(y(i_k h))| \leq \sqrt{\delta}|q(y(i_k h))|, \text{ if } |y(i_k h)| > \epsilon^{\frac{3}{2}}$$

which imply that the requirement of the quantization error bound in (3.20) is guaranteed.

In the remainder of this section, we will show that the proposed quantizer $q(\cdot)$ is finite-level by considering the dynamics of the system and the event-triggered communication scheme.

It follows from (3.1)-(3.3) that

$$\begin{aligned}|y(i_{k+1}h) - q(y(i_kh))| &= |y(i_{k+1}h) - y(i_{k+1}h-h) + y(i_{k+1}h-h) - q(y(i_kh))| \\ &\leq |y(i_{k+1}h) - y(i_{k+1}h-h)| + |y(i_{k+1}h-h) - q(y(i_kh))| \\ &< \sqrt{\delta}|q(y(i_kh))| + |Ce^{Ah}x(i_{k+1}h-h) - Cx(i_{k+1}h-h) \\ &\quad +C\int_{i_{k+1}h-h}^{i_{k+1}h} e^{A(i_{k+1}h-s)}BKq(y(i_kh))ds| + \epsilon^{\frac{3}{2}} \\ &\leq \sqrt{\delta}|q(y(i_kh))| + |C\int_0^h e^{As}ds \cdot BKq(y(i_kh))| + \epsilon^{\frac{3}{2}} \\ &\quad +\|e^{Ah} - I\| \cdot |y(i_{k+1}h-h) - q(y(i_kh)) + q(y(i_kh))| \\ &\leq D_k \end{aligned} \tag{3.32}$$

where $D_k = M_1|q(y(i_kh))| + M_2\epsilon^{\frac{3}{2}}$ with

$$\begin{aligned} M_1 &= \sqrt{\delta} + |C\int_0^h e^{As}ds \cdot BK| + \|e^{Ah} - I\|(1+\sqrt{\delta}) \\ M_2 &= 1 + \|e^{Ah} - I\|. \end{aligned}$$

On the other hand, the communication scheme (3.2) implies that

$$|y(i_{k+1}h) - q(y(i_kh))| \geq \sqrt{\delta}|q(y(i_kh))|. \tag{3.33}$$

One can see that $|y(i_{k+1}h)|$ must lie in one of the following regions:

$$\begin{aligned} r_1 &= [0, \epsilon^{\frac{3}{2}}], \quad r_2 = (\epsilon^{\frac{3}{2}}, (1-\sqrt{\delta})|q(y(i_kh))|) \\ r_3 &= ((1+\sqrt{\delta})|q(y(i_kh))|, |q(y(i_kh))| + D_k]. \end{aligned}$$

We denote the number of quantization levels in the three regions r_1, r_2, r_3 as L_1, L_2 and L_3, respectively.

It can be seen from (3.31) that $L_1 = 2$. Let

$$|q(y(i_kh))| \cdot \rho^{-\frac{L_2}{2}}(1-\sqrt{\delta}) \leq \epsilon^{\frac{3}{2}}. \tag{3.34}$$

One can find that

$$L_2 \geq \max\{2\lceil \log_\rho[(1-\sqrt{\delta})\epsilon^{-\frac{3}{2}}|q(y(i_kh))|]\rceil, 0\} \tag{3.35}$$

where $\lceil\cdot\rceil$ is the ceiling function, which is the integer obtained by rounding up. Let

$$\begin{aligned}|q(y(i_kh))|\cdot\rho^{\frac{L_3}{2}}(1+\sqrt{\delta}) &\geq |q(y(i_kh))|+D_k \\ &= (M_1+1)|q(y(i_kh))|+M_2\epsilon^{\frac{3}{2}} \end{aligned} \tag{3.36}$$

which leads to

$$\begin{aligned}\rho^{\frac{L_3}{2}}(1+\sqrt{\delta}) &\geq M_1+1+\frac{M_2\epsilon^{\frac{3}{2}}}{|q(y(i_kh))|} \\ &\geq M_1+1+\frac{M_2\epsilon^{\frac{3}{2}}}{\frac{1}{2}\epsilon^{\frac{3}{2}}} \\ &= M_1+2M_2+1. \end{aligned} \tag{3.37}$$

Then, we have

$$L_3 \geq 2\lceil\log_\rho \frac{M_1+2M_2+1}{1+\sqrt{\delta}}\rceil. \tag{3.38}$$

The results can be summarized in the following

Proposition 3.3. *The quantizer (3.31) proposed for system (3.10) with event-triggered communication scheme (3.2) has a finite number of quantization levels. And the number of the quantization levels can be upper bounded by*

$$N_k = \underline{L_2} + \underline{L_3} + 2 \tag{3.39}$$

where $\underline{L_2}$ and $\underline{L_3}$ are given by the right side of (3.35) and (3.38), respectively. In addition, the quantization error is bounded by (3.20).

Remark 3.5. *If the constant scalar $\epsilon > 0$ in (3.2) is replaced by a variable scalar $\epsilon(k) > 0$ as in [56], where $\epsilon(k)$ is an appropriate designed decreasing sequence and $\lim_{k\to 0}\epsilon(k) = 0$, then the asymptotic stability could be obtained for system (3.10).*

3.6 A numerical example

In this section, a numerical example taken from [55] is employed to demonstrate the effectiveness of the proposed approach. Consider the following system

$$\begin{cases} \dot{x}(t) = \begin{bmatrix} 0 & 1 \\ -2 & 3 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t) \\ y(t) = \begin{bmatrix} -1 & 4 \end{bmatrix} x(t). \end{cases} \tag{3.40}$$

We take the sampling period $h = 0.01s$. The parameters of the communication scheme (3.2) and the dynamical quantizer (3.31) are given as: $\delta = 0.1, \epsilon = 0.3$. By using Proposition 3.2, we can get a controller $K = -1.17$.

For the purpose of simulation, the initial condition is supposed to be $x(0) = [0.5 \ \ -0.5]^T$. With the obtained controller, the communication scheme (3.2) and the quantizer (3.31), the state response of the resulted system is plotted in Figure 3.2, which shows that the state is well bounded. The transmission release time distribution of the quantized data is illustrated in Figure 3.3. Within the simulation time $T_s = 25s$, the output $y(t)$ is sampled 2500 times, while only 58 sampled signals are quantized and transmitted to the controller for feedback, which implies that a certain proportion of transmission resources can be saved. The average data quantization and transmission time interval is $0.43s$, which is much longer than the sampling period $h = 0.01s$. The desired control performance (uniform ultimate boundedness) is guaranteed, which shows the effectiveness of the proposed approach. According to the proposed quantizer design method presented in Section 3.5, one can obtain an upper bound on quantization levels of the finite-level quantizer and the corresponding upper bound on bit rates for the system in this simulation, which are illustrated in Figure 3.4.

3.7 Conclusion

This chapter has presented an event-triggered output feedback control scheme with a finite-level dynamical quantizer. The current sampled data is used for feedback

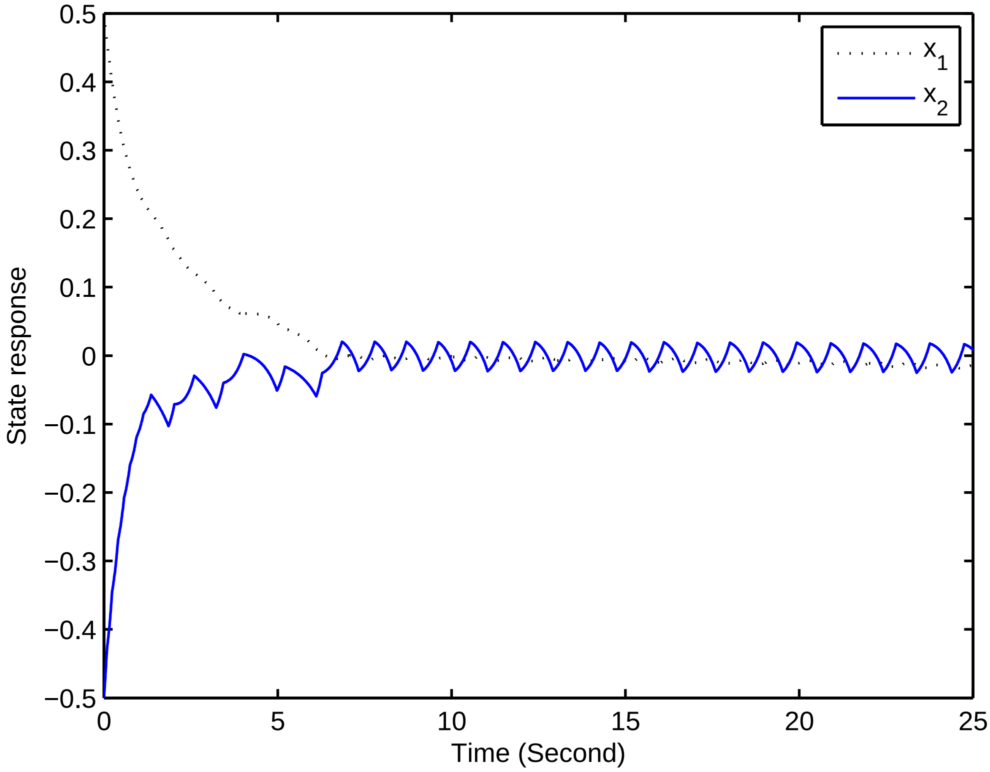

Figure 3.2: State response of the system.

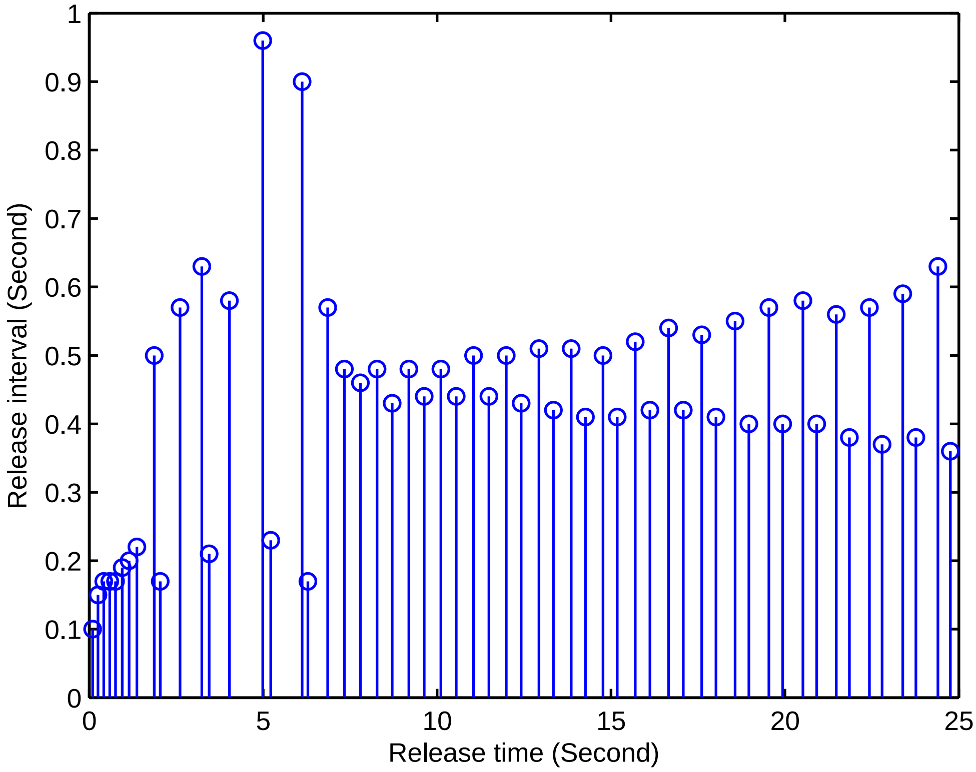

Figure 3.3: Release time distribution of the system in the simulation.

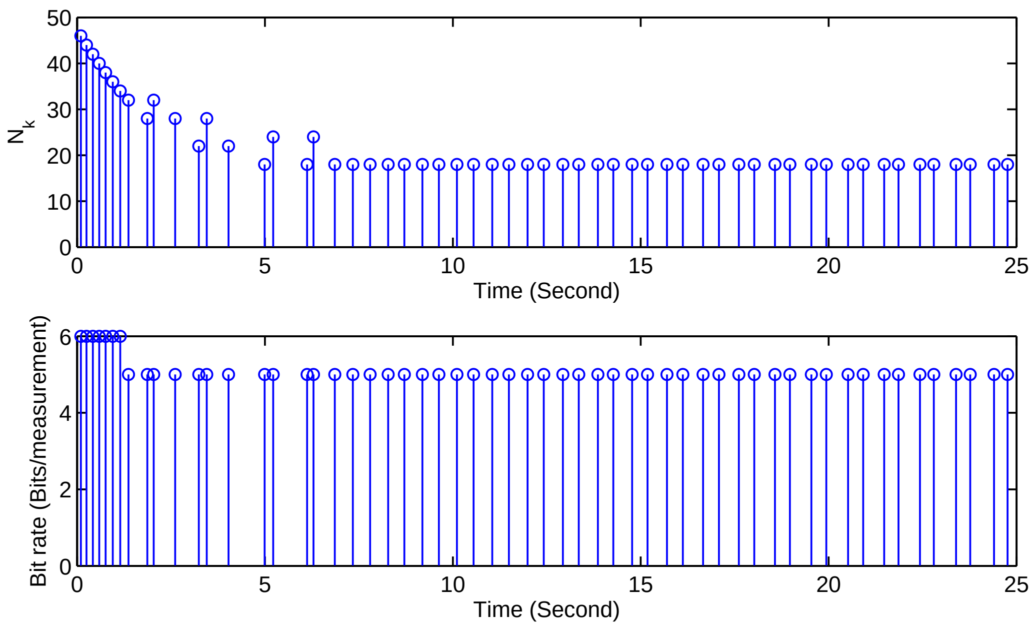

Figure 3.4: Upper bounds of quantization levels and bit rate for the system.

only when the error between the current sampled data and the latest quantized sampled data exceeds a specific threshold according to an event-triggered communication scheme. A dynamical finite-level quantizer has been designed based on the communication scheme. The uniform ultimate boundedness analysis and the corresponding output feedback controller design have also bee presented. A numerical example shows that with the proposed approach, the number of transmissions can be effectively reduced while maintaining the desired control performance, which shows the effectiveness of the obtained results.

Chapter 4

Decentralized event-triggered control for networked control systems with asynchronous sampling

4.1 Introduction

Decentralized control has been attracting increasing attention along with the technological developments in reliable wireless network transmission as well as low-cost microprocessors [81]. There is a number of applications of large scale systems in practical situations. In such a large scale system, a group of physically distributed sensors are employed to measure the system output, which can not be measured via a centralized sensor node. Many results on analysis and design of decentralized control systems have been reported in the literature [82–87].

For the efficient use of the limited transmission resources, for example, network bandwidth and battery power, it is of great importance to introduce the event-triggered transmission mechanism into decentralized control implementations to reduce some unnecessary transmissions. A challenge to such decentralized event-triggered control is that a full output of the system is not available to any of the geographically distributed sensor nodes, which makes the design of event-triggered conditions much more complicated in comparison with the design of a centralized event-triggered threshold. To deal with this problem, several efforts have been made recently [41, 55–57]. Based on a centralized event-triggered sampling approach [40], a group of threshold parameters are introduced to develop a set of sub-event conditions [57], which ensures that the measurement error of the full system state is upper bounded by the centralized event-triggered threshold that guarantees asymptotic stability of the system. It is noted that although each sensor node can locally determine when to generate a new sampled data in [57], all of the state components are required for being transmitted to the central controller synchronously. In this case every sensor requires a single informing message sent from the controller whenever a sub-event threshold is violated. A practical stabilization of a nonlinear system under asynchronous updates is established by using a decentralized event-triggered control strategy [56], where each component of the system's state is event-triggered for sampling when the local measurement error exceeds a pre-given positive scalar. More recently, based on output feedback, a decentralized event-triggered scheme [55] is proposed, where each local event-triggered threshold in a sensor node is established by the current values in the node and a positive scalar. It is worth mentioning that the corresponding event-triggered control system in [55] is modeled by an impulsive system. For weakly coupled distributed control systems, an event-triggered control scheme is developed in [41], where the current state of the subsystem is sampled and released for transmission only when the local measurement error of the subsystem state exceeds a specified state-dependent threshold.

It is noted that in all the mentioned results on decentralized event-triggered control, system outputs have to be measured continuously by some special real-time detection hardware, by which the sampling and transmission can be executed at the moment when an event-triggered threshold is violated. This may pose a critical requirement for the hardware. Another issue is that when such a continuous event-triggered mechanism is applied in decentralized control with asynchronous updates, a separate term of a positive scalar need to be involved in each sub-event threshold condition (as in [55, 57]) to guarantee a nonzero minimum inter-event time for each sensor node. By this way, it generally only leads to a practical stability of the system in [57], while the asymptotic stability is achieved if the group of positive scalars could be adjusted online. In addition, all the results on decentralized event-triggered control are obtained based on a pre-given central controller (see, for example, state feedback [56], output feedback [55]). It is noted that although a nonzero lower bound of inter-sampling interval is guaranteed for each sensor node, the central controller may still receive and process arbitrarily many signals within a certain time period, especially when the number of the sensor nodes are very large. In this case, the controller is required of computing control inputs arbitrarily fast, which is infeasible in practical implementations.

In this chapter, we propose a decentralized event-triggered transmission scheme based on asynchronous sampling. As in decentralized control systems applications [57, 81], for example, control over wireless sensor/actuator networks [57], the system under consideration in this chapter can not be measured by a centralized sensor node. Instead, several sensor nodes have to be employed to measure the system state. Therefore, a complete measurement of the system state is not available for each sensor node to adopt an event-triggered transmission mechanism which is based on the full state vector [40]. Another challenge in this chapter is that the event-triggered transmissions from different sensor nodes are not required to be synchronous as in [57]. Although it brings difficulties in system analysis, asynchronous sampling/transmission can increase the flexibility of control systems and be easier to be implemented in practice than the synchronous one.

In this chapter, the components of system state are grouped into several subvectors, each of which is sampled by a separate time-triggered sensor node. We first present a decentralized event-triggered transmission scheme to check if each sampled data should be transmitted via a network to a decentralized control station. The current sampled data is used to update the control input only when the error between the current sampled data and the latest transmitted sampled data exceeds a pre-designed threshold. An interval time-varying delay is considered for each network transmission. By using a switching Lyapunov-Krasovskii functional, we get an $\mathcal{L}_2$ stability analysis result for the decentralized event-triggered control system with appropriate consideration of input delay characteristics. Based on the stability criterion, a design method is presented to obtain a desired $\mathcal{L}_2$ controller for the resulted system. In addition, for comparison with other existing results, the proposed decentralized event-triggered transmission scheme is applied in the case where a central controller is employed and some corresponding results are obtained.

The main contribution of this chapter is a decentralized event-triggered transmission scheme, which has some advantages over some existing decentralized event-triggered sampling/transmission schemes. On the one hand, the real-time detection hardware is no longer needed since sampling and transmission mechanisms are executed separately in this chapter. On the other hand, there is no separate term of a positive scalar required to be involved in the proposed event-triggered threshold compared with the results in [55,56], which leads to that the asymptotic stability for the decentralized event-triggered systems can be easily achieved. Compared with the decentralized event-triggered control proposed in [57], synchronous transmission is not required of the proposed decentralized event-triggered transmission scheme. Another contribution is an $\mathcal{L}_2$ decentralized controller design method, while all the existing results on decentralized event-triggered control only consider a central controller with the "emulation-based method".

The organisation of this chapter is as follows. Section 4.2 presents a decentralized event-triggered transmission scheme and formulates the decentralized event-triggered $\mathcal{L}_2$ control problem. The $\mathcal{L}_2$ stability analysis and controller design for the decentralized event-triggered system are given in Section 4.3. The decentralized event-triggered control with a central controller is presented in Section 4.4. Simulation examples are given to demonstrate the effectiveness of the proposed approach in Section 4.5. This chapter is concluded in Section 4.6.

4.2 Problem statement

4.2.1 System model

Consider the decentralized control system described by

$$\dot{x}(t) = Ax(t) + \sum_{i=1}^{N} B_i u_i(t) + B_\omega \omega(t) \tag{4.1}$$

where $x(t) \in \mathbb{R}^n$ is the system state; N is the number of control stations; $u_i(t) \in \mathbb{R}^{m_i}$ is control input from the ith control station; $\omega(t) \in \mathcal{L}_2[0, \infty)$ is the exogenous disturbance; A, B_i and B_ω are real constant matrices with appropriate dimensions; and the initial condition of the system (4.1) is given by $x(0) = x_0$.

It is assumed in this chapter that the full system state $x(t)$ are not measured by a single centralized sensor node. Instead, the state variables to be measured are grouped into N physically distributed nodes. $x_{\mathcal{N}_i}(t)$ is a state subvector composed of the state variables in the ith sensor node. It is clear that

$$x(t) = \begin{bmatrix} x_{\mathcal{N}_1}^T(t) & x_{\mathcal{N}_2}^T(t) & \cdots & x_{\mathcal{N}_N}^T(t) \end{bmatrix}^T.$$

For the purpose of easy implementation, every sensor is time-driven, i.e. $x_{\mathcal{N}_i}(t)$ is periodically sampled with a sampling period $h_i > 0$, $i = 1, 2, \ldots, N$. All the N sensors do not have to start sampling at the same time and they can have different sampling periods. In this sense, the whole sampling process is asynchronous. Whether or not the current sampled data is to be transmitted through the wireless network to a control station is decided by each separate sensor node according to a pre-given transmission scheme. It is noted that this kind of asynchronous sampling/transmission mechanism makes system (4.1) have an advantage of modeling many physical processes in reality [81].

For the efficient use of the transmission resources (e.g. battery power and/or network bandwidth), we propose a decentralized event-triggered transmission scheme to reduce the transmission traffic load. As is shown in Figure 4.1, each sensor is collocated with an event-triggered transmitter which is used to determine whether or not the current sampled data is transmitted based on the error between the current sampled data and the latest transmitted sampled data.

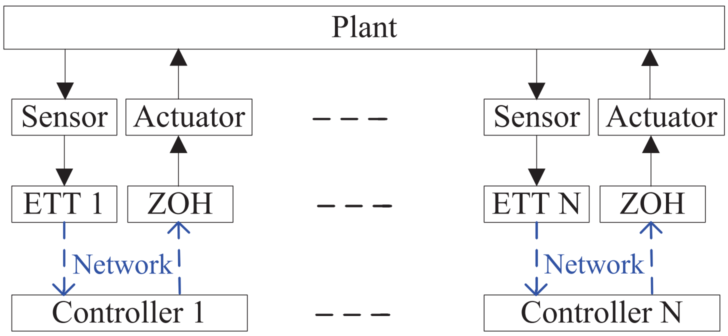

Figure 4.1: A framework of a decentralized event-triggered NCS

The broadcast release time sequence of the ith event-triggered transmitter is denoted as $\{t^i_{k_i}h_i\}^{\infty}_{k_i=1}$, which is generated according to the following event-triggered transmission scheme

$$t^i_{k_i+1}h_i = t^i_{k_i}h_i + \min_{l\in\mathbb{Z}^+}\{lh_i \mid \|x_{\mathcal{N}_i}(t^i_{k_i}h_i + lh_i) - x_{\mathcal{N}_i}(t^i_{k_i}h_i)\|^2 > \delta_i\|x_{\mathcal{N}_i}(t^i_{k_i}h_i + lh_i)\|^2\} \tag{4.2}$$

where $t^i_{k_i}h_i$ is the k_ith transmission time instant of the ith transmitter; $x_{\mathcal{N}_i}(t^i_{k_i}h_i+lh_i)$ is the current sampled data in the ith sensor; $\mathbb{Z}^+$ is the set of positive integers; δ_i satisfying $\delta_i > 0$ is the threshold of the event-triggered transmission scheme.

Remark 4.1. *The decentralized event-triggered sampling scheme in [56] is given as*

$$t^i_{k_i+1} = \min\{t > t^i_{k_i} | (x_i(t) - x_i(t^i_{k_i}))^2 > \eta_i\} \tag{4.3}$$

where $\eta_i > 0$ is introduced to guarantee a nonzero minimum inter-event time. However, the positive scalar term η_i usually only leads to a practical stability of the system, i.e. the trajectory finally stays in a small compact set. It is also shown in [56] that asymptotic stability could be achieved only if the positive scalars could be appropriately adjusted online. In [55], a separate positive scalar term is also introduced in the sub-event threshold to guarantee a nonzero minimum inter-event time, in addition to a measurement dependent term. It can be seen that this kind of separate positive scalar term is no longer needed in this chapter.

Remark 4.2. *The decentralized event-triggered transmission scheme (4.2) is employed to reduce some unnecessary data transmissions. Compared with decentralized event-triggered mechanisms presented in [55, 56], the event-triggered mechanism in this chapter and the sampling process are executed separately. Therefore, the real-time detection hardware is no longer needed.*

Since an event-triggered transmitter is introduced to check whether or not the current sampled data could get access to the network channel, it is expected that the frequency of data transmission through network could be effectively reduced compared with the case where a general time-triggered transmitter is employed. Therefore, the proposed decentralized event-triggered transmission scheme can contribute to improve the network quality of service (QoS) in the proposed NCS framework. With this observation, it is assumed in this chapter that there is no data dropout or packet disorder during the data transmission.

In what follows, we are interested in designing a decentralized controller composed of N control stations, which are in the form of

$$u_i(t^+) = K_i x_{\mathcal{N}_i}(t^i_{k_i} h_i), \quad t \in [t^i_{k_i} h_i + \tau^i_{k_i}, t^i_{k_i+1} h_i + \tau^i_{k_i+1}) \tag{4.4}$$

where $K_i \in \mathbb{R}^{m_i \times n_i}, i = 1, 2, \ldots, N$ are to be determined; $\tau^i_{k_i}$ denotes the network-induced delays from sensor i to control station i and from the control station to the actuator. We have the following assumption on the network-induced delays:

$$0 < \tau_{k_i}^i \le \bar{\tau}_i, \quad i = 1, 2, \ldots, N \tag{4.5}$$

where $\bar{\tau}_i$ is a positive constant.

Remark 4.3. *Traditionally, a central controller is employed to compute control input in a control system. However, the centralized computing capability may fail when a central controller is used in a large scale complex system. To deal with the problem, one can utilize a decentralized controller as given in (4.4), which can be used to compute control input flexibly and increase the computing efficiency.*

4.2.2 Problem formulation

Let

$$B = \begin{bmatrix} B_1 & B_2 & \cdots & B_N \end{bmatrix}, \quad K = diag\{K_1, K_2, \ldots, K_N\}. \tag{4.6}$$

Then system (4.1) with control input (4.4) can be written as

$$\dot{x}(t) = Ax(t) + BK \begin{bmatrix} x_{\mathcal{N}_1}^T(t_{k_1}^1 h_1) & x_{\mathcal{N}_2}^T(t_{k_2}^2 h_2) & \cdots & x_{\mathcal{N}_N}^T(t_{k_N}^N h_N) \end{bmatrix}^T + B_\omega \omega(t),$$
$$t \in [t_k, t_{k+1}) \tag{4.7}$$

where $t_k = \max_{i=1,2,\ldots,N}\{t_{k_i}^i h_i + \tau_{k_i}^i\}$, $t_{k+1} = \min_{i=1,2,\ldots,N}\{t_{k_i+1}^i h_i + \tau_{k_i+1}^i\}$.

Let

$$\eta_i(t) = \begin{cases} t - \max_{l \in \mathbb{Z}^+}\{lh_i | lh_i \le t\}, & \text{if} \quad t_{k_i}^i h_i + \tau_{k_i}^i \le t < t_{k_i+1}^i h_i \\ t - t_{k_i+1}^i h_i + h_i, & \text{if} \quad t_{k_i+1}^i h_i \le t < t_{k_i+1}^i h_i + \tau_{k_i+1}^i \end{cases} \tag{4.8}$$

$$\begin{aligned} e_{\mathcal{N}_i}(t) &= \begin{cases} x_{\mathcal{N}_i}(\max_{l \in \mathbb{Z}^+}\{lh_i | lh_i \le t\}) - x_{\mathcal{N}_i}(t_{k_i}^i h_i), & \text{if} \quad t_{k_i}^i h_i + \tau_{k_i}^i \le t < t_{k_i+1}^i h_i \\ x_{\mathcal{N}_i}(t_{k_i+1}^i h_i - h_i) - x_{\mathcal{N}_i}(t_{k_i}^i h_i), & \text{if} \quad t_{k_i+1}^i h_i \le t < t_{k_i+1}^i h_i + \tau_{k_i+1}^i \end{cases} \\ &= x_{\mathcal{N}_i}(t - \eta_i(t)) - x_{\mathcal{N}_i}(t_{k_i}^i h_i) \end{aligned} \tag{4.9}$$

$$e(t) = \begin{bmatrix} e_{\mathcal{N}_1}^T(t) & e_{\mathcal{N}_2}^T(t) & \cdots & e_{\mathcal{N}_N}^T(t) \end{bmatrix}^T \tag{4.10}$$

$$D_i = diag\{\underbrace{0, \cdots, I_{\mathcal{N}_i \times \mathcal{N}_i}}_{i}, \cdots, 0\}. \tag{4.11}$$

It follows from (4.7)-(4.11) that

$$\dot{x}(t) = Ax(t) + BK\sum_{i=1}^{N} D_i x(t-\eta_i(t)) - BKe(t) + B_\omega \omega(t), \quad t \in [t_k, t_{k+1}). \quad (4.12)$$

One can see from (4.8) that the bound of $\eta_i(t)$ in two cases are $0 \le \eta_i(t) < h_i$ and $h \le \eta_i(t) < h_i + \bar{\tau}_i$, respectively. In order to take into consideration this distribution of $\eta_i(t)$, we represent system (4.12) as

$$\begin{aligned}\dot{x}(t) = Ax(t) + BK\sum_{i=1}^{N} \chi_i(\eta_i(t)) D_i x(t-\eta_i(t)) + BK\sum_{i=1}^{N}(1-\chi_i(\eta_i(t))) D_i x(t-\eta_i(t)) \\ -BKe(t) + B_\omega \omega(t),\ t \in [t_k, t_{k+1}) \end{aligned} \quad (4.13)$$

where the characteristic function $\chi_i(\eta_i(t))$ is 1 if $\eta_i(t) \in [0, h_i)$ and 0 otherwise. In what follows, we will drop the argument of $\chi_i(\eta_i(t))$ for clarity, i.e. $\chi_i(\eta_i(t))$ will be written as χ_i.

We supplement the initial condition of the state on $[-\bar{h}, 0]$ as $x(t) = \varphi(t), t \in [-\bar{h}, 0]$, where $\bar{h} = \max_{i=1,2,\ldots,N}\{h + \bar{\tau}_i\}$; $\varphi(t)$ is a continuous function on $[-\bar{h}, 0]$ and $\varphi(0) = x_0$.

The purpose of this chapter is to design a decentralized controller in the form of (4.4) such that

(i) the sampled-data error dependent closed-loop system (4.12) with $w(t) = 0$ is asymptotically stable;

(ii) the $\mathcal{L}_2$ gain from w to x is less than a given scalar $\gamma > 0$, that is, under zero initial condition, $\|x(t)\|_2 < \gamma \|\omega(t)\|_2$ for any nonzero $\omega(t) \in \mathcal{L}_2[0, \infty)$.

To develop the required results, the following lemmas will be used later.

Lemma 4.1. [65] *For any constant matrix $W \in \mathbb{R}^{n\times n}$, $W = W^T > 0$, scalar $\gamma > 0$, and vector function $\dot{x} : [-\gamma, 0] \longrightarrow \mathbb{R}^n$ such that the following integration is well defined, then*

$$-\gamma \int_{-\gamma}^{0} \dot{x}^T(t+\xi) W \dot{x}(t+\xi) d\xi \le \begin{bmatrix} x^T(t) & x^T(t-\gamma) \end{bmatrix} \begin{bmatrix} -W & W \\ W & -W \end{bmatrix} \begin{bmatrix} x(t) \\ x(t-\gamma) \end{bmatrix}.$$

Lemma 4.2. *For any constant matrices* $R \in \mathbb{R}^{n\times n}$, $Y \in \mathbb{R}^{n\times n}$, $R = R^T > 0$, *scalar* $h > 0$, *function* $\eta(t)$ *satisfying* $0 \le \eta(t) < h$, *and vector function* $\dot{x} : [-h, 0] \longrightarrow \mathbb{R}^n$ *such that the following integrations are well defined, then*

$$-h\int_{t-\eta(t)}^{t} \dot{x}^T(s)R\dot{x}(s)ds - h\int_{t-h}^{t-\eta(t)} \dot{x}^T(s)R\dot{x}(s)ds \le \begin{bmatrix} x^T(t) & x^T(t-\eta(t)) & x^T(t-h) \end{bmatrix} \times \begin{bmatrix} -R & R+Y & -Y \\ * & -2R-Y-Y^T & R+Y \\ * & * & -R \end{bmatrix} \begin{bmatrix} x(t) \\ x(t-\eta(t)) \\ x(t-h) \end{bmatrix} \quad (4.14)$$

holds, if

$$\begin{bmatrix} R & Y \\ Y^T & R \end{bmatrix} \ge 0. \quad (4.15)$$

Proof. The lemma will be proved in the two cases: when $0 < \eta(t) < h$ and when $\eta(t) = 0$, respectively.

Case (i): $0 < \eta(t) < h$. By the Jensens integral inequality, one can get that

$$\begin{aligned} &-h\int_{t-\eta(t)}^{t} \dot{x}^T(s)R\dot{x}(s)ds - h\int_{t-h}^{t-\eta(t)} \dot{x}^T(s)R\dot{x}(s)ds \\ &\le -\frac{h}{\eta(t)} z_1^T(t)Rz_1(t) - \frac{h}{h-\eta(t)} z_2^T(t)Rz_2(t) \\ &\le -(1+\alpha(t))z_1^T(t)Rz_1(t) - (1+\frac{1}{\alpha(t)})z_2^T(t)Rz_2(t) \end{aligned} \quad (4.16)$$

where $z_1(t) := x(t) - x(t-\eta(t))$, $z_2(t) := x(t-\eta(t)) - x(t-h)$, $\alpha(t) = \frac{h-\eta(t)}{\eta(t)} > 0$. It follows from (4.15) that

$$\begin{bmatrix} \sqrt{\alpha(t)}z_1(t) \\ \sqrt{\frac{1}{\alpha(t)}}z_2(t) \end{bmatrix}^T \begin{bmatrix} R & Y \\ Y^T & R \end{bmatrix} \begin{bmatrix} \sqrt{\alpha(t)}z_1(t) \\ \sqrt{\frac{1}{\alpha(t)}}z_2(t) \end{bmatrix} \ge 0$$

which leads to

$$-\alpha(t)z_1^T(t)Rz_1(t) - \frac{1}{\alpha(t)}z_2^T(t)Rz_2(t) \le 2z_1^T(t)Yz_2(t). \quad (4.17)$$

Then one can find from (4.16) and (4.17) that

$$\begin{aligned} &-h\int_{t-\eta(t)}^{t} \dot{x}^T(s)R\dot{x}(s)ds - h\int_{t-h}^{t-\eta(t)} \dot{x}^T(s)R\dot{x}(s)ds \\ &\le -z_1^T(t)Rz_1(t) - z_2^T(t)Rz_2(t) + 2z_1^T(t)Yz_2(t) \\ &= \begin{bmatrix} z_1^T(t) & z_2^T(t) \end{bmatrix} \begin{bmatrix} -R & Y \\ Y^T & -R \end{bmatrix} \begin{bmatrix} z_1(t) \\ z_2(t) \end{bmatrix} \end{aligned} \quad (4.18)$$

which is equivalent to (4.14).

Case (ii): $\eta(t) = 0$. Then (4.14) becomes

$$-h\int_{t-h}^{t} \dot{x}^T(s)R\dot{x}(s)ds \leq \begin{bmatrix} x^T(t) & x^T(t-h) \end{bmatrix} \begin{bmatrix} -R & R \\ R & -R \end{bmatrix} \begin{bmatrix} x(t) \\ x(t-h) \end{bmatrix}$$

which is equivalent to Lemma 4.1 by some integration transformation. The proof is thus completed. □

4.3 $\mathcal{L}_2$ stability analysis and controller design

In this section, we will first derive an $\mathcal{L}_2$ stability criterion for the decentralized event-triggered control system (4.12).

Proposition 4.1. *Given a scalar $\gamma > 0$, with transmission scheme (4.2), the system (4.12) is finite-gain $\mathcal{L}_2$ stable from ω to x with a gain less than γ, if there exist real matrices $P > 0$, $\{R_i > 0\}_{i=1}^N$, $\{S_i > 0\}_{i=1}^N$, $\{Y_i\}_{i=1}^N$, $\{Z_i\}_{i=1}^N$, M_1, M_2 with appropriate dimensions such that for all $\lambda_i \in \{0, 1\}, i = 1, 2, \ldots, N$*

$$\begin{bmatrix} R_i & Y_i \\ * & R_i \end{bmatrix} \geq 0, \quad \begin{bmatrix} S_i & Z_i \\ * & S_i \end{bmatrix} \geq 0 \tag{4.19}$$

$$\Gamma(\lambda_1, \lambda_2, \ldots, \lambda_N) := \begin{bmatrix} \Gamma_{11} & P + A^T M_2 - M_1^T & \Gamma_{13} \\ * & \sum_{i=1}^N (h_i^2 R_i + \bar{\tau}_i^2 S_i) - M_2 - M_2^T & \Gamma_{23} \\ * & * & \Gamma_{33} \end{bmatrix} < 0 \tag{4.20}$$

where

$$\Gamma_{11} = M_1^T A + A^T M_1 + I - \sum_{i=1}^N R_i$$

$$\Gamma_{13} = \begin{bmatrix} \varphi_{11} & \varphi_{12} & \cdots & \varphi_{1N} & -M_1^T BK & M_1^T B_\omega \end{bmatrix}$$

$$\Gamma_{23} = \begin{bmatrix} \varphi_{21} & \varphi_{22} & \cdots & \varphi_{2N} & -M_2^T BK & M_2^T B_\omega \end{bmatrix}$$

$$\Gamma_{33} = diag\{\varphi_{31}, \varphi_{32}, \cdots, \varphi_{3N}, -I, -\gamma^2 I\}$$

$$\varphi_{1i} = \begin{bmatrix} M_1^T BKD_i + \lambda_i(R_i - Y_i) & \lambda_i Y_i + (1-\lambda_i) R_i & 0 \end{bmatrix}$$

$$\varphi_{2i} = \begin{bmatrix} M_2^T BKD_i & 0 & 0 \end{bmatrix}$$

$$\varphi_{3i} = \begin{bmatrix} \phi_i & \lambda_i(R_i - Y_i) + (1-\lambda_i)(S_i - Z_i^T) & (1-\lambda_i)(S_i - Z_i) \\ * & -R_i - S_i & \lambda_i S_i + (1-\lambda_i) Z_i \\ * & * & -S_i \end{bmatrix}$$

$$\phi_i = \delta_i D_i + \lambda_i(Y_i + Y_i^T - 2R_i) + (1-\lambda_i)(Z_i + Z_i^T - 2S_i).$$

Proof. It follows from the decentralized event-triggered transmission scheme (4.2) that for $\forall t \in [t_k, t_{k+1})$

$$e^T(t)e(t) \leq \sum_{i=1}^{N} \delta_i x^T(t-\eta_i(t)) D_i x(t-\eta_i(t)) \tag{4.21}$$

with D_i given by (4.11).

Choose the following Lyapunov-Krasovskii functional candidate

$$\begin{aligned} V(t, x(t)) = x^T(t)Px(t) &+ \sum_{i=1}^{N} h_i \int_{-h_i}^{0} \int_{t+\theta}^{t} \dot{x}^T(s) R_i \dot{x}(s) ds d\theta \\ &+ \sum_{i=1}^{N} \bar{\tau}_i \int_{-h_i-\bar{\tau}_i}^{-h_i} \int_{t+\theta}^{t} \dot{x}^T(s) S_i \dot{x}(s) ds d\theta. \end{aligned} \tag{4.22}$$

Taking the right derivative of $V(t, x(t))$ with respect to t along the trajectory of (4.12) yields

$$\dot{V}(t, x(t)) = 2x^T(t)P\dot{x}(t) + \dot{x}^T(t) \left[\sum_{i=1}^{N} (h_i^2 R_i + \bar{\tau}_i^2 S_i) \right] \dot{x}(t) + f \tag{4.23}$$

where $\dot{V}(t, x(t)) = \limsup_{\triangle t \to 0+} \frac{V(t+\triangle t, x(t+\triangle t)) - V(t, x(t))}{\triangle t}$, and

$$f = -\sum_{i=1}^{N} h_i \int_{t-h_i}^{t} \dot{x}^T(s) R_i \dot{x}(s) ds - \sum_{i=1}^{N} \bar{\tau}_i \int_{t-h_i-\bar{\tau}_i}^{t-h_i} \dot{x}^T(s) S_i \dot{x}(s) ds.$$

Taking into consideration the distribution of $\eta_i(t)$, one can see that

$$\begin{aligned} f = -\chi_i \sum_{i=1}^{N} \Bigg(& h_i \int_{t-\eta_i(t)}^{t} \dot{x}^T(s) R_i \dot{x}(s) ds + h_i \int_{t-h_i}^{t-\eta_i(t)} \dot{x}^T(s) R_i \dot{x}(s) ds \\ & + \bar{\tau}_i \int_{t-h_i-\bar{\tau}_i}^{t-h_i} \dot{x}^T(s) S_i \dot{x}(s) ds \Bigg) - (1-\chi_i) \sum_{i=1}^{N} \Bigg(h_i \int_{t-h_i}^{t} \dot{x}^T(s) R_i \dot{x}(s) ds \\ & + \bar{\tau}_i \int_{t-\eta_i(t)}^{t-h_i} \dot{x}^T(s) S_i \dot{x}(s) ds + \bar{\tau}_i \int_{t-h_i-\bar{\tau}_i}^{t-\eta_i(t)} \dot{x}^T(s) S_i \dot{x}(s) ds \Bigg) \end{aligned} \tag{4.24}$$

By Lemma 4.1 and integration transformation, we have

$$-h_i \int_{t-h_i}^{t} \dot{x}^T(s) R_i \dot{x}(s) ds \leq \begin{bmatrix} x(t) \\ x(t-h_i) \end{bmatrix}^T \begin{bmatrix} -R_i & R_i \\ R_i & -R_i \end{bmatrix} \begin{bmatrix} x(t) \\ x(t-h_i) \end{bmatrix}$$

and

$$-\bar{\tau}_i \int_{t-h_i-\bar{\tau}_i}^{t-h_i} \dot{x}^T(s) S_i \dot{x}(s) ds \leq \begin{bmatrix} x(t-h_i) \\ x(t-h_i-\bar{\tau}_i) \end{bmatrix}^T \begin{bmatrix} -S_i & S_i \\ S_i & -S_i \end{bmatrix} \begin{bmatrix} x(t-h_i) \\ x(t-h_i-\bar{\tau}_i) \end{bmatrix}.$$

It follows (4.19) and Lemma 4.2 that

$$\begin{aligned} & h_i \int_{t-\eta_i(t)}^{t} \dot{x}^T(s) R_i \dot{x}(s) ds + h_i \int_{t-h_i}^{t-\eta_i(t)} \dot{x}^T(s) R_i \dot{x}(s) ds \\ & \leq \begin{bmatrix} x(t) \\ x(t-\eta_i(t)) \\ x(t-h_i) \end{bmatrix}^T \begin{bmatrix} -R_i & R_i - Y_i & Y_i \\ * & Y_i + Y_i^T - 2R_i & R_i - Y_i \\ * & * & -R_i \end{bmatrix} \begin{bmatrix} x(t) \\ x(t-\eta_i(t)) \\ x(t-h_i) \end{bmatrix} \end{aligned}$$

and

$$\begin{aligned} & \bar{\tau}_i \int_{t-\eta_i(t)}^{t-h_i} \dot{x}^T(s) S_i \dot{x}(s) ds + \bar{\tau}_i \int_{t-h_i-\bar{\tau}_i}^{t-\eta_i(t)} \dot{x}^T(s) S_i \dot{x}(s) ds \\ & \leq \begin{bmatrix} x(t-h_i) \\ x(t-\eta_i(t)) \\ x(t-h_i-\bar{\tau}_i) \end{bmatrix}^T \begin{bmatrix} -S_i & S_i - Z_i & Z_i \\ * & Z_i + Z_i^T - 2S_i & S_i - Z_i \\ * & * & -S_i \end{bmatrix} \begin{bmatrix} x(t-h_i) \\ x(t-\eta_i(t)) \\ x(t-h_i-\bar{\tau}_i) \end{bmatrix}. \end{aligned}$$

Then, we have

$$\begin{aligned} f \leq \sum_{i=1}^{N} \Bigg(& \chi_i \begin{bmatrix} x(t) \\ x(t-\eta_i(t)) \\ x(t-h_i) \end{bmatrix}^T \begin{bmatrix} -R_i & R_i - Y_i & Y_i \\ * & Y_i + Y_i^T - 2R_i & R_i - Y_i \\ * & * & -R_i \end{bmatrix} \begin{bmatrix} x(t) \\ x(t-\eta_i(t)) \\ x(t-h_i) \end{bmatrix} \\ & + \chi_i \begin{bmatrix} x(t-h_i) \\ x(t-h_i-\bar{\tau}_i) \end{bmatrix}^T \begin{bmatrix} -S_i & S_i \\ S_i & -S_i \end{bmatrix} \begin{bmatrix} x(t-h_i) \\ x(t-h_i-\bar{\tau}_i) \end{bmatrix} + (1-\chi_i) \\ & \times \begin{bmatrix} x(t) \\ x(t-h_i) \end{bmatrix}^T \begin{bmatrix} -R_i & R_i \\ R_i & -R_i \end{bmatrix} \begin{bmatrix} x(t) \\ x(t-h_i) \end{bmatrix} + (1-\chi_i) \begin{bmatrix} x(t-h_i) \\ x(t-\eta_i(t)) \\ x(t-h_i-\bar{\tau}_i) \end{bmatrix}^T \\ & \times \begin{bmatrix} -S_i & S_i - Z_i & Z_i \\ * & Z_i + Z_i^T - 2S_i & S_i - Z_i \\ * & * & -S_i \end{bmatrix} \begin{bmatrix} x(t-h_i) \\ x(t-\eta_i(t)) \\ x(t-h_i-\bar{\tau}_i) \end{bmatrix} \Bigg) \end{aligned} \tag{4.25}$$

It is clear that there exist real nonsingular matrices M_1 and M_2 such that

$$\begin{aligned} & 2(x^T(t)M_1^T + \dot{x}^T(t)M_2^T)(Ax(t) + BK\sum_{i=1}^{N} D_i x(t-\eta_i(t)) \\ & \qquad -BKe(t) + B_\omega \omega(t) - \dot{x}(t)) = 0. \end{aligned} \tag{4.26}$$

Taking (4.21)–(4.26) into account together, one can get that

$$\dot{V}(t,x(t)) \leq \xi^T(t)\Gamma(\chi_1,\chi_2,\ldots,\chi_N)\xi(t) - x^T(t)x(t) + \gamma^2 w^T(t)w(t) \tag{4.27}$$

where $\Gamma(\lambda_1,\lambda_2,\ldots,\lambda_N)$ is given in (4.20); and

$$\xi(t) = \mathrm{col}\{x(t),\dot{x}(t),x(t-\eta_1(t)),x(t-h_1),x(t-h_1-\bar{\tau}_1),\cdots,x(t-\eta_N(t)),$$
$$x(t-h_N),x(t-h_N-\bar{\tau}_N),e(t),\omega(t)\}$$

It then follows from (4.20) that

$$\dot{V}(t,x(t)) \leq -x^T(t)x(t) + \gamma^2\omega^T(t)\omega(t). \tag{4.28}$$

Since $\dot{V}(t,x(t))$ is continuous in time t, for any $t_k > 0$, we have

$$\int_0^{t_k} \dot{V}(s,x(s))ds \leq \int_0^{t_k} -x^T(s)x(s) + \gamma^2\omega^T(s)\omega(s)ds.$$

It is clear that, under zero initial condition,

$$\int_0^{\infty} x^T(s)x(s)ds \leq \gamma^2 \int_0^{\infty} \omega^T(s)\omega(s)ds \tag{4.29}$$

holds for any nonzero $\omega(t) \in \mathcal{L}_2[0,\infty)$.

When $\omega(t) = 0$, (4.28) becomes

$$\dot{V}(t,x(t)) \leq -x^T(t)x(t) \tag{4.30}$$

from which one can conclude the asymptotic stability of the closed-loop system (4.12). The proof is completed. □

Based on the stability analysis result developed above, we are now in a position to design an $\mathcal{L}_2$ controller in the form of (4.7) such that the sampled-data error dependent closed-loop system (4.12) is finite-gain $\mathcal{L}_2$ stable from ω to x with a gain less than γ.

Proposition 4.2. *Given a scalar $\gamma > 0$, the system (4.12) with decentralized event-triggered transmission scheme (4.2) is finite-gain $\mathcal{L}_2$ stable from ω to x with a gain* *less than γ, if there exist real matrices $\tilde{P} > 0$, $\left\{\tilde{R}_i > 0\right\}_{i=1}^N$, $\left\{\tilde{S}_i > 0\right\}_{i=1}^N$, $\left\{\tilde{Y}_i\right\}_{i=1}^N$, $\left\{\tilde{Z}_i\right\}_{i=1}^N$, $\{F_i\}_{i=1}^N$, $\left\{\tilde{M}_i\right\}_{i=1}^N$ and U with appropriate dimensions such that for all $\lambda_i \in \{0,1\}, i = 1,2,\ldots,N$*

$$\begin{bmatrix} \tilde{R}_i & \tilde{Y}_i \\ * & \tilde{R}_i \end{bmatrix} \geq 0, \quad \begin{bmatrix} \tilde{S}_i & \tilde{Z}_i \\ * & \tilde{S}_i \end{bmatrix} \geq 0 \tag{4.31}$$

$$\begin{bmatrix} \tilde{\Gamma}_{11} & \tilde{P} + \tilde{M}^T A^T - U^T \tilde{M} & \tilde{\Gamma}_{13} & 0 \\ * & \sum_{i=1}^N (h_i^2 \tilde{R}_i + \bar{\tau}_i^2 \tilde{S}_i) - \tilde{M} - \tilde{M}^T & \tilde{\Gamma}_{23} & 0 \\ * & * & \tilde{\Gamma}_{33} & \tilde{\Gamma}_{34} \\ * & * & * & \tilde{\Gamma}_{44} \end{bmatrix} < 0 \tag{4.32}$$

where

$$\tilde{\Gamma}_{11} = U^T A \tilde{M} + \tilde{M}^T A^T U - \sum_{i=1}^N \tilde{R}_i, \quad \tilde{M} = diag\{\tilde{M}_1, \tilde{M}_2, \cdots, \tilde{M}_N\}$$

$$\tilde{\Gamma}_{13} = \begin{bmatrix} \tilde{\varphi}_{11} & \tilde{\varphi}_{12} & \cdots & \tilde{\varphi}_{1N} & -U^T \sum_1^N B_i F_i E_i^T & U^T B_\omega & \tilde{M}^T \end{bmatrix}$$

$$\tilde{\Gamma}_{23} = \begin{bmatrix} \tilde{\varphi}_{21} & \tilde{\varphi}_{22} & \cdots & \tilde{\varphi}_{2N} & -\sum_1^N B_i F_i E_i^T & B_\omega & 0 \end{bmatrix}$$

$$\tilde{\Gamma}_{33} = diag\{\tilde{\varphi}_{31}, \tilde{\varphi}_{32}, \cdots, \tilde{\varphi}_{3N}, -\tilde{M}^T \tilde{M}, -\gamma^2 I, -I\}$$

$$\tilde{\Gamma}_{34} = \begin{bmatrix} \Upsilon_{34}^T & 0 \end{bmatrix}^T, \quad \Upsilon_{34} = diag \begin{bmatrix} \tilde{\varphi}_{41} & \tilde{\varphi}_{42} & \cdots & \tilde{\varphi}_{4N} \end{bmatrix}$$

$$\tilde{\Gamma}_{44} = diag\{-\delta_1^{-1} I, -\delta_2^{-1} I, \cdots, -\delta_N^{-1} I\}$$

$$\tilde{\varphi}_{1i} = \begin{bmatrix} U^T B_i F_i E_i^T + \lambda_i (\tilde{R}_i - \tilde{Y}_i) & \lambda_i \tilde{Y}_i + (1-\lambda_i) \tilde{R}_i & 0 \end{bmatrix}$$

$$\tilde{\varphi}_{2i} = \begin{bmatrix} B_i F_i E_i^T & 0 & 0 \end{bmatrix}$$

$$\tilde{\varphi}_{3i} = \begin{bmatrix} \phi_i & \lambda_i (\tilde{R}_i - \tilde{Y}_i) + (1-\lambda_i)(\tilde{S}_i - \tilde{Z}_i^T) & (1-\lambda_i)(\tilde{S}_i - \tilde{Z}_i) \\ * & -\tilde{R}_i - \tilde{S}_i & \lambda_i \tilde{S}_i + (1-\lambda_i) \tilde{Z}_i \\ * & * & -\tilde{S}_i \end{bmatrix}$$

$$\tilde{\varphi}_{4i} = \begin{bmatrix} E_i^T \tilde{M} & 0 & 0 \end{bmatrix}^T, \quad E_i = \mathrm{col}\{\underbrace{0, \cdots, I_{\mathcal{N}_i \times \mathcal{N}_i}}_{i}, \cdots, 0\}$$

$$\phi_i = \lambda_i (\tilde{Y}_i + \tilde{Y}_i^T - 2\tilde{R}_i) + (1-\lambda_i)(\tilde{Z}_i + \tilde{Z}_i^T - 2\tilde{S}_i).$$

Moreover, the controller parameter matrices in (4.4) are given by $K_i = F_i \tilde{M}_i^{-1}, i = 1,2,\ldots,N$.

Proof. It can be seen from (4.32) that $\tilde{M}$ is nonsingular. Pre- and post-multiply (4.31) by $diag\{\tilde{M}^{-T}, \tilde{M}^{-T}\}$ and by $diag\{\tilde{M}^{-1}, \tilde{M}^{-1}\}$, respectively. Make a congruence transformation to (4.32) by $diag\{\tilde{M}^{-T}, \tilde{M}^{-T}, \ldots, \tilde{M}^{-T}, I\}$.

Let

$$M_1 = U\tilde{M}^{-1}, \quad M_2 = \tilde{M}^{-1}, \quad P = \tilde{M}^{-T}\tilde{P}\tilde{M}^{-1}, \quad R_i = \tilde{M}^{-T}\tilde{R}_i\tilde{M}^{-1}$$
$$F_i = K_i\tilde{M}_i, \quad , S_i = \tilde{M}^{-T}\tilde{S}_i\tilde{M}^{-1}, \quad Y_i = \tilde{M}^{-T}\tilde{Y}_i\tilde{M}^{-1}, \quad Z_i = \tilde{M}^{-T}\tilde{Z}_i\tilde{M}^{-1}$$

It is easy to verify that

$$BKD_i\tilde{M} = B_iF_iE_i^T, \quad BK\tilde{M} = \sum_{i=1}^{N} B_iF_iE_i^T, \quad \tilde{M}^TD_i\tilde{M} = \tilde{M}^TE_iE_i^T\tilde{M}. \quad (4.33)$$

By Schur complement, one can find the conditions in (4.19)-(4.20) are satisfied. The result then follows from Proposition 4.1. □

Remark 4.4. *Proposition 4.2 gives a sufficient condition by which a desired $\mathcal{L}_2$ decentralized controller can be obtained for the decentralized event-triggered control system. One can see that the decentralized event-triggered transmission scheme and $\mathcal{L}_2$ controller can be designed simultaneously by Proposition 4.2, which implies that Proposition 4.2 can serve as an effective tool for co-design of the decentralized event-triggered transmission scheme and $\mathcal{L}_2$ controller, where transmission scheme (4.2) and controller (4.4) are to be designed simultaneously such that some desired communication and control performances are optimal or suboptimal in a way.*

4.4 Decentralized event-triggered control with a central controller

In this section, we will apply the proposed decentralized event-triggered transmission scheme to a linear time-invariant system with a centralized state feedback controller. The obtained results will be compared with some existing decentralized event-triggered control approaches presented in the literature.

Consider the LTI system described by

$$\dot{x}(t) = Ax(t) + Bu(t) + B_\omega\omega(t), \ t \geq 0 \quad (4.34)$$

where $x(t) \in \mathbb{R}^n$, $u(t) \in \mathbb{R}^m$ and $\omega(t) \in \mathcal{L}_2[0, \infty)$ are the system state, control input and the exogenous disturbance, respectively; A, B and B_ω are constant matrices with appropriate dimensions; and the initial condition of the system (4.34) is given by $x(0) = x_0$.

For clear elaboration, we take the following assumptions.

Assumption 4.1. *The system state $x(t)$ is not measured by a centralized sensor node. Instead, each component of the system state is measured by one sensor node from a group of physically distributed sensor nodes.*

Assumption 4.2. *All the n sensor nodes have the same sampling rate. The $x_i(t)$, the ith component of $x(t)$, is periodically sampled by sensor i with a sampling period $h > 0$, $i = 1, 2, \ldots, n$.*

Assumption 4.3. *Each sensor is collocated with an event-triggered transmitter which is used to determine whether or not the current sampled data should be transmitted to the controller based on the error between the current sampled data and the latest transmitted one.*

Assumption 4.4. *The centralized controller is event-driven. Time delays in the signal transmission and controller computation are not considered in this section.*

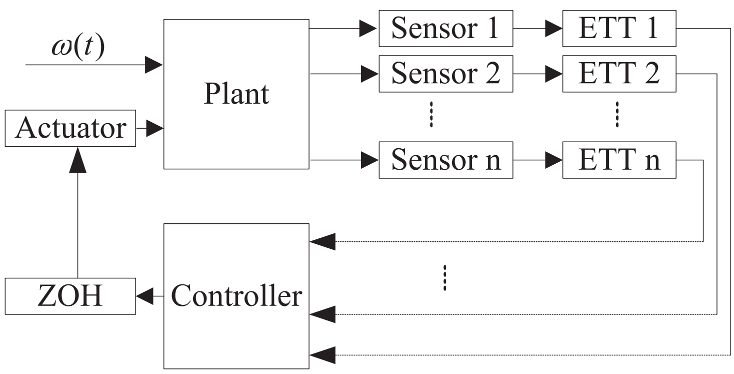

Figure 4.2: Decentralized event-triggered control with a central controller

Figure 4.2 gives a conceptual framework of the decentralized event-triggered control with a central controller. The broadcast release time sequence of the ith event-triggered transmitter is denoted as $\{t^i_{k_i}h\}^{\infty}_{k_i=1}$, which is generated according to the following event-triggered transmission scheme

$$t^i_{k_i+1}h = t^i_{k_i}h + \min_{l\in\mathbb{Z}^+}\{lh|(x_i(t^i_{k_i}h + lh) - x_i(t^i_{k_i}h))^2 > \delta_i x_i^2(t^i_{k_i}h + lh)\} \tag{4.35}$$

where $t^i_{k_i}h$ is the k_ith transmission time instant of the ith transmitter; $\mathbb{Z}^+$ is the set of positive integers; δ_i satisfying $\delta_i > 0$ is the threshold of the event-triggered transmission scheme.

In this section, we are interested in designing a controller in the form of

$$u(t) = K\begin{bmatrix} x(t^1_{k_1}h) & x(t^2_{k_2}h) & \cdots & x(t^n_{k_n}h) \end{bmatrix}^T,\ t \in [t_k, t_{k+1}) \tag{4.36}$$

where $K \in \mathbb{R}^{m\times n}$ is the controller gain to be determined;

$$t_k = \max_{i=1,2,\ldots,n}\{t^i_{k_i}h\}, \quad t_{k+1} = \min_{i=1,2,\ldots,n}\{t^i_{k_i+1}h\}. \tag{4.37}$$

Remark 4.5. *In the literature, a central controller is used in decentralized event-triggered control in [56–58], where each component of the system is event-triggered to be sampled and transmitted to a central controller. It is noted that although a positive lower bound of inter-transmission intervals is evaluated for each sensor, the controller computations may be executed arbitrarily close to each other in practical implementations.*

For $t \in [t_k, t_{k+1})$, let

$$\begin{aligned} \eta^i_k(t) &= t - t_k h - \max_{l\in\mathbb{Z}^+}\{lh|t^i_k h + lh \le t\} && (4.38)\\ e^i_k(t) &= x_i(t - \eta^i_k(t)) - x_i(t^i_{k_i}h), \quad i = 1, 2, \ldots, n. && (4.39)\end{aligned}$$

Then the control input $u(t)$ in (4.36) can be rewritten as

$$u(t) = K(\sum_{i=1}^{n} D_i x(t - \eta^i_k(t)) - e_k(t)), \quad t \in [t_k, t_{k+1}) \tag{4.40}$$

where

$$D_i = diag\{\underbrace{0, \ldots, 1}_{i}, \ldots, 0\}, \quad e_k(t) = [e^1_k(t), e^2_k(t), \ldots, e^n_k(t)]^T \tag{4.41}$$

Then the sampled-data error dependent closed-loop system can be obtained as

$$\dot{x}(t) = Ax(t) + BK\sum_{i=1}^{n} D_i x(t-\eta_k^i(t)) - BKe_k(t) + B_\omega \omega(t), t \in [t_k, t_{k+1}). \quad (4.42)$$

We supplement the initial condition of the system on $[-h, 0]$ as $x(t) = \varphi(t), t \in [-h, 0]$, where $\varphi(t)$ is a continuous function on $[-h, 0]$ and $\varphi(0) = x_0$.

We will first give an $\mathcal{L}_2$ stability criterion for the decentralized event-triggered control closed-loop system (4.42). Then a design method will be presented to obtain a central controller in the form of (4.36).

Proposition 4.3. *Given a scalar $\gamma > 0$, with transmission scheme (4.35), the system (4.42) is finite-gain $\mathcal{L}_2$ stable from ω to x with a gain less than γ, if there exist real matrices $P > 0$, $Q > 0$, $\{M_i = M_i^T\}_{i=1}^n$, $\{N_i\}_{i=1}^n$, Z_1, Z_2 with appropriate dimensions such that for all $\theta_i \in \{0, 1\}, i = 1, 2, \ldots, n$*

$$\begin{bmatrix} P + h\sum_{i=1}^n \theta_i M_i & h\Gamma_1\Theta \\ * & (h\Gamma_2 - I)\Theta + I \end{bmatrix} > 0 \quad (4.43)$$

$$\begin{bmatrix} \Pi_{11} & \Pi_{12} & 0 & \Pi_{14} & -Z_1^T BK & Z_1^T B_\omega \\ * & \Pi_{22} & 0 & \Pi_{24} & -Z_2^T BK & Z_2^T B_\omega \\ * & * & -Q & 0 & 0 & 0 \\ * & * & * & \Pi_{44} & 0 & 0 \\ * & * & * & * & -I & 0 \\ * & * & * & * & * & -\gamma^2 I \end{bmatrix} < 0 \quad (4.44)$$

where

$$\Gamma_1 = \begin{bmatrix} N_1 - M_1 & N_2 - M_2 & \cdots & N_n - M_n \end{bmatrix}$$

$$\Gamma_2 = diag\{M_1 - N_1 - N_1^T, \cdots, M_n - N_n - N_n^T\}, \quad \Theta = diag\{\theta_1 I, \theta_2 I, \ldots, \theta_n I\}$$

$$\Pi_{11} = A^T Z_1 + Z_1^T A + Q + I - \sum_{i=1}^n M_i, \quad \Pi_{12} = A^T Z_2 + P - Z_1^T + \sum_{i=1}^n \theta_i M_i$$

$$\Pi_{14} = -\Gamma_1 + Z_1^T BK \begin{bmatrix} D_1 & D_2 & \cdots & D_n \end{bmatrix}, \quad \Pi_{22} = -Z_2 - Z_2^T$$

$$\Pi_{24} = \Gamma_1\Theta + Z_2^T BK \begin{bmatrix} D_1 & D_2 & \cdots & D_n \end{bmatrix}$$

$$\Pi_{44} = -\Gamma_2 + diag\{\delta_1 D_1, \delta_2 D_2, \ldots, \delta_n D_n\}.$$

Proof. Denote the left side of (4.43) and (4.44) respectively as $\Gamma(\theta_1 h, \theta_2 h, \ldots, \theta_n h)$ and $\Pi(\theta_1 h, \theta_2 h, \ldots, \theta_n h)$. By mathematical induction, one can find that there exist

2^n functions $\alpha_{\underline{\theta_1\theta_2\cdots\theta_n}}(t) \geq 0$ such that

$$\sum_{\theta_1=0}^{1}\sum_{\theta_2=0}^{1}\cdots\sum_{\theta_n=0}^{1}\alpha_{\underline{\theta_1\theta_2\cdots\theta_n}}(t) \neq 0 \tag{4.45}$$

$$\begin{aligned}\Gamma(h-\eta_1(t), h-\eta_2(t), \ldots, h-\eta_n(t)) = &\sum_{\theta_1=0}^{1}\sum_{\theta_2=0}^{1}\cdots\sum_{\theta_n=0}^{1}\alpha_{\underline{\theta_1\theta_2\cdots\theta_n}}(t)\\ &\times\Gamma(\theta_1 h, \theta_2 h, \ldots, \theta_n h).\end{aligned} \tag{4.46}$$

It follows from (4.43), (4.45) and (4.46) that

$$\Gamma(h-\eta_1(t), h-\eta_2(t), \ldots, h-\eta_n(t)) > 0. \tag{4.47}$$

By the similar method and (4.44), one can find that

$$\Pi(\eta(t)) := \Pi(h-\eta_1(t), h-\eta_2(t), \ldots, h-\eta_n(t)) < 0. \tag{4.48}$$

Choose the following Lyapunov-Krasovskii functional candidate

$$\begin{aligned}V(t, x(t)) = x^T(t)Px(t) + \int_{t-h}^{t} x^T(s)Qx(s)ds + \sum_{i=1}^{n}(h-\eta_k^i(t))\begin{bmatrix} x(t) \\ x(t-\eta_k^i(t)) \end{bmatrix}^T\\ \times\begin{bmatrix} M_i & N_i - M_i \\ * & M_i - N_i - N_i^T \end{bmatrix}\begin{bmatrix} x(t) \\ x(t-\eta_k^i(t)) \end{bmatrix}.\end{aligned} \tag{4.49}$$

It follows from (4.47) that there exists a positive scalar $\kappa > 0$ such that $V(t, x(t)) > \kappa x^T(t)x(t)$.

Notice that the decentralized event-triggered transmission scheme (4.2) ensures that

$$e_k^T(t)e_k(t) \leq \sum_{i=1}^{n}\delta_i x^T(t-\eta_i(t))D_i x(t-\eta_i(t)) \tag{4.50}$$

with D_i given in (4.41). Then following the similar line as in the proof of Proposition 2.1, one can complete the proof of this proposition. □

Remark 4.6. *One can see from the $\mathcal{L}_2$ stability analysis result that if a controller gain matrix K in (4.36) is pre-given as in [55–57], a group of parameters for the decentralized event-triggered transmission scheme (4.35) can be obtained by Proposition 4.3 to reduce the transmission traffic. This will be illustrated later by a simulation example.*

Based on the stability criterion given above, we are now in a position to design an $\mathcal{L}_2$ controller in the form of (4.36) such that the sampled-data error dependent closed-loop system (4.42) is finite-gain $\mathcal{L}_2$ stable from ω to x with a gain less than γ.

It can be seen from (4.44) that Z_2 in Proposition 4.3 is nonsingular. Then we have $Z_1 = UZ_2$ with $U = Z_1 Z_2^{-1}$.

Let $\tilde{Z} = Z_2^{-1}$. Partition $\tilde{Z}$ and K respectively as

$$\tilde{Z} = \begin{bmatrix} \tilde{Z}_1^T & \tilde{Z}_2^T & \cdots & \tilde{Z}_n^T \end{bmatrix}^T, \quad K = \begin{bmatrix} K_1 & K_2 & \cdots & K_n \end{bmatrix}. \tag{4.51}$$

Let

$$E_i = D_i \begin{bmatrix} 1 & 1 & \cdots & 1 \end{bmatrix}^T, \quad F_i = K_i \tilde{Z}_i, \quad i = 1, 2, \ldots, n. \tag{4.52}$$

It is easy to verify that

$$\tilde{Z}^T D_i \tilde{Z} = \tilde{Z}^T E_i E_i^T \tilde{Z} \tag{4.53}$$

$$K D_i \tilde{Z} = F_i, \quad K\tilde{Z} = \sum_{i=1}^{n} F_i. \tag{4.54}$$

By some matrix congruence transformation and linearization technique, we can obtain the following design result.

Proposition 4.4. *Given a scalar $\gamma > 0$, the closed-loop system (4.42) is finite-gain $\mathcal{L}_2$ stable from ω to x with a gain less than γ, if there exist real matrices $\tilde{P} > 0$, $\tilde{Q} > 0$, $\left\{\tilde{M}_i = \tilde{M}_i^T\right\}_{i=1}^n$, $\left\{\tilde{N}_i\right\}_{i=1}^n$, $\{F_i\}_{i=1}^n$, $\tilde{Z}$ and U with appropriate dimensions and real constants $\left\{\tilde{\delta}_i > 0\right\}_{i=1}^n$, ν such that for all $\theta_i \in \{0, 1\}, i = 1, 2, \ldots, n$*

$$\begin{bmatrix} \tilde{P} + h\sum_{i=1}^n \theta_i \tilde{M}_i & h\tilde{\Gamma}_1 \Theta \\ * & h\tilde{\Gamma}_2\Theta + h\tilde{\mathcal{P}}(I - \Theta) \end{bmatrix} > 0 \tag{4.55}$$

$$\begin{bmatrix} \tilde{\Pi}_{11} & 0 & \tilde{\Pi}_{13} & \tilde{\Pi}_{14} & 0 \\ * & -\tilde{Q} & 0 & 0 & 0 \\ * & * & -\tilde{\Gamma}_2 & 0 & \tilde{\Pi}_{35} \\ * & * & * & \tilde{\Pi}_{44} & 0 \\ * & * & * & * & \tilde{\Pi}_{55} \end{bmatrix} < 0 \tag{4.56}$$

where

$$\begin{aligned}
&\tilde{\Gamma}_1 = \begin{bmatrix} \tilde{N}_1 - \tilde{M}_1 & \tilde{N}_2 - \tilde{M}_2 & \cdots & \tilde{N}_n - \tilde{M}_n \end{bmatrix} \\
&\tilde{\Gamma}_2 = diag\{\tilde{M}_1 - \tilde{N}_1 - \tilde{N}_1^T, \cdots, \tilde{M}_n - \tilde{N}_n - \tilde{N}_n^T\} \\
&\tilde{\Pi}_{11} = \begin{bmatrix} \Omega & \tilde{P} + \tilde{Z}^T A^T - U^T \tilde{Z} + h\sum_{i=1}^n \theta_i \tilde{M}_i \\ * & -\tilde{Z} - \tilde{Z}^T \end{bmatrix} \\
&\tilde{\Pi}_{13} = \begin{bmatrix} -\tilde{\Gamma}_1 + U^T B \begin{bmatrix} F_1 & F_2 & \cdots & F_n \end{bmatrix} \\ h\tilde{\Gamma}_1 \Theta + B \begin{bmatrix} F_1 & F_2 & \cdots & F_n \end{bmatrix} \end{bmatrix} \\
&\tilde{\Pi}_{14} = \begin{bmatrix} -U^T B \sum_{i=1}^n F_i & U^T B_\omega & \tilde{Z}^T \\ -B\sum_{i=1}^n F_i & B_\omega & 0 \end{bmatrix} \\
&\tilde{\Pi}_{35} = diag\{\tilde{Z}^T E_1, \tilde{Z}^T E_2, \ldots, \tilde{Z}^T E_n\}, \quad \tilde{\Pi}_{55} = diag\{-\tilde{\delta}_1, -\tilde{\delta}_2, \ldots, -\tilde{\delta}_n\} \\
&\tilde{\Pi}_{44} = diag\{\nu^2 I - \nu\tilde{Z} - \nu\tilde{Z}^T, -\gamma^2 I, -I\} \\
&\Theta = diag\{\theta_1 I, \theta_2 I, \ldots, \theta_n I\}, \quad \tilde{\mathcal{P}} = diag\{\tilde{P}, \tilde{P}, \ldots, \tilde{P}\} \\
&\Omega = \tilde{Q} + U^T A\tilde{Z} + \tilde{Z}^T A^T U - \sum_{i=1}^n \tilde{M}_i.
\end{aligned}$$

The controller parameter matrix K and the decentralized event-triggered transmitter parameters $\{\delta_i > 0\}_{i=1}^n$ are respectively given by

$$K = \sum_{i=1}^n F_i \tilde{Z}^{-1}, \quad \delta_i = 1/\tilde{\delta}_i, \; i = 1, 2, \ldots, n. \tag{4.57}$$

Remark 4.7. *Proposition 4.4 gives a sufficient condition by which the decentralized event-triggered transmission scheme and $\mathcal{L}_2$ controller may be designed simultaneously, which will be demonstrated in the simulation example in the following section. This implies that Proposition 4.4 can be used as a tool for co-design of the decentralized event-triggered transmission scheme and a central $\mathcal{L}_2$ controller.*

4.5 An example

Consider the following Batch Reactor system which was used as a benchmark example in [56, 58, 88, 89]:

$$\dot{x}(t) = \begin{bmatrix} 1.380 & -0.208 & 6.715 & -5.676 \\ -0.581 & -4.290 & 0 & 0.675 \\ 1.067 & 4.273 & -6.654 & 5.893 \\ 0.048 & 4.273 & 1.343 & -2.104 \end{bmatrix} x(t) + \begin{bmatrix} 0 & 0 \\ 5.679 & 0 \\ 1.136 & -3.146 \\ 1.136 & 0 \end{bmatrix} u(t). \tag{4.58}$$

We will first use this example to compare the proposed approach in this chapter with two other decentralized event-triggered control strategies presented in [56, 58] in terms of average transmission interval. Then this unstable system will be used to demonstrate the effectiveness of the proposed $\mathcal{L}_2$ controller design approach.

4.5.1 Average transmission interval

For the purpose of comparison, we take the following state feedback controller which is used in [56, 58]:

$$K = \begin{bmatrix} 0.1006 & -0.2469 & -0.0952 & -0.2447 \\ 1.4099 & -0.1966 & 0.0139 & 0.0823 \end{bmatrix}. \tag{4.59}$$

With this controller, by Proposition 4.3, we can get a group of parameters for decentralized event-triggered transmission scheme (4.35) as

$$\delta_1 = 0.1, \quad \delta_2 = 0.9, \quad \delta_3 = 0.4, \quad \delta_4 = 0.5$$

which ensures that system (4.58) with controller (4.59) is asymptotically stable.

For the same control system (4.58)-(4.59), Table 4.1 shows the average transmission time intervals by using the three different decentralized event-triggered transmission schemes, all of which guarantee the asymptotic stability of the corresponding decentralized event-triggered closed-loop system. Compared with the result in [56], one can see that the average release time intervals in three sensors are increased. And the total average release time intervals in the four sensors are increased from $0.307s$ to $0.433s$. Figure 4.3 and Figure 4.4 illustrate the transmission release time instants distribution in the four sensor nodes by the decentralized event-triggered transmission scheme (4.35). The corresponding state response is plotted in Figure 4.5.

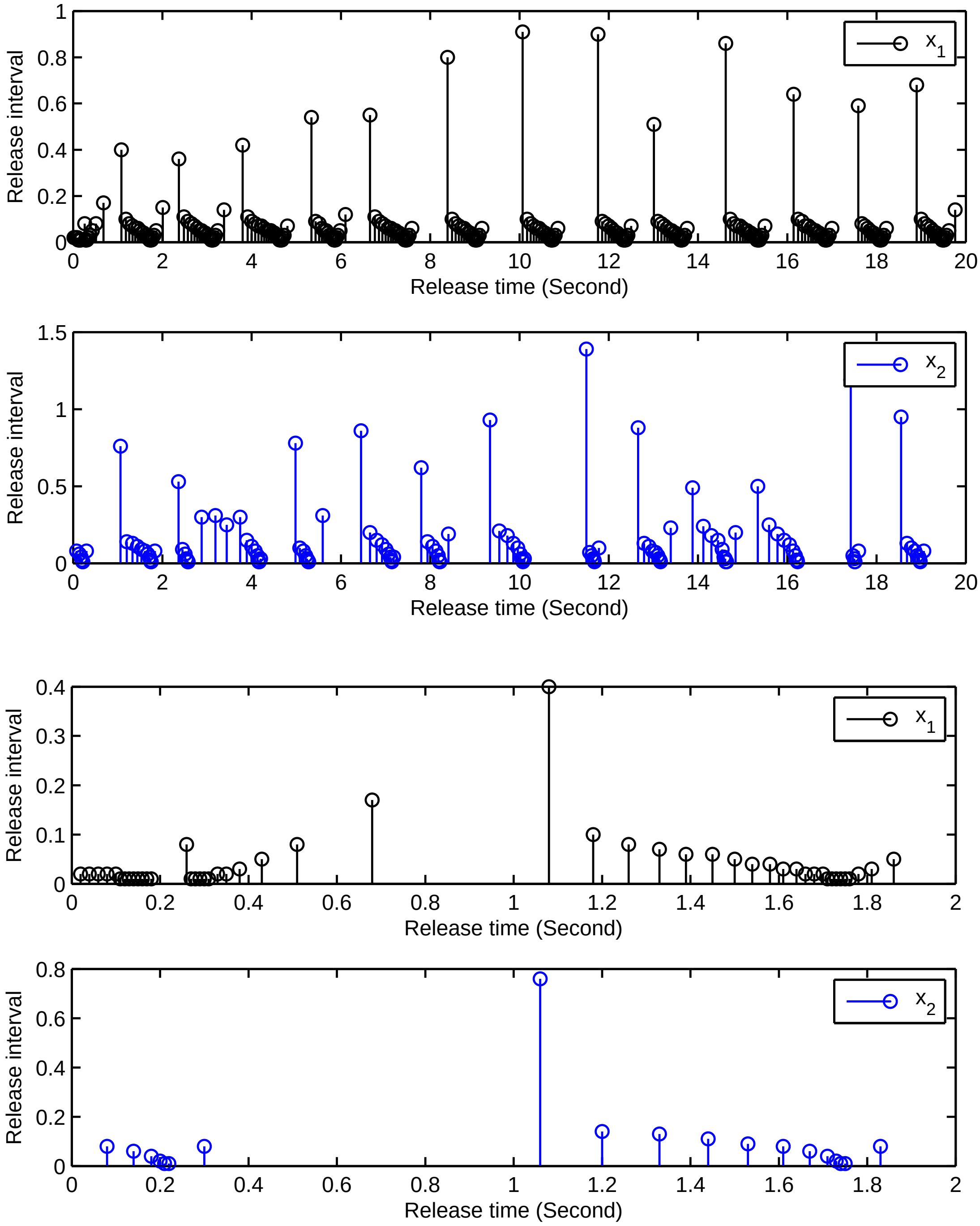

Figure 4.3: Release time intervals of $x_1(t)$ and $x_2(t)$ in Section 4.5.1.

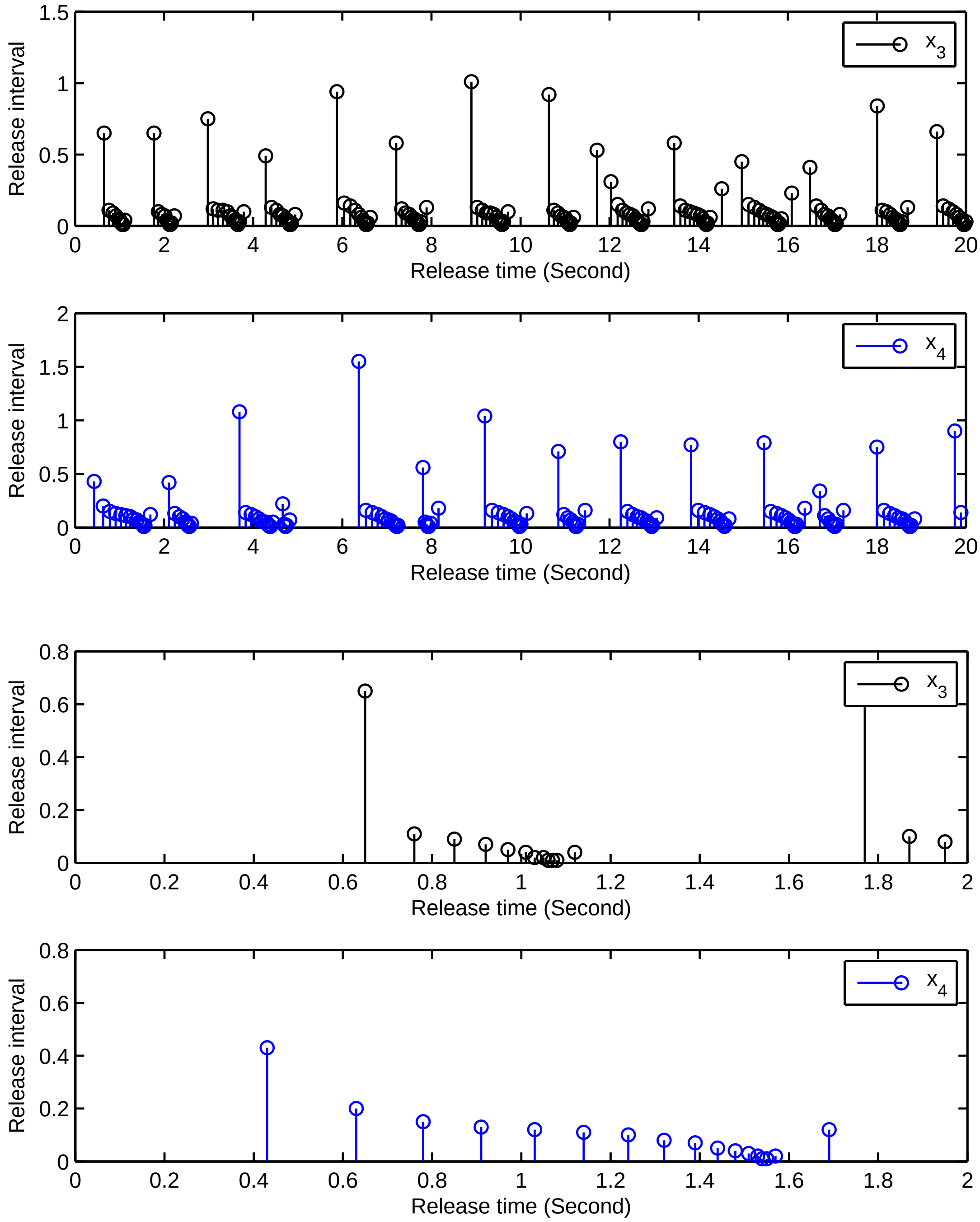

Figure 4.4: Release time intervals of $x_3(t)$ and $x_4(t)$ in Section 4.5.1.

Table 4.1: Average transmission intervals in the simulation

	$\bar{t}_1$	$\bar{t}_2$	$\bar{t}_3$	$\bar{t}_4$	$\sum_{i=1}^{4} \bar{t}_i$
The scheme in [58]	0.025s	0.028s	0.035s	0.034s	0.122s
The scheme in [56]	0.121s	0.073s	0.056s	0.057s	0.307s
The scheme in this chapter	0.061s	0.154s	0.100s	0.118s	0.433s

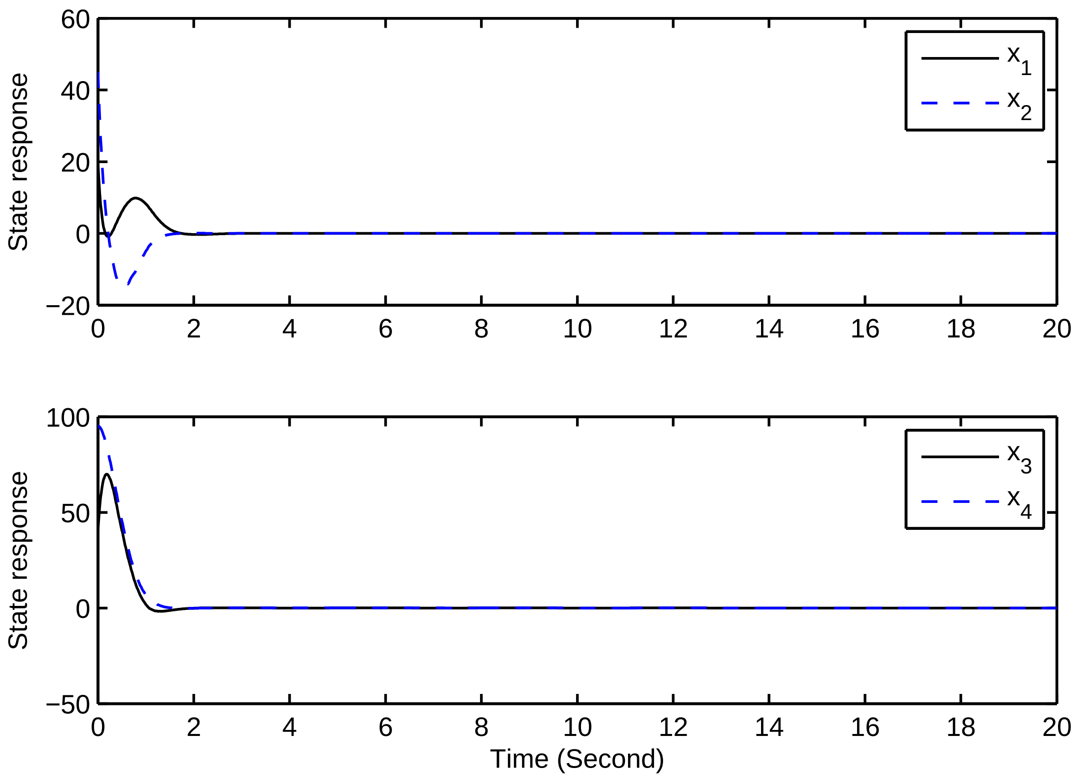

Figure 4.5: State response of the closed-loop system without disturbance.

4.5.2 Design of the central controller and the decentralized event-triggered transmission scheme

In this subsection, we introduce the following disturbance to illustrate the proposed design method presented in Proposition 4.4:

$$\omega(t) = \frac{1}{1+t}, \quad B_\omega = \begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix}^T. \tag{4.60}$$

We choose $U = 0.3I$, the sampling period $h = 10$ms. By using Proposition 4.4, one can find that the system (4.58) with controller (4.36) is finite-gain $\mathcal{L}_2$ stable with a gain less than $\gamma = 20$. A feasible solution to (4.55)-(4.56) leads to the following controller parameter matrix and the decentralized event-triggered transmission scheme parameters

$$K = \begin{bmatrix} 1.0923 & -0.73439 & -0.48149 & -1.8008 \\ 5.4158 & 0.24353 & 1.9154 & -0.67662 \end{bmatrix}$$
$$\delta_1 = 0.015, \quad \delta_2 = 0.014, \quad \delta_3 = 0.014, \quad \delta_4 = 0.015.$$

With controller (4.36) and decentralized event-triggered transmission scheme (4.35), the event-triggered broadcast release time intervals in all the four sensors are shown in Figure 4.6 and Figure 4.7. The state response of closed-loop system is shown in Figure 4.8.

Within the simulation period $T_s = 20s$, each component of the system state is sampled 2000 times while the numbers of the transmitted sampled data are 164, 136, 158, 139, respectively. It is shown that the average release time intervals in the four sensors are respectively $\bar{t}_1 = 0.121s, \bar{t}_2 = 0.146s, \bar{t}_3 = 0.126s, \bar{t}_4 = 0.143s$, all of which are much larger than the sampling period $h = 0.01s$. Compared with the periodic transmission scheme, it is clear that a large proportion of the required transmission resources may be saved, which shows the effectiveness of the approach.

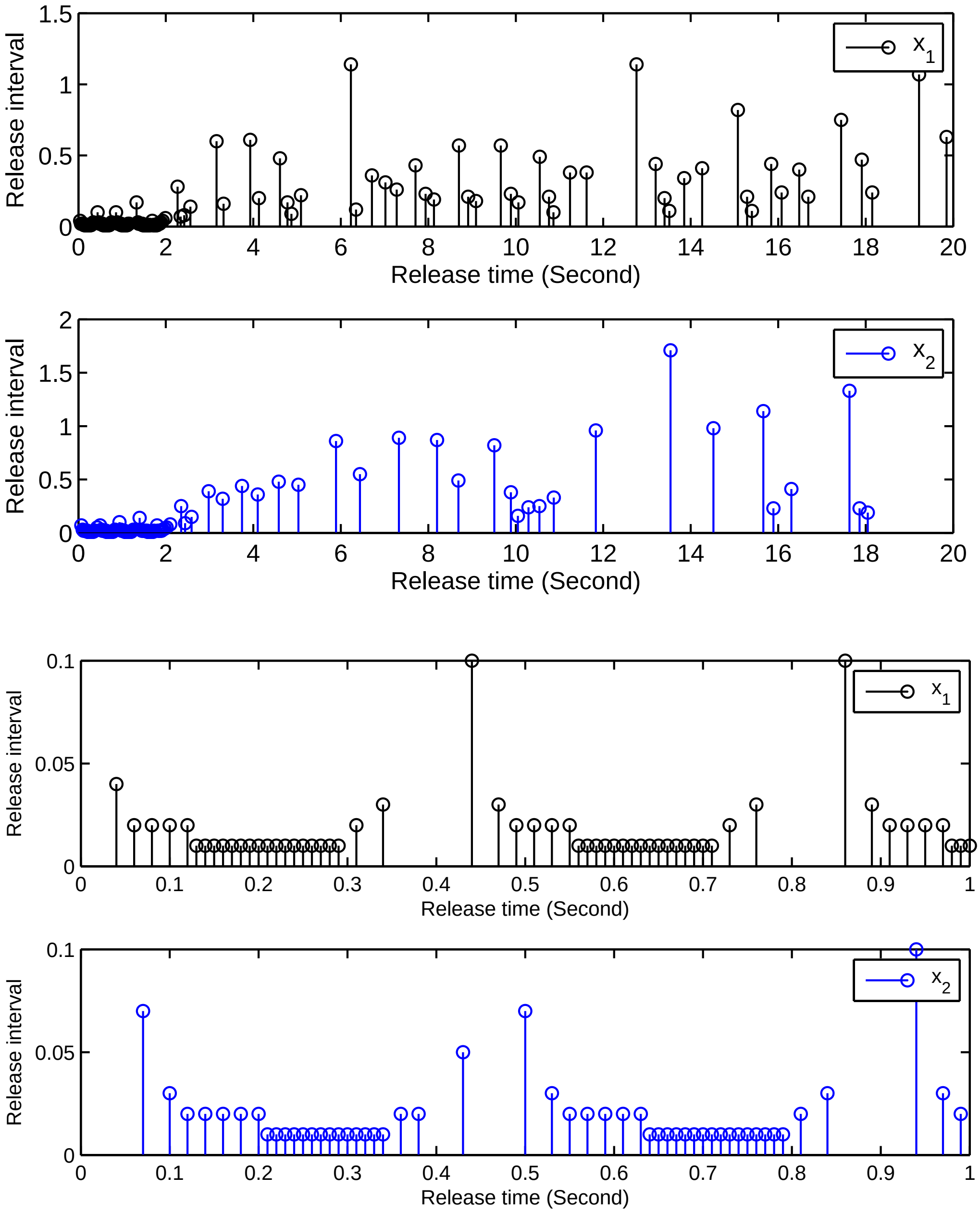

Figure 4.6: Release time intervals of $x_1(t)$ and $x_2(t)$ in Section 4.5.2.

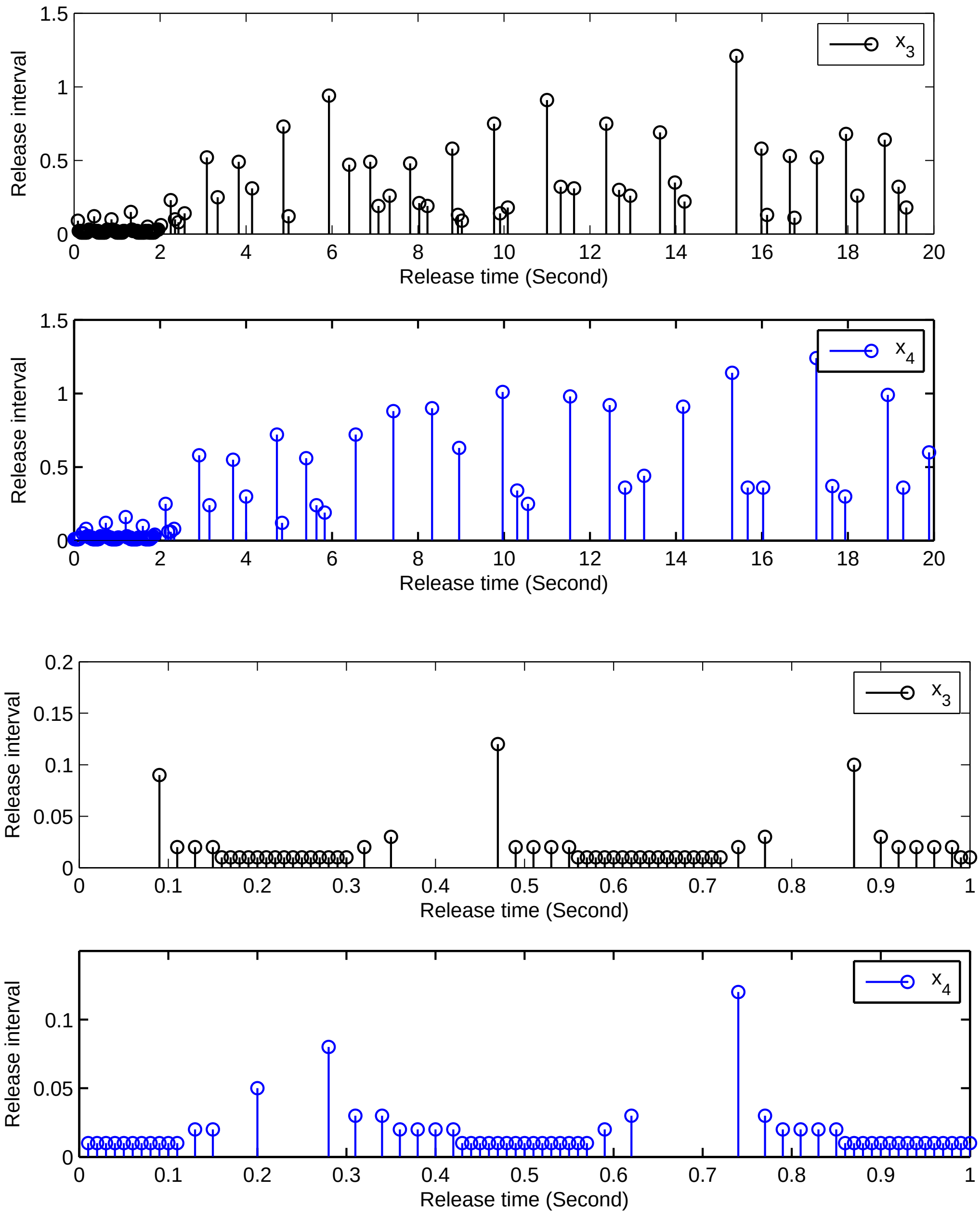

Figure 4.7: Release time intervals of $x_3(t)$ and $x_4(t)$ in Section 4.5.2.

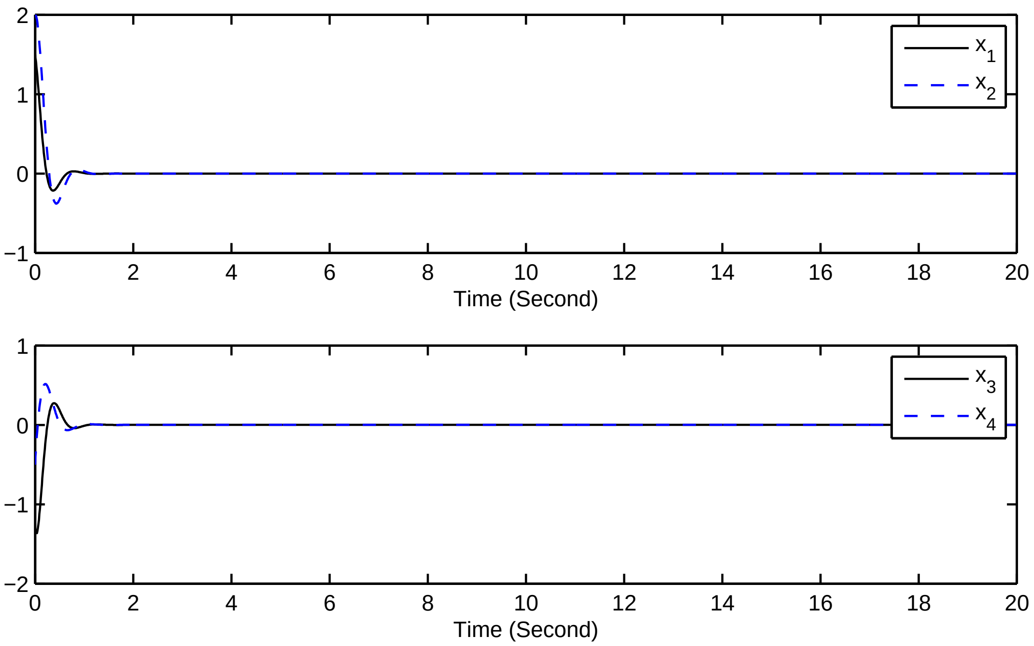

Figure 4.8: State response of the closed-loop system in Section 4.5.2.

4.5.3 Design of the decentralized controller

In this subsection, by using Proposition 4.2, we will design an $\mathcal{L}_2$ decentralized controller in the form of (4.4) for the following system:

$$\dot{x}(t) = Ax(t) + \sum_{i=1}^{2} B_i u_i(t) + B_\omega \omega(t) \tag{4.61}$$

where $\omega(t)$ and B_ω are given by (4.60); and

$$A = \begin{bmatrix} 1.380 & -0.208 & 6.715 & -5.676 \\ -0.581 & -4.290 & 0 & 0.675 \\ 1.067 & 4.273 & -6.654 & 5.893 \\ 0.048 & 4.273 & 1.343 & -2.104 \end{bmatrix}, \ B_1 = B_2 = \begin{bmatrix} 0 & 0 \\ 5.679 & 0 \\ 1.136 & -3.146 \\ 1.136 & 0 \end{bmatrix}.$$

It is assumed that the components of system state $x(t)$ are lumped as

$$x_{\mathcal{N}_1}(t) = \text{col}\{x_1(t), x_2(t)\}, \quad x_{\mathcal{N}_2}(t) = \text{col}\{x_3(t), x_4(t)\}$$

for the purpose of sampling. The sampling periods of the two sensor nodes are assumed to be $h_1 = 10ms$ and $h_2 = 5ms$, respectively. The parameters in decentralized event-triggered transmission scheme (4.2) are given as $\delta_1 = 0.02, \delta_2 = 0.015$.

The upper bounds of network-induced delay are assumed to be $\bar{\tau}_1 = 0.01s$ and $\bar{\tau}_2 = 0.005s$.

In virtue of Proposition 4.2, it can be verified that system (4.61) with the decentralized event-triggered transmission scheme is finite-gain $\mathcal{L}_2$ stable from ω to x with a gain less than $\gamma = 200$. And a desirable decentralized controller in the form of (4.4) for the system can be obtained as

$$K = diag\{K_1, K_2\} = diag\{\begin{bmatrix} 62.195 & -6.1501 \\ 8.318 & -0.30899 \end{bmatrix}, \begin{bmatrix} 269.45 & -228.08 \\ 38.085 & -31.663 \end{bmatrix}\}. \quad (4.62)$$

We take the system initial state as $x_0 = \begin{bmatrix} 1.5 & 2 & -1.3 & -0.5 \end{bmatrix}^T$ for simulation. The state response of the decentralized event-triggered controlled system is ploted in Figure 4.9, which illustrates that the prescribed $\mathcal{L}_2$ control performance is achieved. Figure 4.10 shows the event-triggered transmission time instants distribution of the two sensor nodes in the the simulation. As is illustrated in the figure, some transmission intervals are much longer than the proposed sampling periods, which shows the effectiveness of the decentralized event-triggered scheme.

Within the simulation time $T = 30s$, the state sub-vector $x_{\mathcal{N}_1}(t)$ is sampled 3000 times while only 1534 sampled signals are transmitted to controller station K_1; $x_{\mathcal{N}_2}(t)$ is sampled 6000 times and only 534 of the sampled signals are transmitted via a network to controller station K_2. It can be seen that in this example only a small proportion of the sampled data are required to be fed back to achieve and maintain the prescribed control performance. It is clear the proposed decentralized event-triggered transmission scheme provides a way of selecting the desired feedback data, by which the limited network resources can be substantially saved.

4.6 Conclusion

A decentralized event-triggered transmission scheme has been proposed based on asynchronous sampling. An input delay approach is employed to model the decentralized event-triggered control system with either a decentralized controller or a

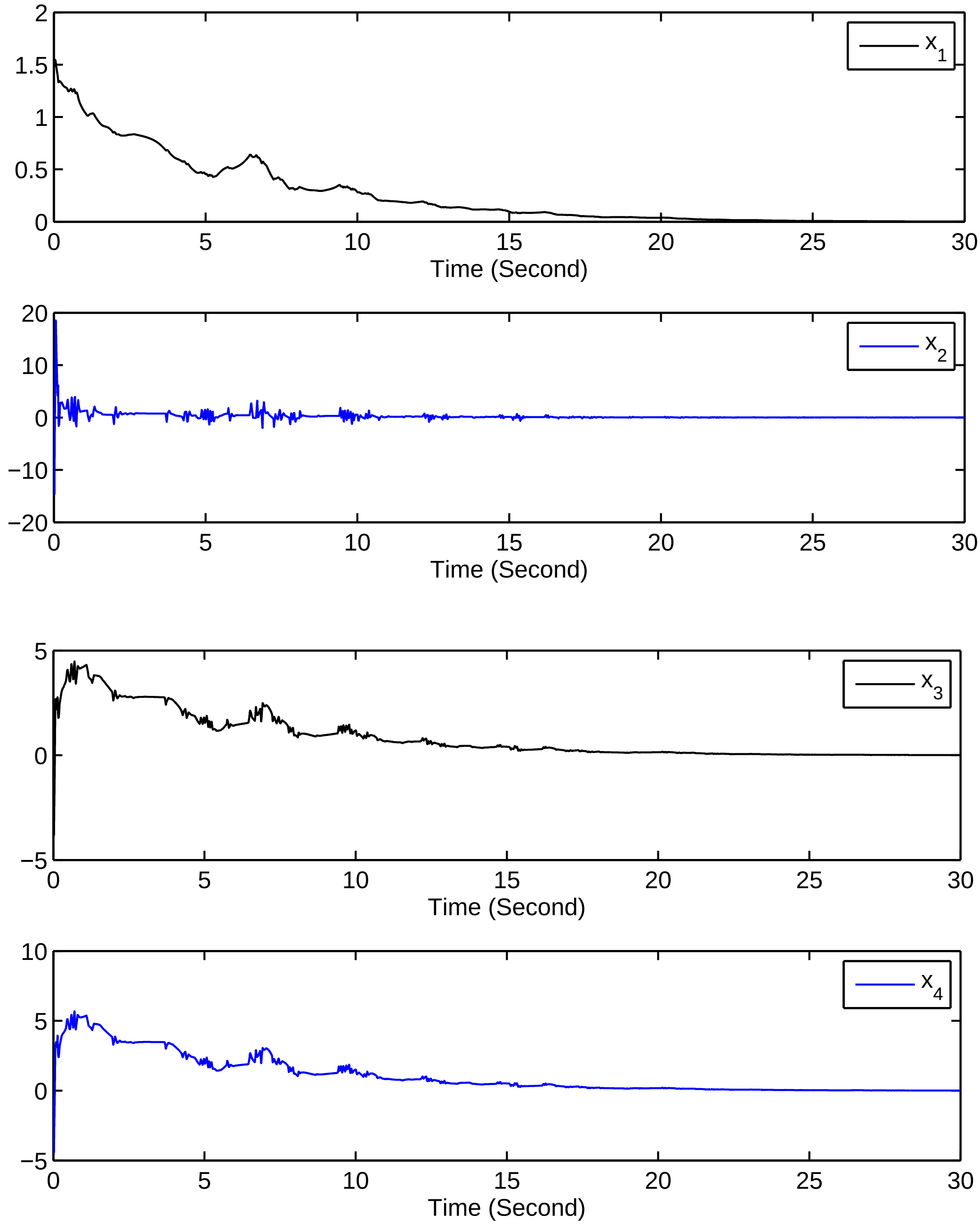

Figure 4.9: State response of the closed-loop system in Section 4.5.3.

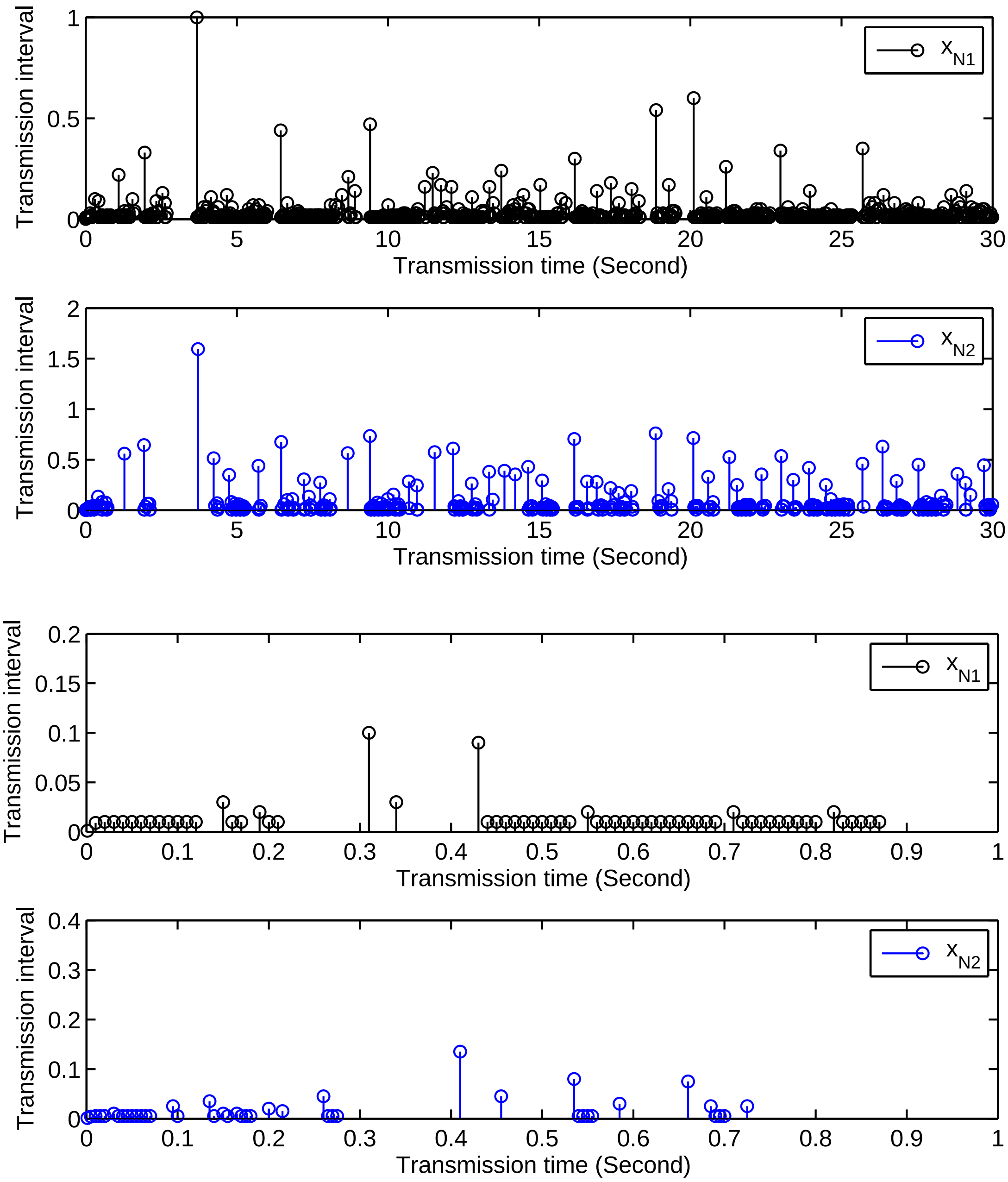

Figure 4.10: Transmission intervals of the system in Section 4.5.3.

central controller. By using a switching Lyapunov-Krasovskii functional, the input delay and network-induced delays are well characterized in the $\mathcal{L}_2$ stability analysis for the event-triggered networked control system with a decentralized controller. The $\mathcal{L}_2$ controller design has been considered in both of the cases. The numerical examples illustrate the effectiveness of the obtained results.

Chapter 5

Event-triggered control for networked control systems with packet dropouts

5.1 Introduction

As we demonstrated in previous chapters, there are some appealing advantages of implementing an event-triggered transmission scheme in a networked control system (NCS). For example, the required network bandwidth of the NCS could be effectively reduced while some desired control performance can still be maintained [42]; the average data transmission interval can be substantially increased and the network traffic load could be reduced, which may improve the network quality of service with respect to alleviating some network-induced problem such as network congestion, network-induced delays, data packet dropouts, etc.

It is noted that almost all of the existing results on event-triggered control are obtained under the assumption that there is no packet dropout during the process of data transmission. However, it is well known that packet dropouts are usually unavoidable over network transmissions especially when a shared network channel is employed, as is stated in Chapter 1. In fact, the issue of analysis and design of event-triggered NCSs is very challenging when random packet dropouts are taken into consideration. Since the sampled data is event-triggered to be transmitted, which implies that each of event-triggered data can bring important information about the plant and it is necessary to use the event-triggered signal for feedback control. Therefore, data packet dropouts may deteriorate system performances more severely in the event-triggered NCSs than in the time-triggered NCSs. What makes the situation worse is that in most of the event-triggered sampling/tranmsission schemes (see, for example, [40–42, 55]), the event is designed based on the error between the current measurement and the latest event-triggered measurement and this relationship of dependence is utilized in the system analysis. Accordingly, when an event-triggered signal is lost in an NCS, the relationship between consecutive successfully transmitted signals is not available and the system analysis becomes more difficult as a result. To deal with packet dropouts in event-triggered NCSs is an important motivation of this chapter.

The results on event-triggered NCSs with data packet dropouts are very few in the existing literature. To deal with packet dropouts in event-triggered distributed networked control systems, a distributed event-triggered broadcasting scheme is first presented in [41] for the distributed system without packet dropouts, then a more conservative event-triggered scheme with a smaller threshold was given to tolerate some limited packet dropouts. It is noted that with the proposed method in [41], when an event-triggered signal gets lost, the following data transmission still depends on the error between current data and the lost data. By this way, the effectiveness of the transmission scheme may be reduced although the measurement error is still bounded by the original threshold. Following the line of [41], the distributed event-triggered control with packet dropouts is considered in [96] with a static event-triggered condition, by which a new sampled data is generated and transmitted when the norm of the local measurement error exceeds a given positive constant. Based on the event-triggered communication scheme, two transmission protocols are proposed in the case when a data packet gets lost: one is to retransmit the lost data after a waiting period and the other one is to transmit the current data after a waiting time, both of which can only make the state of each subsystem converge to a small region due to the static event-triggered condition.

In this chapter, we are concerned with event-triggered control systems with packet dropouts. A compensation scheme is proposed to deal with the packet dropouts occurred in an event-triggered control system. Whether or not the current sampled data should be transmitted via a network to the controller is determined by an event generator according to a pre-given event-triggered communication scheme. Each sampled data is also put in an FIFO (first in, first out) queue implemented by a buffer. The number of sampled signals stored in the queue in this thesis depends on the maximum transmission interval of the event-triggered NCS with packet lossless transmissions. When a data packet gets lost during the network transmission, the transmitter will retrieve the lost data stored in the buffer and retransmit it to the controller. Compared with some existing results, the proposed approach can be directly used to study NCSs with data dropouts while the event-triggered broadcasting scheme given in [41] is based on an original event-triggered scheme (when no dropouts occur) and a modified event-triggered scheme which takes a smaller threshold to tolerate some limited packet dropouts. On the other hand, from the event-triggered control point of view, one can find that an event-triggered measurement usually represents a substantial change occurred in the system state. Therefore, if the event-triggered measurement gets lost during the transmission, it is important to retrieve and retransmit the lost event-triggered data rather than to wait for the next event-triggered one. In addition, an $\mathcal{L}_2$ controller can be designed under consideration of the event-triggered communication scheme. While in most of the existing results, the event-triggered mechanism is first designed based on a pre-given controller, which is designed without considering the event-triggered mechanism.

The organisation of the remaining chapter is as follows. Section 5.2 first presents the model of an event-triggered NCS and a compensation scheme for packet dropouts, then the $\mathcal{L}_2$ control problem for the resulted system with packet dropouts is formulated. The $\mathcal{L}_2$ stability analysis for the event-triggered NCS is carried out in Section 5.3. The stability criterion is then employed in Section 5.4 to design a feasible $\mathcal{L}_2$ controller. A numerical example is given in Section 5.5 to illustrate the effectiveness of the proposed approach. This chapter is concluded in Section 5.6.

5.2 Problem statement

This section will present an event-triggered networked control system with packet dropouts and formulate the $\mathcal{L}_2$ control problem.

5.2.1 The model of a networked control system

A conceptual framework of the NCS considered in the chapter is illustrated in Figure 5.1. The plant to be controlled is modeled by a linear time-invariant system described by

$$\dot{x}(t) = Ax(t) + Bu(t) + B_\omega \omega(t), \quad t \geq t_0 \tag{5.1}$$

where $x(t) \in \mathbb{R}^n$, $u(t) \in \mathbb{R}^m$ and $\omega(t) \in \mathcal{L}_2[0, \infty)$ are the system state, control input and exogenous disturbance, respectively; A, B and B_ω are constant matrices with appropriate dimensions; the initial condition of system (5.1) is given by $x(t_0) = x_0$.

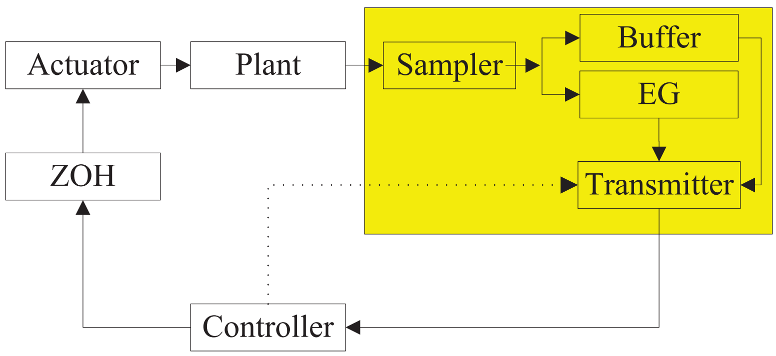

Figure 5.1: An event-triggered NCS with packet dropouts

The system state $x(t)$ is assumed to be fully available for feedback and it is periodically sampled with a sampling period $h > 0$. For the efficient use of the transmission resources, instead of transmitting all the sampled data through a network to a controller, an event generator (EG) is employed in this chapter to decide which sampled data should be transmitted according to the following event-triggered communication scheme

$$i_{k+1}h = i_k h + \min_{l \in \mathbb{Z}^+}\{lh | e^T(r_{k,l}h)\Phi e(r_{k,l}h) \geq \delta x^T(i_k h)\Phi x(i_k h)\} \tag{5.2}$$

where $i_k h$ is the time instant when the kth sampled measurement is released by the event generator; $\mathbb{Z}^+$ is the set of positive integers; the positive matrix Φ is a weighting matrix; δ satisfying $0 < \delta < 1$ is the threshold of the event-triggered communication scheme; and

$$r_{k,l}h = i_k h + lh, \quad e(r_{k,l}h) = x(r_{k,l}h) - x(i_k h). \tag{5.3}$$

In this chapter, we are interested in designing the following controller

$$u(t) = Kx(i_k h), \qquad t \in [i_k h, i_{k+1}h). \tag{5.4}$$

where $K \in R^{m \times n}$ is the controller gain to be determined.

It is noted that the released sampled data may get lost during the network transmission. To deal with the problem, we will propose a compensation scheme for packet dropouts. To convey the idea clearly, the network-induced delays are not taken into consideration in this chapter. Once the controller receives $x(i_k h)$, it will send a reception acknowledgement packet (ACK) back to the transmitter. The size of the ACK is very small and it enjoys a first priority of transmission privilege as in the TCP-like communication protocols [93]. Therefore, it is reasonable to assume that transmission of the ACK is error-free [94].

5.2.2 Compensation of packet dropouts

It can be seen from the event-triggered communication scheme (5.2) that: i) the event-triggered sampled data is usually important for feedback control since the error between two consecutive event-triggered sampled data must exceed some threshold; ii) all of the event-triggered sampled data is correlated through the event threshold and this relationship is usually utilized in analysis of the event-triggered control system. On the other hand, there is no strong relationship between two consecutive transmitted sampled data as in time-triggered systems. Therefore, the problem of packet dropouts in an event-triggered NCS is much more complicated than that in a time-triggered NCS.

In this section, we will propose a compensation scheme for packet dropouts occurred in the event-triggered NCS presented previously. The basic idea of the compensation scheme is that: in the presence of a packet dropout, the transmitter can retrieve the lost transmitted data from a queue of sampled data stored in a buffer and retransmit the sampled data. As shown in Figure 5.1, once the system state is sampled, it is checked by the event generator whether or not the sampled data should be transmitted. At the same time, the sampled data is pushed into an FIFO (first in, first out) queue implemented in the buffer. Therefore, the buffer is updated at each sampling time instant. The number of sampled signals stored in the queue of the buffer is determined by the maximum transmission interval of the event-triggered NCS. Then the transmitter can retrieve the lost data from the buffer when a data packet get lost. In this chapter, we take this number as

$$N := N_k + 1 \tag{5.5}$$

where N_k is the number of sampling periods during the maximum transmission interval of the event-triggered system without packet dropout occurred. The queue length of the buffer given in (5.5) is chosen such that the transmitter could find the lost sampled data in the buffer in the presence of a packet dropout.

As is shown in Figure 5.1, not all of the sampled data are transmitted through a network to the controller although the transmitter can get access to all of the sampled data through the buffer. When an event-triggered sampled signal gets lost during the transmission, the controller will not send the ACK for the transmission. Then the transmitter retrieves the lost data from the buffer and retransmits it to the controller.

Remark 5.1. *It is noted that the queue length of the buffer can not be set infinite long for the transmitter to retrieve the lost data when a packet gets lost. In this chapter, the queue length is predicted as N given by (5.5), which is expected to be enough for the dynamical storage of the sampled data at the normal situation. As we know, packet dropouts occur in an NCS occasionally. The probability of packet dropouts occurred in an event-triggered NCS may become smaller since less data packets are required to be transmitted due to the event-triggered transmission scheme. Therefore, one can see that the probability of the occurrence of N consecutive packet dropouts in an event-triggered NCS can be very small. On the other hand, the occurrence of the small-probability event probably implies that the network is congested or other physical problem occurs, which will not be considered in the thesis.*

Remark 5.2. *When network transmission delays are taken into consideration in the event-triggered control system, the proposed method can still apply by appropriately estimating the delay bounds of the transmission of sampled data and the ACK, respectively, according to the practical situation. Then one can set up transmission priority in the transmitter for the event-triggered and compensated sampled data. The queue length of the buffer should be adjusted accordingly.*

5.2.3 Problem formulation

For the convenience of system analysis, let

$$\tilde{r}_{k,l}h = \begin{cases} i_k h, & l = 0 \\ r_{k,l}h, & 1 \leq l < i_{k+1} - i_k. \end{cases} \tag{5.6}$$

Then there is a one-to-one correspondence between the sampling time instants sequences $\{kh\}_{k=i_1}^{\infty}$ and $\{\tilde{r}_{k,l}h\}_{k=1}^{\infty}$ as the system evolves. The interval $[i_k h, i_{k+1}h)$ can be written as

$$[i_k h, i_{k+1}h) = \bigcup_{j=1}^{l_k} I_{k,j} \tag{5.7}$$

where

$$I_{k,j} = [i_k h + (j-1)h, i_k h + jh), \quad j = 1, 2, \dots, l_k \tag{5.8}$$

$$l_k = i_{k+1} - i_k, \quad k = 1, 2, \dots. \tag{5.9}$$

Now, we define two functions $\eta(t)$ and $e(t)$ on $[i_k h, i_{k+1}h)$ as

$$\eta(t) := \begin{cases} t - i_k h, & t \in I_{k,1} \\ t - i_k h - h, & t \in I_{k,2} \\ \vdots, & \vdots \\ t - i_k h - l_k h + h, & t \in I_{k,l_k} \end{cases} \tag{5.10}$$

$$e(t) := \begin{cases} 0, & t \in I_{k,1} \\ x(\tilde{r}_{k,1}h) - x(i_k h), & t \in I_{k,2} \\ \vdots, & \vdots \\ x(\tilde{r}_{k,l_k-1}h) - x(i_k h), & t \in I_{k,l_k} \end{cases} \tag{5.11}$$

Then one can obtain that

$$x(i_k h) = x(t - \eta(t)) - e(t), \quad t \in [i_k h, i_{k+1}h). \tag{5.12}$$

The sampled-data error dependent closed-loop system can be derived from (5.1)-(5.4) and (5.12) as

$$\dot{x}(t) = Ax(t) + BKx(t - \eta(t)) - BKe(t) + B_\omega \omega(t),\ t \in [i_k h, i_{k+1}h). \tag{5.13}$$

We supplement the initial condition of the state on $[t_0 - h, t_0]$ as $x(t) = \varphi(t), t \in [t_0 - h, t_0]$, where $\varphi(t)$ is a continuous function on $[t_0 - h, t_0]$ and $\varphi(t_0) = x_0$.

The purpose in what follows is to design a controller in the form of (5.4) such that

(i) the system (5.13) with $\omega(t) = 0$ is asymptotically stable;

(ii) the $\mathcal{L}_2$ gain from w to x is less than a given scalar $\gamma > 0$, that is, under zero initial condition, $\|x(t)\|_2 < \gamma \|\omega(t)\|_2$ for any nonzero $\omega(t) \in \mathcal{L}_2[0, \infty)$.

5.3 $\mathcal{L}_2$ stability analysis

In this section, we will establish a stability criterion for the system (5.13) under the communication scheme (5.2).

Proposition 5.1. *Given a scalar $\gamma > 0$, under communication scheme (5.2), the system (5.13) is finite-gain $\mathcal{L}_2$ stable from w to x with a gain less than γ, if there exist real matrices $P > 0, Q > 0, R_1 = R_1^T, R_2, Y_1, Y_2, Y_3, Z_1$ and Z_2 with appropriate dimensions such that*

$$\begin{bmatrix} P + hR_1 & hR_2 - hR_1 \\ * & hR_1 - hR_2 - hR_2^T \end{bmatrix} > 0 \tag{5.14}$$

$$\begin{bmatrix} \Gamma_{11} & \Gamma_{12} & \Gamma_{13} & hY_1^T & -Z_1^T BK & Z_1^T B_w \\ * & \Gamma_{22} & Y_2^T & hY_2^T & -Z_2^T BK & Z_2^T B_w \\ * & * & \Gamma_{33} & hY_3^T & -\Phi & 0 \\ * & * & * & -hQ & 0 & 0 \\ * & * & * & * & -(1-\delta)\Phi & 0 \\ * & * & * & * & * & -\gamma^2 I \end{bmatrix} < 0 \tag{5.15}$$

$$\begin{bmatrix} \Gamma_{11} & \Gamma_{12} + hR_1 & \Gamma_{13} & -Z_1^T BK & Z_1^T B_w \\ * & \Gamma_{22} + hQ & \Gamma_{23} & -Z_2^T BK & Z_2^T B_w \\ * & * & \Gamma_{33} & -\Phi & 0 \\ * & * & * & -(1-\delta)\Phi & 0 \\ * & * & * & * & -\gamma^2 I \end{bmatrix} < 0 \tag{5.16}$$

where

$$\begin{aligned} &\Gamma_{11} = Z_1^T A + A^T Z_1 - R_1 + I - Y_1 - Y_1^T \\ &\Gamma_{12} = -Z_1^T + A^T Z_2 - Y_2 + P \\ &\Gamma_{13} = Y_1^T - Y_3 + R_1 - R_2, \quad \Gamma_{22} = -Z_2 - Z_2^T \\ &\Gamma_{23} = Y_2^T + hR_2 - hR_1, \ \Gamma_{33} = -R_1 + R_2 + R_2^T - \Phi + Y_3 + Y_3^T. \end{aligned} \tag{5.17}$$

Proof. Choose the following Lyapunov-Krasovskii functional candidate

$$\begin{aligned} V(t,x(t)) &= x^T(t)Px(t) + (h-\eta(t))\int_{t-\eta(t)}^{t} \dot{x}^T(s)Q\dot{x}(s)ds \\ &+(h-\eta(t))\begin{bmatrix} x(t) \\ x(t-\eta(t)) \end{bmatrix}^T \begin{bmatrix} R_1 & R_2-R_1 \\ * & R_1-R_2-R_2^T \end{bmatrix} \begin{bmatrix} x(t) \\ x(t-\eta(t)) \end{bmatrix}. \end{aligned} \tag{5.18}$$

Notice that $0 \leq \eta(t) < h$ and $\dot{\eta}(t) = 1$. Taking the right derivative of $V(t,x(t))$ with respect to t along the trajectory of (5.13) yields

$$\begin{aligned} \dot{V}(t,x(t)) &= 2x^T(t)P\dot{x}(t) - \int_{t-\eta(t)}^{t} \dot{x}^T(s)Q\dot{x}(s)ds + 2(h-\eta(t))[x^T(t)R_1 \\ &+x^T(t-\eta(t))(R_2^T-R_1)]\dot{x}(t) + (h-\eta(t))\dot{x}^T(t)Q\dot{x}(t) - \begin{bmatrix} x^T(t) & x^T(t-\eta(t)) \end{bmatrix} \\ &\times \begin{bmatrix} R_1 & R_2-R_1 \\ * & R_1-R_2-R_2^T \end{bmatrix} \begin{bmatrix} x(t) \\ x(t-\eta(t)) \end{bmatrix}. \end{aligned} \tag{5.19}$$

where $\dot{V}(t,x(t)) = \limsup_{\triangle t\to 0+} \frac{V(t+\triangle t, x(t+\triangle t)) - V(t,x(t))}{\triangle t}$.

Using the Jensen's inequality [47], one can get

$$-\int_{t-\eta(t)}^{t} \dot{x}^T(s)Q\dot{x}(s)ds \leq -\eta(t)v^TQv \tag{5.20}$$

with $v = \frac{\int_{t-\eta(t)}^{t}\dot{x}(s)ds}{\eta(t)}$. And $v|_{\eta(t)=0} := \lim_{\eta(t)\to 0^+} v = \dot{x}(t)$.

Notice that for any real matrices Y_1, Y_2, Y_3, Z_1 and Z_2 with appropriate dimensions, the following equalities hold.

$$2(x^T(t)Y_1^T + \dot{x}^T(t)Y_2^T + x^T(t-\eta(t))Y_3^T)(-x(t) + x(t-\eta(t)) + \eta(t)v) = 0, \tag{5.21}$$

$$2(x^T(t)Z_1^T + \dot{x}^T(t)Z_2^T)(Ax(t) + BKx(t-\eta(t)) - BKe(r_{k,n}h) + B_w w(t) - \dot{x}(t)) = 0. \tag{5.22}$$

It follows from the event-triggered communication scheme (5.2) that for $\forall t \in [i_k h, i_{k+1}h)$, the following measurement error condition holds:

$$\begin{aligned} e^T(t)\Phi e(t) &\leq \delta x^T(i_k h)\Phi x(i_k h) \\ &= \delta(x(t-\eta(t)) - e(t))^T \Phi (x(t-\eta(t)) - e(t)). \end{aligned} \tag{5.23}$$

One can get from (5.19)-(5.22) and (5.23) that

$$\dot{V}(t,x(t)) \leq \xi^T(t)\Xi\xi(t) - x^T(t)x(t) + \gamma^2 w^T(t)w(t) \tag{5.24}$$

where

$$\xi(t) = \begin{bmatrix} x^T(t) & \dot{x}^T(t) & x^T(t-\eta(t)) & v^T & e^T(t) & w^T(t) \end{bmatrix}^T$$

$$\Xi = \begin{bmatrix} \Gamma_{11} & \Xi_{12} & \Xi_{13} & \eta(t)Y_1^T & -Z_1^T BK & Z_1^T B_w \\ * & \Xi_{22} & \Xi_{23} & \eta(t)Y_2^T & -Z_2^T BK & Z_2^T B_w \\ * & * & \Xi_{33} & \eta(t)Y_3^T & -\delta\Phi & 0 \\ * & * & * & -\eta(t)Q & 0 & 0 \\ * & * & * & * & -(1-\delta)\Phi & 0 \\ * & * & * & * & * & -\gamma^2 I \end{bmatrix}$$

with

$$\Gamma_{11} = Z_1^T A + A^T Z_1 - R_1 + I - Y_1 - Y_1^T$$

$$\Xi_{12} = \Gamma_{12} + (h-\eta(t))R_1, \quad \Xi_{22} = \Gamma_{22} + (h-\eta(t))Q$$

$$\Xi_{13} = \Gamma_{13} + Z_1^T BK, \quad \Xi_{23} = Z_2^T BK + Y_2^T + (h-\eta(t))(R_2 - R_1)$$

$$\Xi_{33} = -R_1 + R_2 + R_2^T + \delta\Phi + Y_3 + Y_3^T.$$

It follows from (5.15) and (5.16) that $\Xi < 0$ holds for any $\eta(t) \in [0, h)$. By (5.24), for $\forall t \in [i_k h, i_{k+1} h)$, we have

$$\dot{V}(t, x(t)) \leq -x^T(t)x(t) + \gamma^2 w^T(t)w(t). \tag{5.25}$$

Since $\dot{V}(t, x(t))$ is continuous in time t, for any integer $k > 0$, we have

$$\int_{t_0}^{i_k h} \dot{V}(s, x(s))ds \leq \int_{t_0}^{i_k h} -x^T(s)x(s) + \gamma^2 w^T(s)w(s)ds.$$

Then one can find that, under zero initial condition,

$$\int_{t_0}^{\infty} x^T(s)x(s)ds \leq \gamma^2 \int_{t_0}^{\infty} w^T(s)w(s)ds \tag{5.26}$$

holds for any nonzero $\omega(t) \in \mathcal{L}_2[0, \infty)$.

When $\omega(t) = 0$, (5.25) becomes

$$\dot{V}(t, x(t)) \leq -x^T(t)x(t), \quad t \in [kh, kh+h) \tag{5.27}$$

from which one can conclude the asymptotic stability of the closed-loop system (5.13). The proof is thus completed. □

5.4 Controller design

Based on the stability analysis result obtained previously, this section is to present an $\mathcal{L}_2$ controller design method for system (5.13) under the event-triggered communication scheme (5.2).

Proposition 5.2. *For given real constants $\delta > 0$, $\gamma > 0$, under the event-triggered communication scheme (5.2) with $\Phi = \tilde{Z}^{-T}\tilde{\Phi}\tilde{Z}^{-1}$, the system (5.13) is finite-gain $\mathcal{L}_2$ stable from w to x with a gain less than γ, if there exist real matrices $\tilde{P} > 0, \tilde{Q} > 0, \tilde{\Phi} > 0, \tilde{R}_1 = \tilde{R}_1^T, \tilde{R}_2, \tilde{Y}_1, \tilde{Y}_2, \tilde{Y}_3, \tilde{Z}$, $\tilde{K}$ with appropriate dimensions and a scalar α such that*

$$\begin{bmatrix} \tilde{P} + h\tilde{R}_1 & h\tilde{R}_2 - h\tilde{R}_1 \\ * & h\tilde{R}_1 - h\tilde{R}_2 - h\tilde{R}_2^T \end{bmatrix} > 0 \tag{5.28}$$

$$\begin{bmatrix} \Upsilon_{11} & \Upsilon_{12} & \Upsilon_{13} & h\tilde{Y}_1^T & -\alpha B\tilde{K} & \alpha B_w & \tilde{Z}^T \\ * & \Upsilon_{22} & \tilde{Y}_2^T & h\tilde{Y}_2^T & -B\tilde{K} & B_w & 0 \\ * & * & \Upsilon_{33} & h\tilde{Y}_3^T & -\tilde{\Phi} & 0 & 0 \\ * & * & * & -h\tilde{Q} & 0 & 0 & 0 \\ * & * & * & * & -(1-\delta)\tilde{\Phi} & 0 & 0 \\ * & * & * & * & * & -\gamma^2 I & 0 \\ * & * & * & * & * & * & -I \end{bmatrix} < 0 \tag{5.29}$$

$$\begin{bmatrix} \Upsilon_{11} & \Upsilon_{12} + h\tilde{R}_1 & \Upsilon_{13} & -\alpha B\tilde{K} & \alpha B_w & \tilde{Z}^T \\ * & \Upsilon_{22} + h\tilde{Q} & \Upsilon_{23} & -B\tilde{K} & B_w & 0 \\ * & * & \Upsilon_{33} & -\tilde{\Phi} & 0 & 0 \\ * & * & * & -(1-\delta)\tilde{\Phi} & 0 & 0 \\ * & * & * & * & -\gamma^2 I & 0 \\ * & * & * & * & * & -I \end{bmatrix} < 0 \tag{5.30}$$

where

$$\begin{aligned} \Upsilon_{11} &= \alpha A\tilde{Z} + \alpha\tilde{Z}^T A^T - \tilde{R}_1 - \tilde{Y}_1 - \tilde{Y}_1^T \\ \Upsilon_{12} &= -\alpha\tilde{Z} + \tilde{Z}^T A^T - \tilde{Y}_2 + \tilde{P} \\ \Upsilon_{13} &= \tilde{Y}_1^T - \tilde{Y}_3 + \tilde{R}_1 - \tilde{R}_2 \\ \Upsilon_{22} &= -\tilde{Z} - \tilde{Z}^T, \quad \Upsilon_{23} = \tilde{Y}_2^T + h\tilde{R}_2 - h\tilde{R}_1 \\ \Upsilon_{33} &= -\tilde{R}_1 + \tilde{R}_2 + \tilde{R}_2^T - \tilde{\Phi} + \tilde{Y}_3 + \tilde{Y}_3^T. \end{aligned} \tag{5.31}$$

Moreover, the controller parameter matrix K can be obtained by $K = \tilde{K}\tilde{Z}^{-1}$.

Proof. It can be seen from (5.29) that $\tilde{Z} + \tilde{Z}^T > 0$, which implies that matrix $\tilde{Z}$ is nonsingular. Perform congruence transformations to (5.28), (5.29) and (5.30) by $\mathrm{diag}(\tilde{Z}^{-1}, \tilde{Z}^{-1})$, $\mathrm{diag}(\tilde{Z}^{-1}, \tilde{Z}^{-1}, \tilde{Z}^{-1}, \tilde{Z}^{-1}, \tilde{Z}^{-1}, I, I)$ and $\mathrm{diag}(\tilde{Z}^{-1}, \tilde{Z}^{-1}, \tilde{Z}^{-1}, \tilde{Z}^{-1}, I, I)$, respectively. Let

$$
\begin{aligned}
&P = \tilde{Z}^{-T}\tilde{P}\tilde{Z}^{-1}, \quad Q = \tilde{Z}^{-T}\tilde{Q}\tilde{Z}^{-1}, \quad R_1 = \tilde{Z}^{-T}\tilde{R}_1\tilde{Z}^{-1}, \quad R_2 = \tilde{Z}^{-T}\tilde{R}_2\tilde{Z}^{-1} \\
&Y_1 = \tilde{Z}^{-T}\tilde{Y}_1\tilde{Z}^{-1}, \quad Y_2 = \tilde{Z}^{-T}\tilde{Y}_2\tilde{Z}^{-1}, \quad Y_3 = \tilde{Z}^{-T}\tilde{Y}_3\tilde{Z}^{-1}, \quad \Phi = \tilde{Z}^{-T}\tilde{\Phi}\tilde{Z}^{-1} \\
&Z_1 = \alpha\tilde{Z}^{-1}, \quad Z_2 = \tilde{Z}^{-1}, \quad K = \tilde{K}Z^{-1}.
\end{aligned}
$$

Then it can be verified by Schur complement that (5.14), (5.15) and (5.16) in Proposition 5.1 are satisfied, which ends the proof. □

5.5 An illustrative example

In this section, we will give an example to illustrate the effectiveness of the proposed method developed in previous sections.

The following example considers the attitude control of a satellite in orbit, which is studied in [73–75]. The model is established by two rigid bodies connected by a flexible boom with torque constant k and viscous damping constant f. The diagram of the satellite and the model can be found in [73, 74]. The equations of motion are

$$J_1\ddot{\theta}_1 + f(\dot{\theta}_1 - \dot{\theta}_2) + k(\theta_1 - \theta_2) = u, \tag{5.32}$$

$$J_2\ddot{\theta}_2 + f(\dot{\theta}_2 - \dot{\theta}_1) + k(\theta_2 - \theta_1) = w \tag{5.33}$$

where θ_1 denotes the angle of the main satellite with respect to the star and θ_2 is the angle between the star sensor and the instrument module; J_1 and J_2 are inertias; u and w are the control torque and disturbance torque, respectively. Let the state vector of the system be

$$x = \begin{bmatrix} \theta_1 & \theta_2 & \dot{\theta}_1 & \dot{\theta}_2 \end{bmatrix}^T.$$

Then one can get the state equation

$$\dot{x} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\frac{k}{J_1} & \frac{k}{J_1} & -\frac{f}{J_1} & \frac{f}{J_1} \\ \frac{k}{J_2} & -\frac{k}{J_2} & \frac{f}{J_2} & -\frac{f}{J_2} \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ \frac{1}{J_1} \\ 0 \end{bmatrix} u + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{1}{J_2} \end{bmatrix} w. \tag{5.34}$$

Based on physical analysis of the boom, it is assumed in [73] that the parameters k and f may vary with the temperature but are bounded by

$$0.09 \leq k \leq 0.4, \quad 0.038\sqrt{\frac{k}{10}} \leq f \leq 0.2\sqrt{\frac{k}{10}}. \tag{5.35}$$

In this section, we choose $k = 0.3$, $f = 0.1\sqrt{\frac{k}{10}}$ and $J_1 = J_2 = 1$. It is assumed that the sampling period $h = 10$ms, the event-triggered transmission scheme parameter $\delta = 0.05$. By using Proposition 5.2, one can find that the system (5.34) with controller (5.4) is finite-gain $\mathcal{L}_2$ stable with $\gamma = 7.2$. And a feasible solution to (5.28)-(5.30) leads to the following event-triggered communication scheme weighting matrix and controller parameter matrix

$$\begin{aligned} \Phi &= \begin{bmatrix} 230.02 & 14.075 & 101.33 & 483 \\ 14.075 & 8.3609 & 2.6556 & 32.954 \\ 101.33 & 2.6556 & 50.526 & 211.78 \\ 483 & 32.954 & 211.78 & 1030.5 \end{bmatrix} \\ K &= \begin{bmatrix} -28.09 & -0.48195 & -14.852 & -58.548 \end{bmatrix}. \end{aligned}$$

For the purpose of simulation, the initial condition and the disturbance torque are assumed to be

$$x(0) = [-0.5 \ 1.3 \ 0.3 \ -0.3]^T, \quad w(t) = \frac{1}{1+t}$$

respectively. With controller (5.4) and transmission scheme (5.2), the state responses of system (5.34) are shown in Figure 5.2. Figure 5.3 demonstrates the inter-event intervals distribution in the simulation, which is generated according to the proposed event-triggered communication scheme. The sampled signals that are event-triggered to be transmitted via a network may get lost randomly in the simulation. The distribution of dropouts of the event-triggered sampled data is shown in Figure 5.4. The average inter-event time interval in the simulation is 0.1344s, which is much longer than the sampling period 0.01s. Within the simulation period $T_s = 25s$, the system state is sampled 2500 times, while only 186 of the samples are used for feedback control. Compared with the periodic transmission scheme as that in [95], it is clear that a large proportion of the required network resources may be saved.

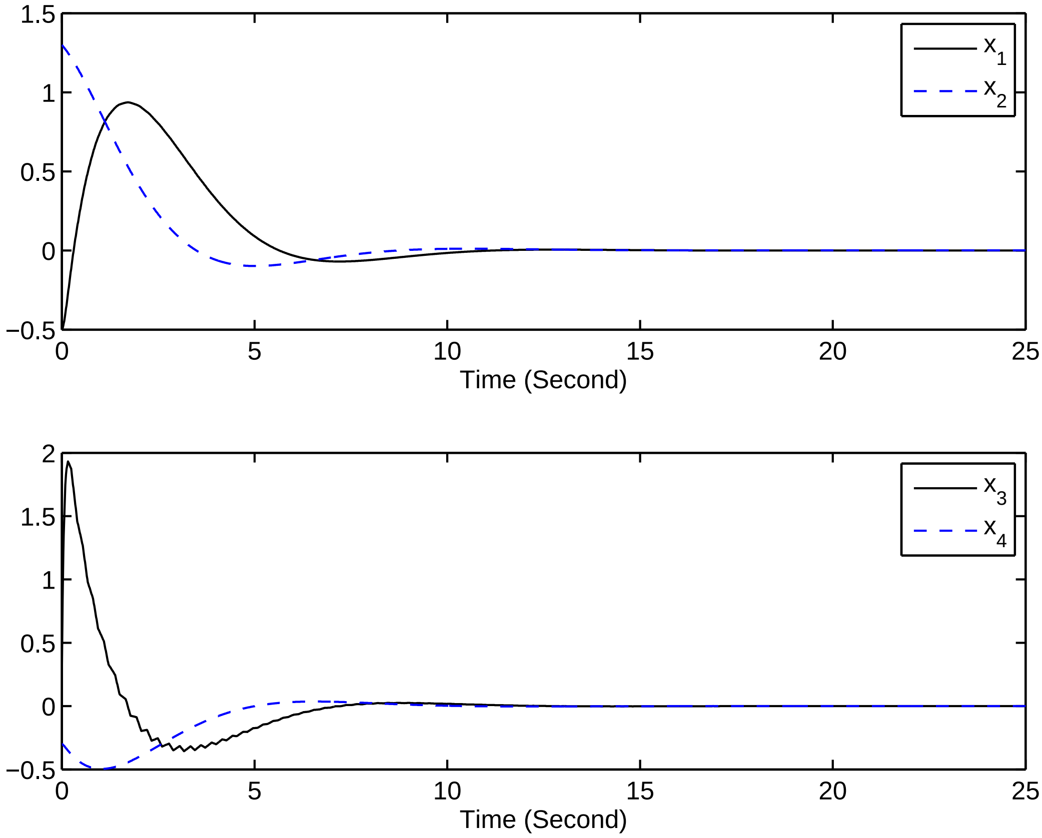

Figure 5.2: State responses of the event-triggered NCS

5.6 Conclusion

A compensation scheme has been proposed in this chapter to deal with the packet dropouts occurred in an event-triggered networked control system. Whether or not the current sampled data should be transmitted via a network to the controller is

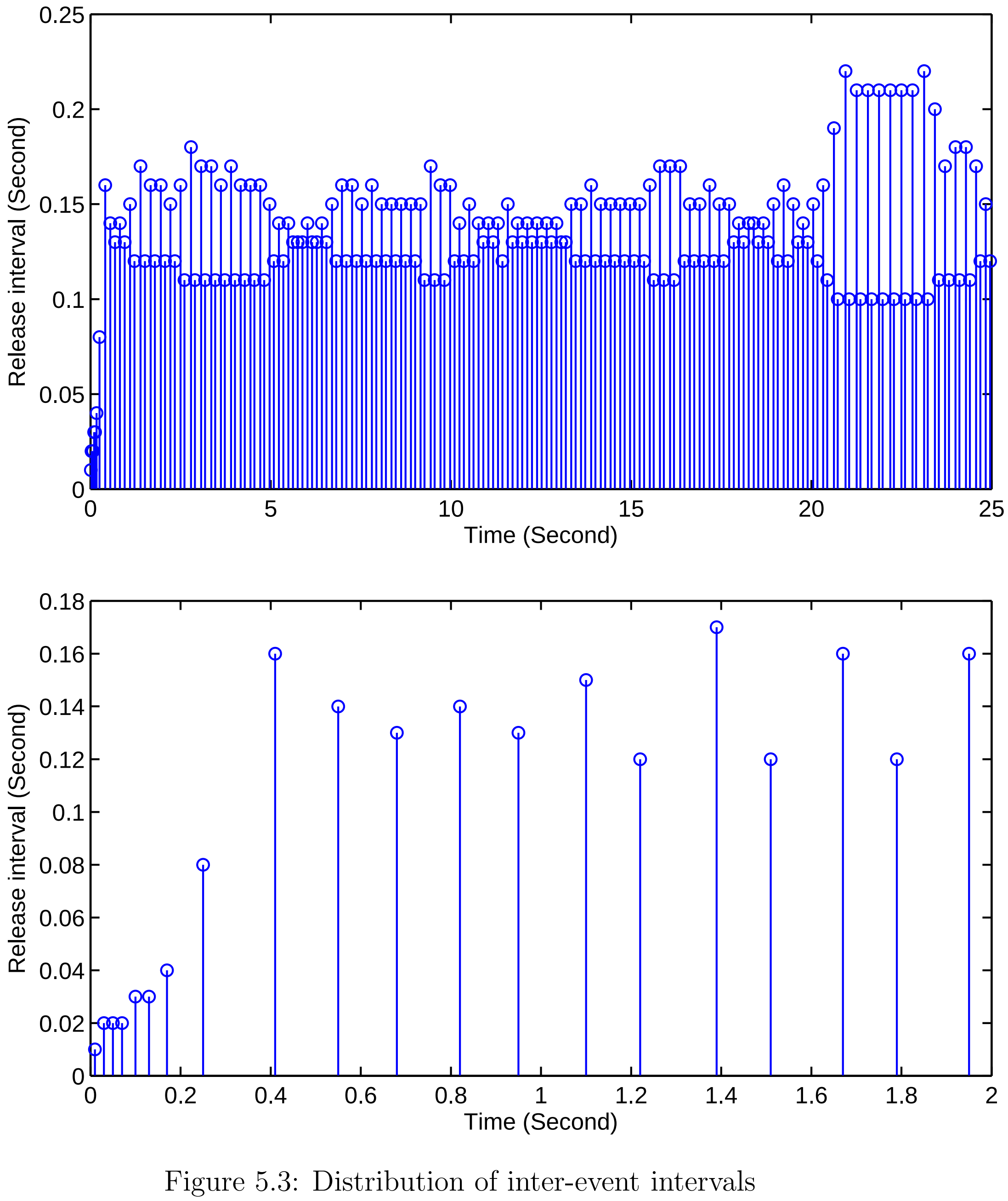

Figure 5.3: Distribution of inter-event intervals

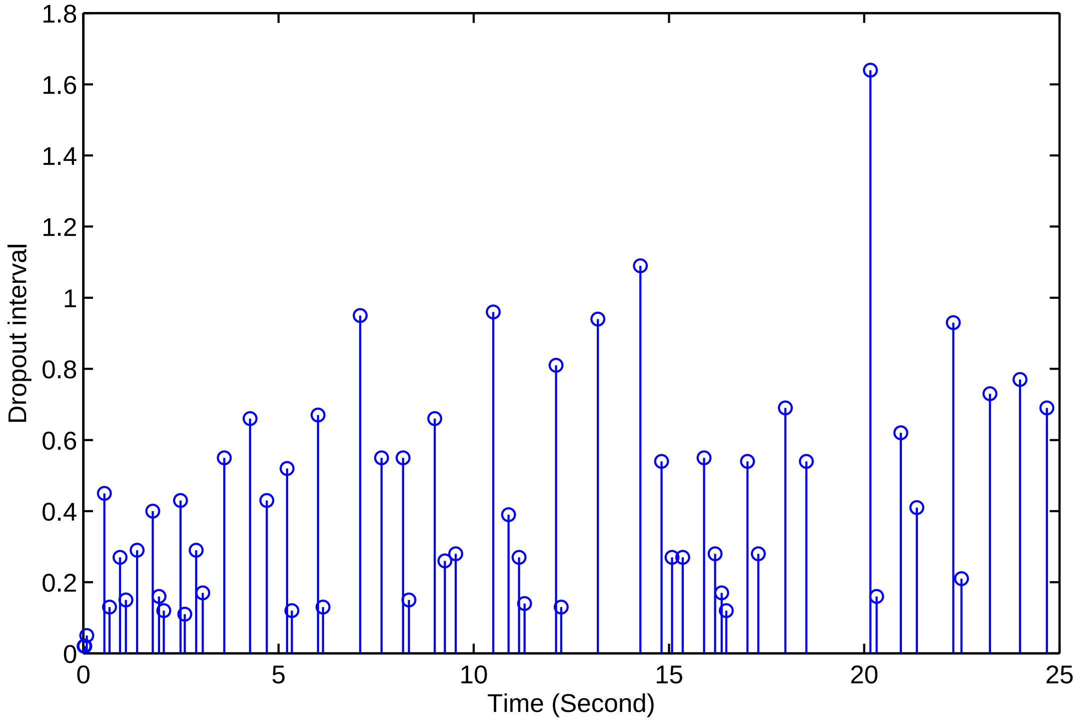

Figure 5.4: Dropouts intervals of the event-triggered sampled data

determined by an event generator according to a pre-given event-triggered communication scheme. A buffer is employed to store a queue sampled data in an FIFO manner. When such an event-triggered measurement gets lost during the transmission, the transmitter retrieves the lost data from a buffer and retransmit it. An $\mathcal{L}_2$ stability criterion has been obtained for the event-triggered control system by using the Lyapunov-Krasovskii functional method. Based on that, a state feedback controller has been designed such that the resulted system is finite-gain $\mathcal{L}_2$ stable. A numerical example demonstrates the effectiveness of the proposed approach.

Chapter 6

Event-triggered output feedback control for Takagi-Sugeno fuzzy systems

6.1 Introduction

Takagi-Sugeno (T-S) model fuzzy systems have been extensively studied in the past decades since T-S fuzzy models can be used to effectively represent a large class of nonlinear systems. Many controller design problems for T-S model fuzzy systems have been considered in the literature [91, 129–131]. The most frequently used control design method is the so-called parallel distribution compensation (PDC), by which the controller shares the same fuzzy premise variables and membership functions with the T-S fuzzy plant. It is shown that the PDC works well for the traditional point-to-point T-S model fuzzy systems, where signals are assumed to be transmitted instantly and accurately between system components.

However, the general PDC may not apply when the fuzzy plant and the controller are connected by a communication network [90, 132, 134]. In this case, information about the premise variables could not be updated synchronously because of the network artifacts such as network-induced delays and packet dropouts, which leads to that the PDC can not be applied for fuzzy controller design. Several efforts have been made to cope with the case. By giving upper error bounds of the asynchronous membership functions, a fuzzy controller design method is proposed for networked-based T-S fuzzy systems [132]. The PDC is applied to networked T-S fuzzy systems under a strict assumption that the fuzzy controller could obtain the system state based on the system mechanism and control input [134]. It is noted that the controller still can not get the exact system state information if the plant is exposed to some unexpected disturbance.

As is shown in previous chapters, the limited transmission resources can be effectively saved by adopting an event-triggered transmission scheme in a control system. It is therefore worthwhile to apply the event-triggered control to the widely used T-S model fuzzy systems. In fact, the problem of event-triggered output feedback control for a discrete-time T-S model fuzzy system is an interesting and challenging issue. On the one hand, most of the results on event-triggered control in the literature are obtained for continuous-time systems. To the best of our knowledge, there is few works that consider the event-triggered discrete-time T-S model fuzzy systems. On the other hand, implementing an event-triggered transmission scheme implies that the controller can not regularly receive output measurement from the fuzzy plant, which makes that the traditional PDC control method may not work. The two aspects motivate this chapter.

In this chapter, an event-triggered transmission scheme is proposed for the discrete-time T-S model fuzzy system. The current output measurement is transmitted to a fuzzy controller only when the output error exceeds a predetermined threshold. In this way, some of the output measurement is not available to the controller, which implies that the controller can not share the same information about the premise variables with the fuzzy plant at each time step. Therefore, the traditional PDC can not apply in this case. To deal with the problem, a dynamical output feedback controller is proposed to generate control input regularly. Based on appropriately chosen premise variables and fuzzy sets, the fuzzy controller can operate separately to generate control input for the T-S model fuzzy plant. Moreover, a certain level of H_∞ control performance can be guaranteed for the resulted fuzzy system. A method to obtain a group of controller parameter matrices will be given.

The organisation of this chapter is as follows. Section 6.2 presents the event-triggered output feedback T-S model fuzzy system and formulates the H_∞ output feedback control problem. The stability and H_∞ performance analysis for the event-triggered fuzzy system is given in Section 6.3. Section 6.4 presents a fuzzy output feedback controller design method. A simulation example is given to demonstrate the effectiveness of the proposed approach in Section 6.5. This chapter is concluded in Section 6.6.

6.2 Problem statement

Consider the following discrete-time T-S model with s plant rules:

Plant Rule i: IF $\xi_1(x(k))$ is F_1^i and $\xi_2(x(k))$ is F_2^i and $\ldots$ and $\xi_r(x(k))$ is F_r^i, THEN

$$\begin{cases} x(k+1) = A_i x(k) + B_{1i}\omega(k) + B_{2i}u(k) \\ y(k) = C_i x(k) + D_i\omega(k) \\ z(k) = L_{1i}x(k) + L_{2i}\omega(k) + L_{3i}u(k) \end{cases} \tag{6.1}$$

where F_j^i is a fuzzy set and s is the number of IF-THEN rules; $x(k) \in \mathbb{R}^n$, $\omega(k) \in \mathbb{R}^p$, $z(k) \in \mathbb{R}^q$, and $y(k) \in \mathbb{R}^m$ are the state, disturbance, regulated output, and the measurement output respectively; A_i, B_{1i}, B_{2i}, C_i, D_i, L_{1i}, L_{2i} and L_{3i} are system matrices with appropriate dimensions; $\xi_1(x(k)), \ldots, \xi_r(x(k))$ are the premise variables. Then the T-S fuzzy system can be compactly represented by

$$\begin{cases} x(k+1) = A(h)x(k) + B_1(h)\omega(k) + B_2(h)u(k) \\ y(k) = C(h)x(k) + D(h)\omega(k) \\ z(k) = L_1(h)x(k) + L_2(h)\omega(k) + L_3(h)u(k) \end{cases} \tag{6.2}$$

where

$$\begin{bmatrix} A(h) & B_1(h) & B_2(h) \\ C(h) & D(h) & 0 \\ L_1(h) & L_2(h) & L_3(h) \end{bmatrix} = \sum_{i=1}^{s} h_i \begin{bmatrix} A_i & B_{1i} & B_{2i} \\ C_i & D_i & 0 \\ L_{1i} & L_{2i} & L_{3i} \end{bmatrix}$$

$$h_i = \frac{\prod_{j=1}^{r} \mu_{ij}[\xi_j(x(k))]}{\sum_{l=1}^{s} \prod_{j=1}^{r} \mu_{lj}[\xi_j(x(k))]}, \quad i = 1, \ldots, s, \quad h := (h_1, h_2, \ldots, h_s) \in \Xi$$

in which $\mu_{ij}[\xi_j(x(k))]$ is the grade of membership of $\xi_j(x(k))$ in F_j^i and Ξ is a set of basis functions satisfying

$$h_i \geq 0, \; i = 1, \; \ldots, s, \quad \sum_{i=1}^{s} h_i = 1. \tag{6.3}$$

The measurement output $y(k)$ is proposed to be transmitted through a network channel to a fuzzy controller. In order to alleviate the traffic burden and to save the limited network resources, we employ an event-triggered transmitter in this chapter to determine whether or not the current measurement $y(k)$ should be transmitted. The basic idea is that $y(k)$ is released by the transmitter only if the error between $y(k)$ and $y(t_k)$ exceeds a pre-given threshold, where $y(t_k)$ is the latest transmitted output measurement. More specifically, the event-triggered transmission scheme can be expressed as

$$t_{k+1} = t_k + \min_{l \in \mathbb{Z}^+} \{ l \mid (y(t_k + l) - y(t_k))^T (y(t_k + l) - y(t_k)) \geq \delta y^T(t_k) y(t_k) \} \tag{6.4}$$

where $\mathbb{Z}^+$ is the set of positive integers; δ satisfying $0 < \delta < 1$ is the threshold of the event-triggered transmission scheme; t_k is the kth transmission time instant. It is clear that $\{y(t_k)\}_{k=1}^{\infty}$ is a subsequence of $\{y(k)\}_{k=1}^{\infty}$.

A typical control design method for the T-S model fuzzy systems usually takes a well-known PDC fuzzy controller, which shares the same premise variables and membership functions with the T-S fuzzy plant all the time. However, the PDC can not be applied to the proposed output-based event-triggered T-S fuzzy model in this chapter. Since the premise variables in (6.1) is not available to the controller due to the fact that only part of the output measurement is transmitted to the controller.

In this chapter, we are interested in designing a fuzzy controller with s controller rules described by:

Controller Rule j: IF $\zeta_1(\hat{x}(k))$ is G_1^j and … and $\zeta_r(\hat{x}(k))$ is G_r^j, THEN

$$\begin{cases} \hat{x}(k+1) = A_{cj}\hat{x}(k) + B_{cj}y(t_k) \\ u(k) = C_{cj}\hat{x}(k), \quad t_k \le k < t_{k+1} \end{cases} \tag{6.5}$$

where A_{cj}, B_{cj} and C_{cj} are to be determined. Then the fuzzy controller can be compactly written as

$$\begin{cases} \hat{x}(k+1) = A_c(\hat{h})\hat{x}(k) + B_c(\hat{h})y(t_k) \\ u(k) = C_c(\hat{h})\hat{x}(k), \quad t_k \le k < t_{k+1} \end{cases} \tag{6.6}$$

with

$$\begin{bmatrix} A_c(\hat{h}) & B_c(\hat{h}) \\ C_c(\hat{h}) & 0 \end{bmatrix} = \sum_{j=1}^{s} \hat{h}_j \begin{bmatrix} A_{cj} & B_{cj} \\ C_{cj} & 0 \end{bmatrix}, \quad \hat{h} := \left(\hat{h}_1, \hat{h}_2, \ldots, \hat{h}_s\right) \in \Xi. \tag{6.7}$$

For any $t_k \le k < t_{k+1}$, let

$$\epsilon(k) = y(k) - y(t_k). \tag{6.8}$$

Then we have

$$y(t_k) = y(k) - \epsilon(k), \quad t_k \le k < t_{k+1}. \tag{6.9}$$

By considering (6.2)-(6.9), we can get the following output-error dependent closed-loop control system:

$$\bar{x}(k+1) = \mathcal{A}(\bar{h})\bar{x}_k + \mathcal{B}_1(\bar{h})\omega(k) + \mathcal{B}_2(\bar{h})\epsilon(k), \quad t_k \le k < t_{k+1} \tag{6.10}$$

$$z(k) = \mathcal{L}_1(\bar{h})\bar{x}_k + \mathcal{L}_2(\bar{h})\omega(k) + \mathcal{L}_3(\bar{h})\epsilon(k) \tag{6.11}$$

where $\bar{x}(k) = \mathrm{col}\{x(k), \hat{x}(k)\}$, $\bar{h} := (h, \hat{h})$

$$\mathcal{A}(\bar{h}) = \begin{bmatrix} A(h) + B_2(h)D_c(\hat{h})C(h) & B_2(h)C_c(\hat{h}) \\ B_c(\hat{h})C(h) & A_c(\hat{h}) \end{bmatrix}$$

$$\mathcal{B}_1(\bar{h}) = \begin{bmatrix} B_1(h) + B_2(h)D_c(\hat{h})D(h) \\ B_c(\hat{h})D(h) \end{bmatrix}, \quad \mathcal{B}_2(\bar{h}) = \begin{bmatrix} -B_2(h)D_c(\hat{h}) \\ -B_c(\hat{h}) \end{bmatrix}$$

$$\mathcal{L}_1(\bar{h}) = \begin{bmatrix} L_1(h) + L_3(h)D_c(\hat{h})C(h) & L_3(h)C_c(\hat{h}) \end{bmatrix}$$

$$\mathcal{L}_2(\bar{h}) = L_2(h) + L_3(h)D_c(\hat{h})D(h), \quad \mathcal{L}_3(\bar{h}) = -L_3(h)D_c(\hat{h}).$$

The purpose of this chapter is to design a fuzzy controller in the form of (6.5) such that

(i) the system (6.10) with $\omega(k) = 0$ is asymptotically stable for any fuzzy basis function $h, \hat{h} \in \Xi$;

(ii) the L_2-gain from the disturbance signal to the regulated output of the closed-loop system is less than a given scalar $\gamma > 0$, that is, under zero initial condition, $\|z_k\|_2 \le \gamma \|\omega_k\|_2$ for any nonzero $\omega(k) \in l_2[0, \infty)$.

In the sequel, we will refer to the system satisfying (i) and (ii) as asymptotically stable with an H_∞ performance γ.

6.3 Stability and H_∞ performance analysis

In this section, we will present a stability criterion for the output-error dependent closed-loop system (6.10)-(6.11).

Proposition 6.1. *For a given scalar $\gamma > 0$, under event-triggered transmission scheme (6.4), the system (6.10)-(6.11) is asymptotically stable with an H_∞ performance γ, if there exists a real matrix $P > 0$ such that for any $h, \hat{h} \in \Xi$*

$$\begin{bmatrix} \Pi_{11} & \Pi_{12} \\ * & \Pi_{22} \end{bmatrix} < 0 \tag{6.12}$$

where

$$\Pi_{11} = diag\{-P, -I, -\delta I\}, \quad \Pi_{22} = diag\{-P, -I, -\gamma^2 I\}$$
$$\Pi_{12} = \begin{bmatrix} \mathcal{A}(\bar{h})P & \mathcal{B}_2(\bar{h}) & \mathcal{B}_1(\bar{h}) \\ \mathcal{L}_1(\bar{h})P & \mathcal{L}_3(\bar{h}) & \mathcal{L}_2(\bar{h}) \\ \delta\mathcal{C}(h)P & 0 & \delta D(h) \end{bmatrix}, \quad \mathcal{C}(h) = \begin{bmatrix} C(h) & 0 \end{bmatrix}.$$

Proof. It follows from the event-triggered transmission scheme (6.4) that for any $t_k \le k < t_{k+1}$, the following measurement error condition holds:

$$\epsilon^T(k)\epsilon(k) < \delta y^T(k) y(k). \tag{6.13}$$

When $\omega(k) \equiv 0$, (6.10) becomes

$$\bar{x}(k+1) = \mathcal{A}(\bar{h})\bar{x}_k + \mathcal{B}_2(\bar{h})\epsilon(k). \tag{6.14}$$

Take $V_k = \bar{x}^T(k)P^{-1}\bar{x}(k)$ as the Lyapunov function. Then the increment of V_k along the solution of system (6.14) is

$$\Delta V_k|_{(6.14)} = \zeta(k)^T \left\{ \begin{bmatrix} \mathcal{A}^T(h) \\ \mathcal{B}_2^T(h) \end{bmatrix} P^{-1} \begin{bmatrix} \mathcal{A}(h) & \mathcal{B}_2(h) \end{bmatrix} - \begin{bmatrix} P^{-1} & 0 \\ 0 & 0 \end{bmatrix} \right\} \zeta(k) \tag{6.15}$$

with $\zeta(k) = \mathrm{col}\{\bar{x}(k), \epsilon(k)\}$. By some congruence transformation, one can see from (6.12) that

$$\begin{bmatrix} \mathcal{A}^T(h) & \mathcal{C}^T(h) \\ \mathcal{B}_2^T(h) & 0 \end{bmatrix} \begin{bmatrix} P^{-1} & 0 \\ 0 & \delta I \end{bmatrix} \begin{bmatrix} \mathcal{A}(h) & \mathcal{B}_2(h) \\ \mathcal{C}(h) & 0 \end{bmatrix} - \begin{bmatrix} P^{-1} & 0 \\ 0 & I \end{bmatrix} < 0 \tag{6.16}$$

which implies that

$$\Delta V_k|_{(6.14)} < \epsilon^T(k)\epsilon(k) - \delta y^T(k)y(k). \tag{6.17}$$

Considering (6.13), one can conclude the asymptotic stability of system (6.14).

Let

$$J_N = \sum_{k=0}^{N-1} \left[z(k)^T z(k) - \gamma^2 \omega(k)^T \omega(k) \right]$$

where N is an arbitrary positive integer. Under zero initial condition, for any nonzero $\omega(k) \in l_2[0, \infty)$,

$$\begin{aligned} J_N &= \sum_{k=0}^{N-1} \left[z(k)^T z(k) - \gamma^2 \omega(k)^T \omega(k) \right] \\ &= \sum_{k=0}^{N-1} \left[z(k)^T z(k) - \gamma^2 \omega(k)^T \omega(k) + \Delta V_k|_{(6.10)} \right] - V_N \end{aligned}$$

where $\Delta V_k|_{(6.10)}$ defines the increment of V_k along the solution of system (6.10). It can be verified that

$$\begin{aligned} J_N &\le \sum_{k=0}^{N-1} \left[z(k)^T z(k) - \gamma^2 \omega(k)^T \omega(k) + \Delta V_k|_{(6.10)} \right] \\ &\le \eta^T(k) \left[\Pi_{12}^T \Pi_{11}^{-1} \Pi_{12} + \Pi_{22} \right] \eta(k) \end{aligned}$$

with $\eta(k) = \mathrm{col}\{\zeta(k), \omega(k)\}$. By Schur complement, one can get from (6.12) that $\Pi_{12}^T \Pi_{11}^{-1} \Pi_{12} + \Pi_{22} < 0$. Then it is clear that $J_N < 0$ for any N, which implies that $\|z_k\|_2 \le \gamma \|\omega(k)\|_2$ for any nonzero $\omega(k) \in l_2[0, \infty)$. □

6.4 H_∞ controller design

Based on the stability analysis result, we will give a design method to obtain a fuzzy controller in the form of (6.5) in this section.

Proposition 6.2. *For a given scalar $\gamma > 0$, if there exist matrices $\{E_i\}_{i=1}^s$, $\{F_i\}_{i=1}^s$, $\{Q_i\}_{i=1}^s$, $X > 0$, $Y > 0$ such that for all $i, j \in \{1, \dots, s\}$*

$$\Upsilon(ij) = \begin{bmatrix} \Upsilon_{11} & \Upsilon_{12} \\ * & \Upsilon_{22} \end{bmatrix} < 0 \tag{6.18}$$

with

$$\Upsilon_{11} = diag\{-\Omega, -I, -\delta I\}, \quad \Upsilon_{22} = diag\{-\Omega, -I, -\gamma^2 I\}$$
$$\Upsilon_{12} = \begin{bmatrix} XA_i + E_jC_i & \Psi_{ij} & -E_j & \Theta_{ij} \\ A_i & A_iY + B_{2i}F_j & 0 & B_{1i} \\ L_{1i} & L_{1i}Y + L_{3i}F_j & 0 & L_{2i} \\ \delta C_i & \delta C_i Y & 0 & \delta D_i \end{bmatrix}, \quad \Omega = \begin{bmatrix} X & I \\ I & Y \end{bmatrix}$$
$$\Psi_{ij} = XA_iY + E_jC_iY + XB_{2i}F_j + Q_j, \quad \Theta_{ij} = XB_{1i} + E_jD_i$$

hold, then there exists a fuzzy controller in the form of (6.5) such that the error dependent closed-loop system (6.10)-(6.11) under transmission scheme (6.4) is asymptotically stable with an H_∞ performance γ. Furthermore, two nonsingular constant matrices M and N can always be obtained such that

$$MN^T = I - XY$$

The controller parameter matrices can be obtained by

$$A_{cj} = M^{-1}Q_jN^{-T}, \quad B_{cj} = M^{-1}E_j \tag{6.19}$$

$$C_{cj} = F_jN^{-T}, \quad j = 1, 2, \dots, s. \tag{6.20}$$

Proof. It follows from (6.18) that $\Omega > 0$. Performing congruence transformation to $\Omega > 0$ by $[X^{-1} \;\; -I]^T$, one can obtain that

$$-X^{-1} + Y > 0$$

which implies that $I - XY$ is nonsingular. Hence there always exist two nonsingular matrices M and N such that $MN^T = I - XY$.

Denote P^{-1} and P in Proposition 6.1 as

$$P^{-1} = \begin{bmatrix} X & M \\ M^T & U \end{bmatrix}, \; P = \begin{bmatrix} Y & N \\ N^T & V \end{bmatrix} \tag{6.21}$$

respectively. Let

$$\Xi = \begin{bmatrix} X & I \\ M^T & 0 \end{bmatrix} \tag{6.22}$$

It follows from (6.18) that

$$\sum_{i=1}^{s} \sum_{j=1}^{s} h_i \hat{h}_j \Upsilon(ij) < 0 \tag{6.23}$$

Performing congruence transformation to (6.23) by $diag\{\Xi^{-1}, I, I, \Xi^{-1}, I, I\}$, one can find that the conditions in (6.12) are satisfied by considering (6.19)-(6.20). The result then follows from Proposition 6.1. □

It is noted that the matrix inequalities in Proposition 6.2 are not strict LMIs. Inspired by [90], we propose the following algorithm to solve the matrix inequalities in Proposition 6.2.

Algorithm 6.1. *Solving (6.18) in Proposition 6.2.*

Step 1. Extract from (6.18) the following inequalities

$$\begin{bmatrix} -Y & A_i Y + B_{2i} F_j \\ * & -Y \end{bmatrix} < 0, \quad i, j \in \{1, \ldots, s\}. \tag{6.24}$$

Step 2. Set $\kappa := 0$. Take λ, which can be a sufficiently small positive scalar, as the step increment of κ.

Step 3. Solve the following LMIs to get a group of feasible solutions $\{F_i\}_{i=1}^s$ and Y.

$$\begin{bmatrix} -Y + \kappa I & A_i Y + B_{2i} F_j \\ * & -Y \end{bmatrix} < 0, \quad i, j \in \{1, \ldots, s\}. \tag{6.25}$$

Step 4. Use the obtained $\{F_i\}_{i=1}^{s}$ *and* Y *to solve (6.18). If the LMI (6.18) is feasible, output the feasible solution* $\{E_i\}_{i=1}^{s}$, $\{F_i\}_{i=1}^{s}$, $\{Q_i\}_{i=1}^{s}$, X, Y *and exit. Otherwise update* κ *by* $\kappa = \kappa + \lambda$, *and go to Step 3.*

Remark 6.1. *It can be seen that one can get a group of feasible solutions to the nonlinear matrix inequalities (6.18) by implementing Algorithm 6.1, in which the LMIs could be solved by using the Matlab LMI Control Toolbox. However, it is worth mentioning that this method introduces some conservatism due to the limitation on some matrix variables.*

6.5 An example

In this section, a simulation example will be provided to demonstrate the effectiveness of the proposed approach. Consider the following discrete-time T-S model fuzzy system, which can be used to model the chaotic Lorenz system [91, 92] with sampling period $T_s = 0.002s$:

Plant Rules:

IF $x_k^{(1)}$ is M_1, THEN

$$\begin{aligned} x_{k+1} &= A_1 x_k + B_{11}\omega_k + B_{21}u_k \\ y_k &= C_1 x_k + D_1\omega_k \\ z_k &= L_{11}x_k + L_{21}\omega_k + L_{31}u_k \end{aligned}$$

IF $x_k^{(1)}$ is M_2, THEN

$$\begin{aligned} x_{k+1} &= A_2 x_k + B_{12}\omega_k + B_{22}u_k \\ y_k &= C_2 x_k + D_2\omega_k \\ z_k &= L_{12}x_k + L_{22}\omega_k + L_{32}u_k \end{aligned}$$

where

$$A_1 = \begin{bmatrix} 1-\sigma T_s & \sigma T_s & 0 \\ \eta T_s & 1-T_s & -M_1 T_s \\ 0 & m_1 T_s & 1-bT_s \end{bmatrix}, \qquad A_2 = \begin{bmatrix} 1-\sigma T_s & \sigma T_s & 0 \\ \eta T_s & 1-T_s & -M_2 T_s \\ 0 & m_2 T_s & 1-bT_s \end{bmatrix}$$

$$B_{11} = \begin{bmatrix} 0.001 \\ 0.002 \\ 0.008 \end{bmatrix}, \quad B_{12} = \begin{bmatrix} 0.001 \\ 0.0015 \\ 0.007 \end{bmatrix}, \quad B_{21} = B_{22} = \begin{bmatrix} 0.1 \\ 0 \\ 0 \end{bmatrix}$$

$$L_{11} = \begin{bmatrix} -0.1 & -0.1 & 0.3 \end{bmatrix}, \quad L_{12} = \begin{bmatrix} -0.05 & -0.05 & 0.2 \end{bmatrix}$$

$$L_{21} = 0.001, \ L_{22} = 0.0015, \quad L_{31} = 0.5, \ L_{32} = 0.4$$

$$C_1 = \begin{bmatrix} 1 & -1 & 3 \\ -1 & 5 & 1 \end{bmatrix}, \ C_2 = \begin{bmatrix} 1 & -1 & 3.5 \\ -1 & 4.5 & 1 \end{bmatrix}$$

$$D_1 = \begin{bmatrix} 0.05 \\ 0.01 \end{bmatrix}, \ D_2 = \begin{bmatrix} 0.04 \\ 0.01 \end{bmatrix}$$

with $[m_1, m_2] = [-20, 30]$ and $(\sigma, \eta, b) = (10, 28, 8/3)$. The membership functions, h_1 and h_2, are described respectively by

$$h_1\left(x_k^{(1)}\right) = \frac{\exp(-(\frac{30-x_k^{(1)}}{50})^2)}{\exp(-(\frac{30-x_k^{(1)}}{50})^2) + \exp(-(\frac{20+x_k^{(1)}}{50})^2)}$$

and

$$h_2\left(x_k^{(1)}\right) = 1 - M_1\left(x_k^{(1)}\right).$$

We choose the parameter of event-triggered transmission scheme (6.4) as $\delta = 0.08$, H_∞ performance level $\gamma = 9.8$. By using Proposition 6.2 and Algorithm 6.1, we obtain a fuzzy controller in the form of (6.5) with the following parameter matrices

$$A_{c1} = \begin{bmatrix} -0.0905 & 0.3063 & 0.0353 \\ 2.2253 & 0.4623 & -0.0534 \\ 0.0438 & -0.0410 & 0.9391 \end{bmatrix}, \ B_{c1} = \begin{bmatrix} -1.0539 & 0.4842 \\ 1.0622 & -4.1929 \\ -3.0750 & -0.8861 \end{bmatrix}$$

$$A_{c2} = \begin{bmatrix} -0.2499 & 0.3169 & 0.0040 \\ 2.747 & 0.4299 & -0.0213 \\ -0.0038 & -0.018782 & 0.93875 \end{bmatrix}, \ B_{c2} = \begin{bmatrix} -1.005 & 0.42153 \\ 1.0352 & -4.1309 \\ -2.9949 & -0.7871 \end{bmatrix}$$

$$C_{c1}(h) = \begin{bmatrix} -1.5252 & 0.0246 & -0.0009 \end{bmatrix}, \ C_{c2}(h) = \begin{bmatrix} -1.44 & 0.0237 - 0.0045 \end{bmatrix}.$$

In this example, $\hat{h}_1$ is supposed to be

$$\hat{h}_1\left(\hat{x}_k^{(1)}\right) = \frac{\exp(-(\frac{40-\hat{x}_k^{(1)}}{50})^2)}{\exp(-(\frac{30-\hat{x}_k^{(1)}}{50})^2) + \exp(-(\frac{10+\hat{x}_k^{(1)}}{50})^2)}.$$

The initial condition and the disturbance signal are assumed to be

$$x(0) = [1 \ \ -1 \ \ -1]^T, \quad \omega(k) = 1/(1+k)$$

respectively. Figure 6.1 and Figure 6.2 illustrate the state response of the fuzzy plant and controller, respectively. Figure 6.3 shows the release time distribution of the event-triggered transmitter. It can be seen that the controlled fuzzy system is stable with the prescribed H_∞ performance level although only a small proportion of the measurement output is triggered to be transmitted to the controller. Within the simulation time, the measurement is generated 400 times while only 121 of them are transmitted to the fuzzy controller, which implies that a certain amount of transmission resources may be saved.

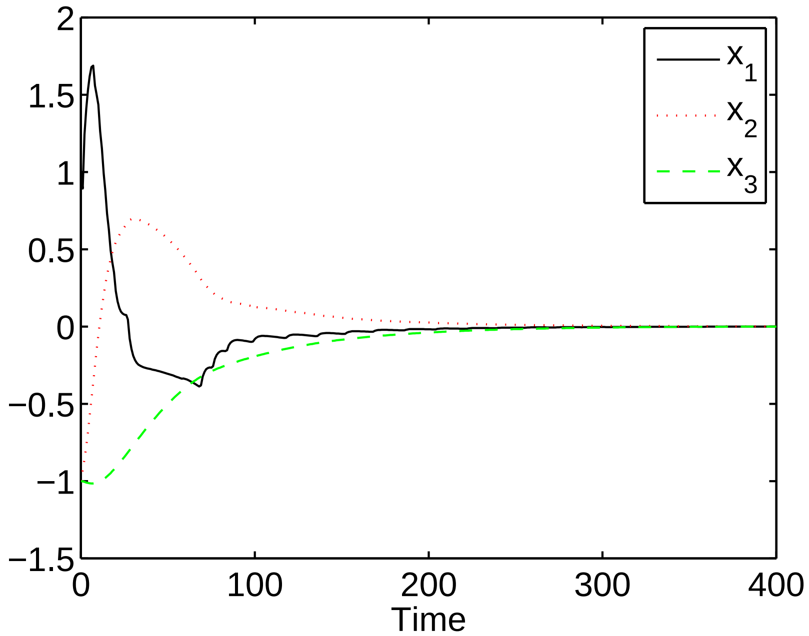

Figure 6.1: State response of the controlled system.

6.6 Conclusion

This chapter has studied the event-triggered output feedback control problem for a class of discrete-time T-S model fuzzy systems. An event-triggered transmission

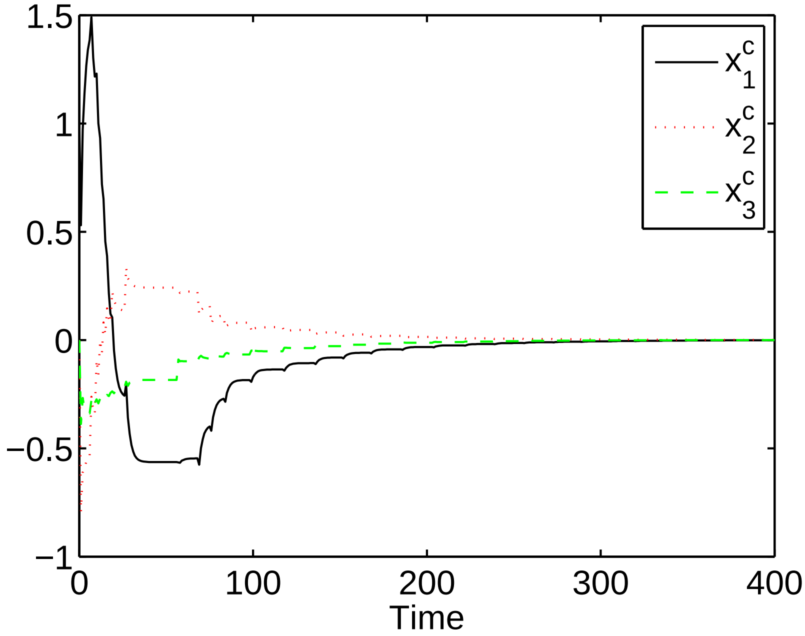

Figure 6.2: State response of the fuzzy controller.

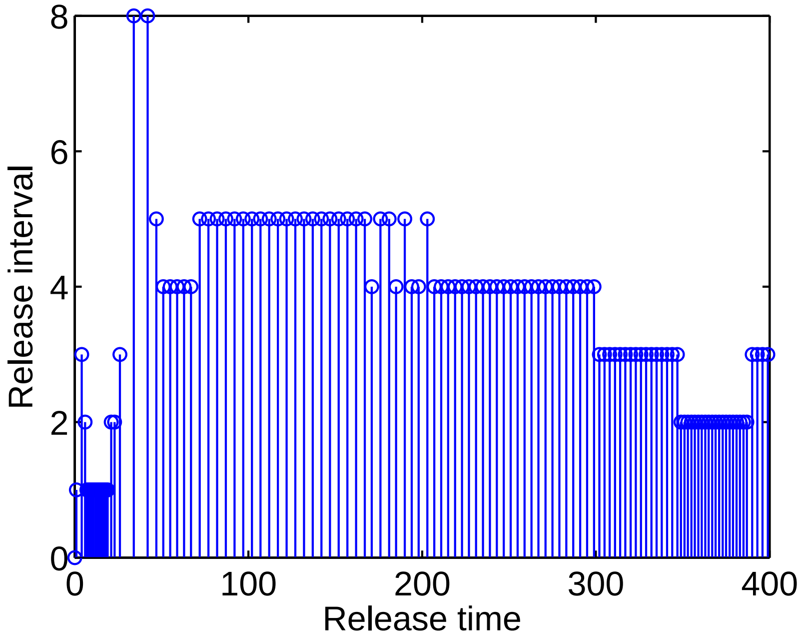

Figure 6.3: Release time distribution of the measurement outpout.

scheme has been proposed to save the limited transmission resources by reducing some unnecessary transmissions. The current measurement is transmitted to the controller only when a certain threshold is violated, which leads to that controller may not receive enough information about the premise variables of the fuzzy plant. By appropriately choosing controller premise variables and fuzzy sets, we have developed a comparatively separate fuzzy controller, which is used to generate control input regularly. Based on an obtained stability analysis criterion, a controller design method has been presented to obtain the controller parameter matrices. A numerical example has been given to show that with the proposed approach, the transmission frequency of system feedback data can be reduced while a certain level of H_∞ disturbance attenuation performance is maintained.

Chapter 7

Conclusions and future work

7.1 Conclusions

This thesis has studied several event-triggered control problems for networked control systems (NCSs). Event-triggered transmission schemes have been developed for NCSs to save the limited network resources, network bandwidth and/or battery power, by efficiently reducing some unnecessary transmissions. The resource efficient event-triggered transmission scheme has been studied in NCSs with different network characteristics such as signal quantization, network-induced delays, packet dropouts, respectively. And it has also been applied in a class of nonlinear systems represented by Takagi-Sugeno (T-S) fuzzy models. More specially, the following problems have been studied in this thesis.

- An $\mathcal{L}_2$ controller design method has been developed for NCSs with quantized measurement, where an event-triggered transmitter is proposed to determine which quantized measurement should be transmitted via a network for feedback. An input delay method together with discrete error bounds guaranteed by the event-triggered transmission scheme is employed to model the inter-event dynamics. Furthermore, an interactive design of finite-level quantizer and event-triggered communication scheme has been presented in a new NCS framework, where an output feedback controller is designed to ensure that the state of the closed-loop system is uniformly ultimately bounded.

- A novel decentralized event-triggered transmission scheme based on asynchronous sampling has been proposed for a class of NCSs, where the system state can only be measured by several spatially distributed sensor nodes instead of a centralized sensor node. Network-induced delays and input delays resulted from the transmission scheme are well depicted by using a switching Lyapunov-Krasovskii functional method. An $\mathcal{L}_2$ decentralized controller design method has been developed for the decentralized event-triggered control system. Some merits and the effectiveness of the proposed approach have been illustrated by simulation examples.

- A compensation scheme of packet dropouts has been proposed for an event-triggered NCS with packet dropouts. Whether or not the current sampled data should be transmitted via a network to the controller is determined by an event generator according to a pre-given event-triggered communication scheme. Each sampled data is also pushed in an FIFO (first in, first out) queue implemented in a buffer. When an event-triggered sampled data gets lost during the network transmission, the transmitter retrieves the lost data from the buffer and retransmit it. A state feedback controller is designed such that the resulted event-triggered control system is finite-gain $\mathcal{L}_2$ stable.

- The event-triggered output feedback control problem for discrete-time T-S model fuzzy systems has been studied. The current measurement output is transmitted to a fuzzy controller only when a certain threshold is violated, which leads to that controller may not receive enough information about the premise variables of the fuzzy plant. By appropriately choosing controller premise variables and fuzzy sets, we have developed a comparatively separate fuzzy controller, which is used to generate control input regularly. A controller design method has been investigated for obtaining a group of controller parameter matrices.

7.2 Future work

As shown in previous chapters, a considerable amount of network resources can be saved by using an event-triggered transmission schemes in a networked control system (NCS) while some desired control performance can be achieved. Although several problems have been studied in the framework of event-triggered networked control systems in this thesis, there are some related research areas worth paying attention in the near future, especially in the following aspects.

- Event-triggered scheduling for inter-connected networked control systems. Since the number of transmissions can be substantially reduced in an NCS by adopting an event-triggered transmission scheme while a certain level of control performance is maintained, it makes a sense to apply the event-triggered transmission scheme in inter-connected network control systems to efficiently allocate network resources among the connected subsystems. In this case, by using an event-triggered transmission scheme, the required transmissions of each subsystem can be effectively reduced, which leads to less network occupation. It is therefore of significance to study how to fully utilize the shared network by developing some appropriate scheduling policy for the whole interconnected NCS.

  The distributed event-triggered control for an NCS consisting of N linear time-invariant interconnected subsystems is studied in [96], where each subsystem broadcast its state over a network according to a static or time-dependent trigger condition. However, the scheduling of data transmissions among the subsystems is not considered. On the other hand, some network-induced problem such signal quantization, networked-induced delays, packet dropouts is not taken into consideration.

- Co-design of controller, quantizer and event-triggered transmission scheme in networked control systems. It can be seen that in an NCS, the controller, quantizer, and transmission scheme can directly influence system's control performances and communication performances. It is of great importance to study the co-design problem of them in an NCS under a suitable pre-given overall system performance which is a composite of some control performances and communication performances. This problem will be much more challenging and interesting when it is studied in non-ideal network environments.

An event-triggered transmission scheme and $\mathcal{L}_2$ control co-design problem is studied for sampled-data control systems [135], where a co-design algorithm is presented to obtain the parameters of the event-triggered transmission scheme and the controller gain simultaneously. It is noted that the co-design performance index introduced in [135] should be examined within a pre-given simulation time period, which may not apply in some practical situations. It is worth mentioning that the co-design performance given in [135] is mainly concerned with the communication performance, while the desired control performance is pre-given.

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