A New Strategy for Far-Field Power Transmission in Orthotropic Plates.

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Vibration intensity (VI) technique is an essential tool for locating and ranking vibration sources and sinks on structures. It can quantify vibration fields by plotting a vector map of energy transmission on the structures. In this paper, a new strategy, by changing coordinate systems of plate equations, is used to develop an intensity equation from shear force component both in x and y directions. The formulation is carried out in frequency domain considering flexural waves. Orthotropic plate theory, far-field conditions, Fourier transform, and finite difference approximation are considered.

1. Introduction

The vibration intensity (VI) is now considered to be a significant measurement tool for structure-borne sound. It yields not only the information of locations of vibration sources and sinks, but also the technique of estimating the reflection coefficients (edge effects), mechanical impedances of the structures. On the other hand, it can be effectively employed to identify the propagation paths of vibration energy transmission. This is possible by plotting x- and y- components of intensity vector on a number of points on the structures. As a result proper damping treatment can be employed to the area of energy transfer.

The VI technique is well established. Most of the earlier methods using VI are useful for simple structures, typically beams and thin plates in flexure [1 - 6]. Some of these formulations are in time domain [2] and others are in frequency domain [3, 4]. Other than this contact method, non-contact such as acoustic holography [7 - 10] and optical measurements [11 - 14] are also available. Numerical analysis using a finite element approach is a good alternative [15 – 16]. Recently, VI is employed for flexural waves in complex structures such as in orthotropic plates for far-field conditions [17] as an approximate method and in general field conditions [18] as an exact method. It requires simultaneous acquisition of all field signals at the same time and an ensemble averaging should be performed. Proper attention should be provided in instrumentation so as to minimize measurements errors.

It is observed, through the literature search, that little research had been undertaken in orthotropic plates using VI. The orthotropic plates such as corrugated plates, beam-stiffened plates, and plate-grid structures are the most important components in industries. It is of the utmost importance to control noise and vibration of such structures. It is, therefore, necessary to get a useful measurement method for those structures. In this paper, VI is used to develop an intensity formulation for flexural waves applicable for thin naturally orthotropic plates from shear force component only using a new method. Through this paper, a preliminary result of shear force contribution of total power will be focused on. Later, contributions from moments will be considered along with the shear force part to obtain total energy transfer. Individual contribution on energy transfer both from shear force and moments is sometimes useful [4]. This research yields a new strategy to formulate a measurement model for vibration energy transmission in orthotropic plates.

2. Theoretical Analysis

2.1 Orthotropic plate equation

This study considers a thin homogeneous orthotropic plate (Figure 1) with small deflections compared to the uniform thickness. The idea of thin plate results when the thickness of the plate, h, is small enough compared to other dimensions. In thin plate flexural wave equation, the influences of rotary inertia and shear deformation are neglected. This approximation is valid when $h \ll \lambda$, the flexural wavelength [19].

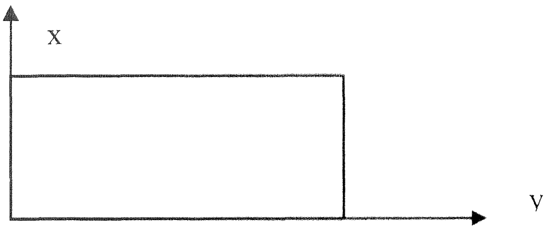

Figure 1 Co-ordinate system of naturally orthotropic plate

The step and analysis of classical orthotropic plate theory can be found in many literatures [19, 20]. The orthotropic plate equation for harmonic flexural vibration can be obtained as [18],

$$(D_x \frac{\partial^4 w}{\partial x^4} + 2H \frac{\partial^4 w}{\partial x^2 \partial y^2} + D_y \frac{\partial^4 w}{\partial y^4}) = m'' \omega^2 w \tag{1}$$

Where, ω is the angular frequency, m'' is the mass per unit area of the plate, D_x and D_y are called the flexural rigidities and quantity D_{xy} is the torsional rigidity of the plate, twice H is called the effective torsional rigidity of the orthotropic plate (equation 2), and w is the transverse deflection in z-direction.

$$H = D_x \nu_y + 2G_{xy} \tag{2}$$

where G_{xy} is shear modulus of elasticity and ν_y is the Poission's ratio. Due to the complexity of analysis with H and different rigidity constants in equation (1), many researchers in the area of orthotropic plates consider an approximation of H such as $H = \sqrt{D_x D_y}$ in the orthotropic plate equations. This approximation gives very good results for the analysis of orthotropic plates [19]. In this case, if the coordinate system changes to a new system such that x is unchanged and y is changed to $y' = y(D_x / D_y)^{1/4}$, the flexural wave equation for orthotropic plate results in the same form as that of an isotropic plate [19]. Consequently, an exact solution of orthotropic plate problem is possible. This modified coordinate for the plate (Figure 1) takes another system of (x, y'). In the following section, this idea is used to modify the orthotropic plate equation (1) to obtain a far-field wave equation.

2.2. Modified plate equation

In an earlier analysis [17], the authors used dimensionless parameters [20] to model the orthotropic plate equation for approximate far-field formulation. Both x and y coordinates were transformed to non-dimensionless parameters. In this case, only the y coordinate is modified which is not dimensionless. As the plate flexural deformation depends on both x and y' coordinates, the related terms in equation (1) can be obtained by partial differentiation with respect to y'.

$$\frac{\partial w}{\partial y} = \frac{\partial w}{\partial y'}(\frac{D_x}{D_y})^{1/4}, \frac{\partial^2 w}{\partial y^2} = \frac{\partial^2 w}{\partial y'^2}(\frac{D_x}{D_y})^{1/2}, \frac{\partial^4 w}{\partial y^4} = \frac{\partial^4 w}{\partial y'^4}(\frac{D_x}{D_y}) \tag{3 a, b, c}$$

Using the idea of equation (3 a, b, c) in equation (1), it is possible to obtain the orthotropic plate equation incorporating the new coordinate system (x, y').

$$(\frac{\partial^4 w}{\partial x^4} + 2 \frac{\partial^4 w}{\partial x^2 \partial y'^2} + \frac{\partial^4 w}{\partial y'^4}) = \frac{m'' \omega^2}{D_x} w \tag{4}$$

This equation (4) behaves like the isotropic plate equation. If k is a flexural wave number such that $k^4 = \frac{m'' \omega^2}{D_x}$, the above plate equation may take a new factorized form as

$$(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y'^2})^2 w = k^4 w \tag{5}$$

The bracketed term may be denoted by ∇ (such that $\nabla = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y'^2}$), the Laplace operator. Although a different transformation is considered in this analysis, the definition of flexural wave number, k, is the same in present analysis and in [17] but ∇ takes a new form. Further simplification of equation (5) is possible and can be transformed as

$$(\nabla + k^2)(\nabla - k^2)w = 0 \tag{6}$$

The equation (6) can be transferred to two equations as

$$(\nabla + k^2)w = 0, \quad and \quad (\nabla - k^2)w = 0 \tag{7 a, b}$$

Equation (7a) represents the condition of far-field where a free propagating wave exists. A complete solution of this equation may be possible. The equation (7b), on the other hand, is the condition of near-field [19] as the disturbances decay exponentially from sources and boundaries.

2.3. Modified shear force equation in the far-field conditions

The equation of shear force in x-direction of orthotropic plate (Figure 1) can be obtained [20] as

$$Q_x = -\frac{\partial}{\partial x}\left(D_x \frac{\partial^2 w}{\partial x^2} + H\frac{\partial^2 w}{\partial y^2}\right) \tag{8}$$

Again, incorporating the new coordinate system (x, y'), the shear force equation (8) may change to another form as

$$Q_x = k^2 D_x \frac{\partial w}{\partial x} \tag{9}$$

The equation (9) is the equation of shear force for the condition of far-field.

Far-field equation for shear force in the y-direction can be obtained in a similar manner. The equation of shear force in y-direction can be presented in terms of spatial derivatives with D_y and H as [20]

$$Q_y = -\frac{\partial}{\partial y}\left(D_y \frac{\partial^2 w}{\partial y^2} + H\frac{\partial^2 w}{\partial x^2}\right) \tag{10}$$

If the new coordinate system is incorporated in the above equation, the shear force takes a different form with modified spatial derivatives. If the coordinate system changes back to original coordinate (x, y) using equation (3a), and putting the value of wave number, k, it is possible to get a new form of y component shear force as

$$Q_y = \omega\sqrt{m'' D_y}\,\frac{\partial w}{\partial y} \tag{11}$$

If the value of wave number is put in the equation (9), a similar equation can be obtained like that of equation (11). It is the far-field equation of shear force in y-direction.

3. Shear force power

Now-a-days it is a common practice to use FFT analyzer for the detection of power flow in structures. Multiple signals can be accommodated in analyzer and analysis is performed in frequency domain, which replaces the time averaging steps of power flow formulation in time domain. In the frequency domain, the complex power from shear force is the cross-spectrum of velocity and force component and is given by the following relation as

$$P_{xS}(f) = G_{vF}(f) \tag{12}$$

where P_{xs} (f) is the complex power in x-direction of the plate, G_{vF} (f) is the cross-spectrum of transverse velocity and shear force.

Differentiating the equation (9) with respect to time and taking Fourier transform, it is possible to get the x-component of shear force equation in terms of transverse velocity as

$$\hat{Q} = \frac{D_x k^2}{j\omega}\frac{\partial \hat{v}}{\partial x} \tag{13}$$

The shear force components in equations (9) and (11) are based on transverse displacement, w. The spatial derivative of shear force equation (13) can be obtained using finite difference approximation (Figure 2). The x-component of VI from shear force can thus be obtained as

$$P_{xS} = < -\hat{v}^* \hat{Q}_x > \tag{14}$$

where < > represents ensemble average. The cross-spectrum of the equation (12) can be defined as in equation (14) [21]. The negative sign is included to make power flow positive in a positive direction [2].

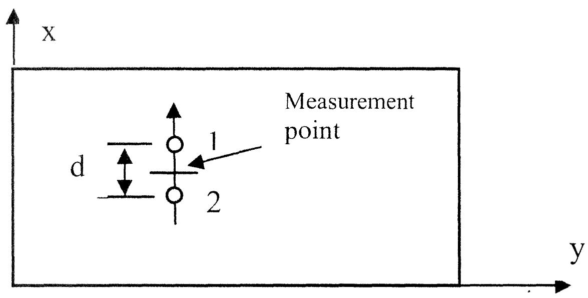

Figure 2: Two-transducer array, measuring vibration power transmission form point 2 to 1 at a distance d, the spacing

Using finite difference approximation, the transverse velocity and spatial derivatives of transverse velocity in equations (13 and 14) can be obtained as (Figure 2)

$$\hat{v} = \frac{\hat{v}_1 + \hat{v}_2}{2}, \quad \frac{\partial \hat{v}}{\partial x} = \frac{\hat{v}_1 - \hat{v}_2}{d} \tag{15a, b}$$

Hence, the complex far-field power from shear force in x-direction of the plate is as follows

$$P_{xs} = <-(\frac{\hat{v}_1 + \hat{v}_2}{2})^* \frac{D_x k^2}{j\omega}(\frac{\hat{v}_1 - \hat{v}_2}{d})> \tag{16}$$

Evaluating the ensemble averages term by term, the final complex form of structural intensity in the x-direction by shear force only would be obtained as

$$P_{xS}(f) = \frac{D_x k^2}{2j\omega d}[(G_{22} - G_{11}) + 2j\mathrm{Im}\{G_{12}\}] \tag{17}$$

where d is the distance between two successive points, ω, the angular frequency, G_{12}, the cross-spectrum of the velocity signals at points 1 and 2: G_{22} and G_{11}, two auto-spectrum of the velocity signals. The real part of the above complex power flow equation (17) presents the power transmitting in the far-field of the orthotropic plates by shear force in the x-direction and it could be written as

$$I_x(f) = \frac{\sqrt{D_x m''}}{d}\mathrm{Im}\{G_{12}\} \tag{18}$$

In the above equation, I is used to represent VI because it is active component of complex power and to differentiate it from the complex power, P. Similarly, the y-component of shear force considering cross-spectrum of velocity signal can be achieved as

$$I_y(f) = \frac{\sqrt{D_y m''}}{d}\mathrm{Im}\{G_{12}\} \tag{19}$$

Since accelerometers are widely used in vibration measurements, it is therefore customary to use acceleration signals instead of velocity signals in the formulation of vibration power. In the frequency domain, the relation between velocity and acceleration is $\hat{v} = \frac{\hat{a}}{j\omega}$. This leads to $G_{vv} = \frac{G_{aa}}{\omega^2}$ for power spectral densities of acceleration and velocity where $\hat{v}$ and $\hat{a}$ are the signals of velocity and acceleration in the frequency domain respectively. The following cross-spectrum G_{ij} refers to the signals of acceleration rather than velocity. By using the acceleration based cross-spectrum, the structural intensity equation (18) could be rewritten as

$$I_x(f)=\frac{\sqrt{D_x m''}}{d\omega^2}\mathrm{Im}\{G_{12}\} \tag{20}$$

The orthogonal component (y component) of structural intensity from shear force can thus be achieved by exchanging the letter x to y.

4. Discussions

In this paper, cross-spectrum density function is used to formulate far-field power flow equation from shear force in the frequency domain for orthotropic plates in x and y directions. A completely different approach is used to get the same relation as that obtained before [17]. By changing the orthotropic plate equation to a form similar to that of isotropic plate, the solution converges to exact [19]. A dual channel FFT analyzer may be used to take simultaneous acquisition of field data. Coefficients of spatial derivatives in orthotropic plate equation in bending are different from those for an isotropic plate. These derivatives are: D_x, D_y and H (equation 1). Only D applies for the case of isotropic plate. This enables the modification of isotropic plate equation to provide usable relations to solve practical applications in industry for NVH problems. This is not possible directly in the case of orthotropic plates. As a result, researchers and engineers working in this area used some assumptions such as ($H=\sqrt{D_x D_y}$) [19, 20 17] and obtained good results. As the orthotropic plates such as corrugated plates (rectangular, trapezoidal, sinusoidal) and beam stiffened plates are widely used in industries, it is necessary to simplify the theoretical formulations into useable equations for FFT analyser's usage. Therefore it yields a significant advantage in providing solutions with small errors in NVH problems.

It is stated above that the flexural wave equation in orthotropic plate is completely different to that of isotropic plate. In the latter case, flexural rigidity (D) comes out from each spatial term of the equations of shear force and bending and twisting moments as a common factor. As a result, it is very simple to replace far-field wave condition in shear force and modified moment relations [1]. It is not directly possible for orthotropic plates because of different rigidity values in their spatial terms such as D_x, D_y and H.

Vector plot of VI represents its magnitude and direction at a point (resultant of x and y components). From source, vibration power is flowing out, meaning that all intensity vectors are going out from a point (location of attached electrical motor, for example). Sink (viscus damper for example), on the other hand, absorbs energy. In this location VI vectors flow to this point. Therefore vector map shows the location of sources and sinks. The magnitude of VI can be presented by its numerical or dB values, indicating its ranking. The propagation paths can also be identified from this plot.

The model (equation 20) is applicable for orthotropic plates with uniform thickness. The applicability of uniform orthotropic plate theory in rib-plates, and corrugated plates depends on the flexural wavelength. It is established that the flexural wavelength of these plates should be considerable greater than one repeating section of rib or corrugation [19]. This is very important in high frequency ranges when the associated wavelength is shorter. By using a filtering technique in data acquisition, it is possible to remove the frequency range where wavelength may not be considerably greater than the distance of one repeating section. Alternatively the idea of elastic equivalence [20] can be useful for the model to technically orthotropic plates such as corrugated plates, ribbed plates, plate grids, beam reinforced plates and similar other plates. This technique transfers a technically orthotropic plate to naturally orthotropic plate of uniform thickness [20]. Therefore this model can be easily applied using FFT analyzer to estimate vibration power flow in industrial applications.

5. Conclusions

A two-transducer technique for VI calculation is proposed considering a different approach. A change in the coordinate system enables the plate flexural wave equation to simplify to obtain a far-field condition. This VI formulation is from shear force only for both x and y directions incorporating a cross-spectrum method. The two transducer model helps to use FFT analyzer for practical data acquisition in the frequency domain. For complete analysis, it is necessary to consider the contribution of moments, however, at this stage, it is put forward as a preliminary result.

6. References

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