

ABSTRACT

The principles of using mathematical models to describe processes involved in the movement of water in soils are surveyed from the literature. Various models are considered within a classification system based on the degree of empiricism or mechanism of the approach. Empirical models are compared and contrasted with mechanistic models and the role of these models in agricultural practice is discussed. A new empirical mathematical model to describe the uptake of water by plant roots is developed through a sink term and combined with well established models including the Richards' equation to provide a paradigm for the movement of water throughout the soil/plant system. Methods of solution of the model are considered and a finite difference method is employed to provide a computer implementation of the solutions under a range of initial and boundary conditions. The computer simulation was found to be easily adapted to a variety of field situations. In particular, the introduction of the 'evaporation front' concept and its embodiment in the new sink term, provide insights into the criteria for scheduling irrigations, laying the basis for field verification and investigation. The use of this mathematical model for determining an optimal irrigation regime is discussed in relation to conventional scheduling methods.

**IRRIGATION SCHEDULING - A MATHEMATICAL MODEL FOR
WATER MOVEMENT IN CROPPED SOILS INCORPORATING
SINK TERM AND EVAPORATION FRONT**

by

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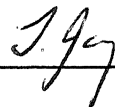
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DECLARATION

This thesis contains no material which has been accepted for any award at any institution. Furthermore, to the best of my knowledge, these works do not contain any material previously published or written by another author except where duly referenced in the text of this thesis.


Terry Janz.