International Journal of Reliability and Applications
Vol. 5, No. 2, pp. 47-57, 2004

Optimal Reliability Improvement for Used Items Sold with Warranty

G. Chattopadhyay *
Department of Mechanical, Manufacturing and Medical Engineering
Queensland University of Technology, Brisbane Q 4000, Australia

D.N.P. Murthy
Department of Mechanical Engineering,
The University of Queensland, St. Lucia, Brisbane 4072, Australia and
Norwegian University of Science and Technology, Trondheim, Norway

Abstract. The market for used products is becoming more competitive and dealers of used products use warranty to promote sales as well as to provide assurance to customers. Offering warranty results in additional costs associated with warranty servicing. This cost can be reduced through actions such as overhaul and upgrade that improves the reliability of the item. This is worthwhile only if the cost of improvement is less than the reduction in the warranty servicing cost. This paper deals with two models to decide on the reliability improvement strategies for used items sold with FRW policy.

Key Words : *Free replacement warranty, Reliability improvement, Used product.*

1. INTRODUCTION

Because of rapid technological changes new products are appearing in the market at an ever-increasing pace. The sales often occur with a trade-in. As a result, the market for used product is also growing and becoming more competitive. Consumers of used products are demanding warranty as part of product assurance and dealers have used warranty as a tool for promoting sales. Recently, lawmakers in many countries have enacted laws to protect the consumer against early failure of used products through mandatory warranty requirements.

A warranty is a contractual obligation incurred by a dealer in connection with the sale of an item that defines the redress action should the item fail within the warranty period specified in the contract. Offering warranty improves sales but results in additional costs

* Corresponding Author
E-mail address: g.chattopadhyay@qut.edu.au

associated with the servicing of the warranty. The expected warranty cost is a function of the warranty terms, the servicing strategy used by the dealer and the reliability of the used item. This in turn depends on the age of the item and the usage and maintenance history.

The literature on warranties for new products is vast. A taxonomy for warranty policies can be found in Blischke and Murthy (1994), which deals with the cost analysis of different warranty policies. Murthy and Djamaludin (2002) provide a review on new product warranties. A discussion of various issues (such as legal, marketing, accounting etc) related to warranty can be found in Blischke and Murthy (1996). In contrast the literature on warranties for used products is very limited. Murthy and Chattopadhyay (1999) proposed a taxonomy for used product warranties.

One way of reducing the warranty servicing cost for used items is through actions such as overhaul and upgrade that improve the reliability of the item. Ruey and Lo (2001) discuss optimal preventive-maintenance warranty policy for repairable products. Zuo et. al. (2000) discuss replacement-repair policy for multi-state deteriorating products under warranty. Davis (1952) deals with the reliability characteristics of reconditioned bus engines and found that the reliability improves after each reconditioning. This then provides an opportunity to improve the reliability of used items through such actions and to reduce the warranty servicing cost. Improving reliability allows the dealer the option to offer better warranty terms and sell the item at a higher price. However, these actions cost money and are worthwhile only if the expected savings in the warranty servicing cost or the increase in the sale price exceed the costs associated with the improvement.

Reliability improvement through overhaul and upgrades has received some attention in the reliability literature. Malik (1979) introduced a "degree of improvement" in the failure rate and called it the "improvement factor". Kijima et al (1988) and Kijima (1989) used the idea of virtual age where the improvement results in a reduction in the age of the system from a reliability point of view.

In this paper, we study the following two problems.

Problem 1: Here the sale price and warranty terms are specified and do not depend on the reliability improvement. The reliability improvement is used to reduce the expected warranty servicing cost.

Problem 2: The sale price depends on the warranty offered. Reliability improvement provides an opportunity to offer better warranty terms and this impacts on the profit. As a result, both reliability improvement and warranty period are decision variables to be selected optimally to maximise the expected profit.

The outline of the paper is as follows: Section 2 deals with model formulation. Analysis of models is carried out in Section 3. Finally, contribution of this paper is summarised in Section 4 along with list of areas for future work.

2. MODEL FORMULATION

We assume that the reliability of a used item depends solely on its age and ignore the effect of usage and maintenance history. Let $F_n(t)$ denote the failure distribution of a new item and we assume that the failure distribution of a used item of age A is given by

$$F(t) = \frac{F_n(A+t) - F_n(A)}{1 - F_n(A)}, \; t \ge 0 \tag{1}$$

This implies that all earlier failures (till age A) were rectified through minimal repair. Let $r_n(t)$ and $r(t)$ denote the failure rates associated with $F_n(t)$ and $F(t)$ respectively. Let L denote the useful life of the item.

2.1 Warranty Policy and Servicing

The item is sold with a FRW policy. This requires the dealer to rectify all failures over the warranty period (W) at no cost to the buyer. The warranty expires after a period W subsequent to the sale.

We consider the case where the dealer rectifies all repairs over the warranty period through minimal repair. The expected cost of each such repair is $\bar{c}$.

2.2 Reliability Improvement

The effect of upgrade actions is to improve the reliability of the item prior to its sale. We consider two different models for reliability improvement.

Model I-1

Here the effect of improvement is to make it effectively younger. As a result, the failure rate changes from $\Lambda(t) = r_n(t)$, the failure rate associated with $F_n(t)$ to $\Lambda_0(t,\tau) = \Lambda(t-\tau)$ with $0 \le \tau \le \bar{\tau} < A$. τ depends on the level of improvement. As such, it is a decision variable to be selected optimally. The cost of improvement is a function of the age (A) and the improvement level τ and is modelled by

$$C_0(\tau; A) = c\tau^{\psi} A^{\xi} \tag{2}$$

for $0 \le \tau \le \bar{\tau} < A$ and the parameters c, ψ, $\xi > 0$. This implies that the cost of improvement increases as A and/or τ increases.

Model I-2

After improvement, the failures occur according to a point process with intensity function given by $\Lambda_0(t,\theta)$ with $\Lambda(t-A) < \Lambda_0(t,\theta) < \Lambda(t)$ with

$$\Lambda_0(t,\theta) = \theta\Lambda(t-A) + (1-\theta)\Lambda(t) \tag{3}$$

where θ depends on the level of improvement. As such, it is a decision variable constrained to $0 \leq \theta \leq 1$ and to be selected optimally. Note that $\theta = 0$ implies no improvement and $\theta = 1$ implies restoring the item back to new. The cost of improvement is given by (2) with θ replacing τ.

2.3 Problem 1

Let $N(W; A, \tau)$ denote the number of failures under warranty with reliability improvement τ. When the reliability improvement is given by Model I-1, the expected warranty servicing cost is

$$J(\tau; A) = C_0(\tau; A) + \bar{c}E[N(W; A, \tau) \tag{4}$$

τ is the decision variable to be selected optimally to minimise the $J(\tau; A)$ given by (4). For reliability improvement given by Model I-2, the objective function is given by (4) with τ replaced by θ.

2.4 Problem 2

The sale price S is a function of the age A and the warranty period W and is modelled by

$$S(W; A) = S_0\{1-(A/L)\}(1+k_1W)^b \tag{5}$$

where S_0 is the sale price of new item, L is the useful life of a new item and $b > 0$. The parameter $k_1 > 0$. The sale price decreases as A increases and increases as W increases. Note that $W < (L-A)$.

The purchase price to dealer for an item of age A is a function of the age of the item and is modelled by:

$$P(A) = k_2S_0(1-A/L) \tag{6}$$

with $0 < k_2 < 1$ and models the immediate loss in resale value subsequent to the sale of a new item.

The expected profit, with reliability improvement given by Model I-1, is given by

$$J(W;A,\tau) = S(W;A) - C_0(\tau;A) - \bar{c}E[N(W;A,\tau)] - P(A) \tag{7}$$

When reliability improvement is given by Model I-2 the expected profit is given by (7) with τ replaced by θ.

3. MODEL ANALYSIS AND OPTIMISATION

We assume the following:

(i) Every failure results in an immediate claim.
(ii) The time for each repair is negligible so that it is ignored.
(iii) All claims are valid.

This implies that the failures over the warranty period occur according to a non-homogeneous Poisson process with intensity function $\Lambda_0(t,\tau)$ or $\Lambda_0(t;\theta)$ depending on the reliability improvement model.

For the FRW policy, the expected number of failures over the warranty period is given by

$$E[N(W;A,\tau)] = \int_A^{A+W} \Lambda(t-\tau)dt \tag{8}$$

when the reliability improvement is given by Model I-1, and by

$$E[N(W;A,\theta)] = \int_A^{A+W} \Lambda_0(t,\theta)dt \tag{9}$$

when the reliability improvement is given by Model I-2 where $\Lambda_0(t,\theta)$ is given by (3).

3.1 Problem 1

For reliability improvement given by Model I-1, using (8) in (4) yields

$$J(\tau;A) = C_o(\tau;A) + \bar{c} \int_A^{A+W} \Lambda(A-\tau)dt \tag{10}$$

and when the improvement is given by Model I-2 we have using (9) in (7)

$$J(\theta;A) = C_o(\theta;A) + \bar{c} \int_A^{A+W} [(1-\theta)\Lambda(t) + \theta\Lambda(t-A)]dt \tag{11}$$

Let $\Lambda(t)$ be given by

$$\Lambda(t) = \lambda\beta(\lambda t)^{(\beta-1)} \tag{12}$$

From (10) we have

$$J(\tau;A) = c\tau^{\psi} A^{\xi} + \bar{c}\lambda^{\beta}\{(A+W-\tau)^{\beta} - (A-\tau)^{\beta}\} \tag{13}$$

The optimal τ^* $(0 < \tau^* < \bar{\tau})$ can be obtained by the usual first order condition (if it is an interior point of the admissible region) and is given by

$$\tau^* = \left\{ \frac{\bar{c}\lambda^{\beta}\beta\{(A+W-\tau)^{\beta-1} - (A-\tau)^{\beta-1}}{c\psi A^{\xi}} \right\}^{\frac{1}{(\psi-1)}} \tag{14}$$

If not, τ^* is either zero (implying no reliability improvement) or $= \bar{\tau}$ (implying maximum reliability improvement).
From (11) we have

$$J(\theta;A) = c\theta^{\psi} A^{\xi} + \bar{c}\lambda^{\beta}[(1-\theta)\{(A+W)^{\beta} - A^{\beta}\} + \theta W^{\beta}] \tag{15}$$

The optimal θ* (if it is an interior point of the region $0 \le \theta < 1$) is given by

$$\theta^* = \left\{ \frac{\bar{c}\lambda^{\beta}\{(A+W)^{\beta} - A^{\beta} - W^{\beta}\}}{c\psi A^{\xi}} \right\}^{\frac{1}{(\psi-1)}} \tag{16}$$

If not then $\theta^* = 0$ or $\bar{\theta}$.

Numerical Example

Let $\beta = 2.3$ and $\lambda = 0.443$. Let $\bar{c}$ = \$150, c = 100, $\psi = 0.95$ and $\xi = 0.3$.

Model I-1: Table 1 gives the τ^* and the corresponding expected total cost, $J(\tau^*;A)$ for a range of W and A.

Table 1. $J(\tau^*; A)$ and τ^* for different combinations of W and A

W	A						
	1	2	3	4	5	6	7
0.5	35.53 0.00	76.16 0.00	122.78 0.00	173.99 0.00	228.98 0.00	287.21 0.00	348.33 0.00
1.0	90.49 0.00	174.98 0.00	270.65 0.00	375.04 0.00	486.69 0.00	604.65 0.00	728.20 0.00
1.5	166.65 0.00	297.77 0.00	444.64 0.00	604.01 0.00	773.90 0.00	952.98 0.00	1140.24 0.00
2.0	217.40 0.90	356.00 1.80	515.78 2.55	690.50 3.18	875.05 3.82	1068.02 4.47	1268.44 5.13

For W = 0.5 to 1.5, $\tau^* = 0$. This implies that if the warranty is less than or equal to 1.5 years it is not worth subjecting the item to any improvement process since the reduction in the expected warranty cost is less than the cost of the improvement process. As W increases, $\tau^* > 0$, this age reduction is more for higher values of A. Costs of upgrade for 1 to 7 year items sold with a 2-year warranty are given in Table 2. It shows that upgrade decision is worthwhile compared to selling without upgrade.

Table 2. Comparison of cost for upgrade versus no upgrade ($W = 2$)

A	τ *	Upgrade cost	Expected Warranty cost + upgrade cost	Expected Warranty cost without upgrade
1	0.90	90.47	217.40	265.48
2	1.80	215.18	356.00	445.64
3	2.54	338.21	515.78	645.69
4	3.18	455.21	690.50	861.73
5	3.82	579.42	875.05	1091.34
6	4.47	710.42	1068.02	1332.85
7	5.13	847.69	1268.44	1585.02

Model I-2:
The corresponding results for Model I-2 are given in Table 3.

Table 3. $J(\theta^*, A)$ and θ^* for different combinations of W and A

W	A						
	1	2	3	4	5	6	7
0.5	35.53 0.00	76.16 0.00	122.78 0.00	173.99 0.00	228.98 0.00	287.21 0.00	348.33 0.00
1.0	90.49	174.98	270.65	375.04	486.69	604.65	728.20

		0.00	0.00	0.00	0.00	0.00	0.00
1.5	127.89 0.90	297.77 0 00	444.64 0.00	604.01 0.00	773.90 0.00	952.98 0.00	1140.24 0.00
2.0	137.78 0.90	369.54 0.90	645.69 0.00	861.73 0.00	1091.34 0.00	1332.85 0.00	1585.02 0.00

Note that $\theta^* = 0$ for $W = 0.5$ and 1.0, for all A. When $W = 1.5$, $\theta^* > 0$ for $A = 1$ and zero for $A \geq 2$. Finally, for $W = 2$, $\theta^* > 0$ for $1 \leq A \leq 2$, zero for $A \geq 3$. The comparison between upgrade versus no upgrade (when $\theta^* > 0$) is given in Table 4.

Table 4. Comparison of cost for upgrade versus no upgrade

A	W	θ^*	Upgrade cost	Expected Warranty cost + upgrade cost	Expected Warranty cost without upgrade
1	1.5	0.90	90.47	127.89	166.65
1	2.0	0.90	90.47	137.78	265.48
2	2.0	1.80	222.77	369.54	445.64

3.2 Problem 2

The analysis of this problem is very similar to that for Problem 1. Hence we simply present the expressions for expected profit and omit the details of the derivation.
For Model I-1 $\Lambda(t)$ is given by (12). As a result, the expected profit is given by

$$J(W,\tau;A) = S(W;A) - c\tau^{\psi} A^{\xi} - \bar{c}\lambda^{\beta}\{(A+W-\tau)^{\beta} - (A-\tau)^{\beta}\} - P(A) \quad (18)$$

The corresponding profit for Model I-2 is given by

$$J(W,\theta;A) = S(W;A) - c\theta^{\psi} A^{\xi} - \bar{c}\lambda^{\beta}[(1-\theta)\{(A+W)^{\beta} - A^{\beta}\} + \theta W^{\beta}] - P(A) \quad (20)$$

The optimal W and τ (or θ) can be obtained from the usual first order conditions if the solution is an interior point. However, W is restricted to a set of discrete values. In this case, $J(W,\tau;A)$ [or $J(W,\theta;A)$] is computed for the different values of W and the optimal τ (or θ) is determined as in Problem 1. Then using a simple search, the optimal W is obtained.

Numerical Example

Let $S_0 = 15{,}000$, $L = 15$, $k_1 = 0.02$, $k_2 = 0.075$ and $b = 1.05$..

Model I-1:

Table 5 gives τ and the corresponding expected profit, $J(W,\tau;A)$ for a range of W and A. The minimum warranty is 0.5 year and we look at larger warranties that are multiples of the minimum warranty.

Table 5. $J(W,\tau;A)$ and τ for different combinations of W and A

W	A						
	1	2	3	4	5	6	7
0.5	3611.50 0.00	3310.38 0.00	3003.25 0.00	2691.54 0.00	2376.05 0.00	2057.31 0.00	1735.69 0.00
1.0	3703.65 0.00	3348.15 0.00	2981.47 0.00	2606.08 0.00	2223.41 0.00	1834.45 0.00	1439.88 0.00
1.5	3783.00 0.89	3370.64 1.25	2933.64 0.00	2492.75 0.00	2041.33 0.00	1580.73 0.00	1111.95 0.00
2.0	3871.19 0.90	3440.54 1.80	2992.13 2.70	2530.75 3.60	2058.93 4.50	1578.27 5.40	1089.89 6.30

The optimal decision for $A = 1$ or 2 is to upgrade and sell with a 2-year warranty. For $A = 3$ to 7 years the optimal decision is not to upgrade and to offer the minimum warranty (i.e., $W = 0.5$). This indicates that upgrade of older items is not worthwhile and best to sell with minimum warranty. This is in line with the practice followed in second-hand car market.

Model I-2:

The corresponding results for Model I-2 are given in Table 6.

Table 6. $J(W,\theta;A)$ and θ for different combinations of W and A

W	A						
	1	2	3	4	5	6	7
0.5	3604.47 0.00	3303.84 0.00	2997.22 0.00	2686.01 0.00	2371.02 0.00	2052.79 0.00	1731.67 0.00
1.0	3689.51 0.00	3335.02 0.00	2969.35 0.00	2595.96 0.00	2213.31 0.00	1825.35 0.00	1431.80 0.00
1.5	3792.11 0.90	3342.23 0.00	2915.36 0.00	2475.99 0.00	2026.10 0.00	1567.02 0.00	1099.76 0.00
2.0	3922.22 0.90	3400.46 0.90	2834.31 0.00	2328.27 0.00	1808.66 0.00	1277.15 0.00	734.98 0.00

The optimal decision for $A = 1$ to 2 is to upgrade and sell with warranty period W = 2 years. For $A = 3$ to 7 years, the optimal decision is not to upgrade and sell with a 0.5-year warranty.

4. CONCLUSIONS

The paper deals with the modelling and analysis to determine the optimal upgrade action given the warranty period (Problem 1) and the optimal upgrade and warranty period (Problem 2) when the warranty period is a decision variable. The models are applicable for the sale of used products such as cars, helicopters, consumer durables, electrical and electronic goods. It allows the second-hand dealers to decide on the optimal decisions for selling their products.

The models proposed in this paper are very simple and can be extended in several directions. We discuss a few such extensions.

1. Build more refined models that incorporate past usage and maintenance history. This will require 2-D models with one dimension representing age and the other the usage. Kim and Rao (2000) discuss two-attribute free-replacement warranties based on a bivariate exponential distribution.
2. We have confined our attention to the simple 1-D FRW policy. The optimal upgrade decisions for other warranty policies (such as Money back Guarantee (MBG), cost limits, deductibles etc) are yet to be studied.
3. The sale model is deterministic. A natural extension is to model it through a probability of sale, which is a function of age, price, upgrade action and warranty terms.
4. The repair cost is a random variable and hence repair decisions are often based on cost limit strategies. The incorporation of this into the model will make it more realistic.
5. In real life not all the claims are exercised. The modelling of such actions has received some attention in the context of new products but not yet studied for used items.

REFERENCES

Blischke, W. R. and Murthy, D.N.P (1994). *Warranty Cost Analysis*, Marcel Dekker, Inc., New York

Blischke, W. R. and Murthy, D.N.P (1996). *Product warranty Handbook*, Marcel Dekker, Inc., New York

Davis, D.J. (1952). An analysis of some failure data, *Journal of American statistical Association*, **47**, 113-150.

Kijima, M., Morimura, H. and Sujuki, Y. (1988). Periodical replacement problem without assuming minimal repair, *European Journal of Operational Research*, **37**, 194-203.

Kijima, M. (1989). Some results for repairable systems with general repair, *Journal of Applied Probability*, **26**, 89-102.

Kim, H.G. and Rao, B.M. (2000). Expected warranty cost of two-attribute free-replacement warranties based on a bivariate exponential distribution, *Computers and Industrial Engineering*, **38**, 4, 425-434.

Malik, M.A.K. (1979). Reliable preventive maintenance policy, *AIIE Transactions*, **11**, 221-228.

Murthy, D.N.P. and Djamaludin, I. (2002). New product warranty: A literature review, *International Journal of Production Economics*, **79**, 3, 231-260.

Murthy, D.N.P. and Chattopadhyay, G.N. (1999). Warranties for second-hand products, *Proceedings of the Ninth International Conference of Flexible Automation and Intelligent Manufacturing (FAIM)*, Tilburg, Netherlands, June 1999, 1145-1159.

Ruey Huei Yeh and Lo, H.C. (2001). Optimal preventive-maintenance warranty policy for repairable products, *European Journal of Operational Research*, **134**, 1, 59-69.

Zuo, Ming J., Liu, Bin and Murthy, D.N.P. (2000). Replacement-repair policy for multi-state deteriorating products under warranty, *European Journal of Operational Research*, **123**, 3, 519-530.