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Beneficial Damage: A New Concept for Fracture Toughness Enhancement and Applications to Railway Engineering

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PREFACE

Railway infrastructure and rollingstock are perhaps good examples for the description of multi-body system dynamics. They contain a large number of engineering components, each with varying characteristics of materials, shapes and sizes. They are subjected to very complex state of random loading generated by mechanical and environmental conditions. Random (variable amplitude) loading leads to fatigue of each component in the system. Although fatigue under variable amplitude loading is not well understood, it is a common knowledge that cracking initiates and propagates within and on the surface of each component under this loading. Inspection of critical components for crack detection is well developed and there are several patented non-destructive testing systems available in the market. The asset management of these components with embedded and surface cracks is an expensive issue for the industry. Surface cracks in rails, axles and wheels are removed through grinding or profiling at very high costs to the industry. The subsurface crack management, on the other hand, is not well understood. Any attempt to control the operating conditions of the rollingstock (speed, axle load, contact patch – through premium bogies) leads to complications and inefficiency. Alternate methods of crack control would offer significant advantage to the railway industry. This paper describes one such potential concept – beneficial damage to enhance the fracture toughness of the materials containing embedded cracks. Although it is very early days, this "blue sky" concept could offer alternate solution with the potential emergence of new technologies to induce "beneficial damage". This alternate solution might help the railways to manage cracked railway components without sacrificing the efficiency of the transport system. The research on beneficial damage described in this paper is in its infancy and we at the Centre for Railway Engineering hope that it would mature into a good research topic and find novel applications in railway engineering.

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Beneficial Damage: A New Concept for Fracture Toughness Enhancement and Applications to Railway Engineering

1. Introduction

When we talk about damage, we always connect it with plasticity, fatigue, cracking, or creep of materials because damage in our mind is related to degradation and failure of the materials. However, not all damage always leads to further damage or failure. Some damage in specific situations provides improvement to the behaviours of materials. Such damage is defined as beneficial damage in this paper. Although the term "beneficial damage" seems a paradox, an attempt will be made to develop this concept further through an extensive literature review.

Damage is defined using a scalar parameter, known as damage index, which is a number that varies between zero and one. Damage index zero represents perfect material with no damage. Damage index one represents fully damaged material that can't resist any loading.

Although the term "Beneficial damage" was not explicitly used, the concept of microcrack shielding was first used in ceramic materials research. From the perspective of mechanics, ceramics is brittle and susceptible to fracture when a macrocrack is formed. Fracture toughness for this kind of material is usually low and hence is a very critical parameter that defines its fracture loading leading to failure. The making of ceramics more ductile with increased level of fracture toughness is attended by material scientists. One practical way of fracture toughness enhancement is to introduce pre-designed microcracks of the order of 5-100 μm or other kinds of microdefects in the materials during the production processes.

Type, density, spacing, and size of the introduced microdefects/ microcracks positively contribute to the fracture toughness enhancement of the brittle material. It would be useful to understand the process of fracture toughness enhancement with pre-designed microcracks. Such an understanding could potentially lead to its application to other engineering fields, for example railway engineering.

The railheads suffer from rail/wheel rolling contact fatigue. In the early days when the railheads were made from low hardness, low yield stress steel, they were subjected mainly to wear. These railheads often exhibit worn profiles which, together with plastic deformation of the railhead profile, alter the contact geometry and conditions negatively. Therefore, in the modern high speed tracks, the pearlitic rail steel with high hardness, high yield stress is widely used. The railheads made from this steel are wear resistant and do not give rise to large plastic deformation of the railhead profiles; hence, desirable contact geometry and conditions are maintained. However, railhead checks initiate and develop instead of the rolling contact wear. Fully propagated checks could lead to vertical breakage of the railhead. Currently reprofiling and rail grinding strategies are employed to prevent further crack growth of these surface-initiated checks at high cost of maintenance. This paper examines a novel method of enhancing the fracture toughness of railheads with embedded cracks. The enhanced fracture toughness would enable trains to operate at normal speed with no requirement of reducing axle loads. Although the method of beneficial damage introduced in this paper appears attractive, its application is affected by the non availability of technology to induce damage.

2. Basic concept of damage

2.1 What is damage?

Damage of materials is the progressive physical process by which they break. At the microscale it can be interpreted as the creation of microsurfaces of discontinuities such as breaking of atomic bonds and plastic enlargement of microcavities. At the mesoscale, it can be defined as growth and coalescence of microcracks or microvoids which together initiate one crack. At the macroscale, damage occurs as the crack grows (Lippmann, 1996).

Damage at microscale and mesoscale are studied by means of damage variables based on the concept of continuously damaged media. Damage at macroscale is usually studied using fracture mechanics in which only crack tip parameters are mainly examined.

For different materials damage appears in different styles. In polymers, damage occurs by the breakage of bonds between the long chains of molecules; in composites, it represents debonding between the fibre and matrix, breaking of fibre, and delamination of braiding layers. In ceramics, it appears as microdecohesions between inclusion and matrix, nucleation and growth of microcrack and/or microvoid. In concrete, it occurs when decohesion between aggregates and cement occurs. In rock, damage is found as nucleation and growth of subcracks and small voids. In wood, damage appears as debonding of the cellulosic cells.

Damage manifests itself in various ways in brittle and ductile materials. In brittle materials, a crack is initiated at the mesoscale without a large amount of plastic strain. The ratio of plastic strain to elastic strain remains below unity for this type of material. The cleavage forces are below the slip forces but are higher than the debonding forces. In ductile materials, damage occurs when plastic deformations reach a certain threshold value. It results from the nucleation of cavities due to growth and coalescence of plastic slips.

2.2 Representation of damage

Damage in the microscale is highly localised. However in macroscale it is randomly distributed and is statistically homogeneous. In the continuum damage mechanics, the damage variable is assumed as a continuous function, which is defined for every point in the material to characterise the damage level. Quantities of the damage variable represent averages on a certain volume, usually defined as a representative volume element (RVE). Within an RVE, damage and other material parameters are averaged out. As an example, Fig.1 shows the definition of homogeneous damage variable. The rectangular prism denotes a RVE from the damaged material. The area of the intersection of the plane within the RVE is denoted by S. The effective area of the intersection of all defects in S is denoted by δS. Damage variable is then defined as

$$D = \frac{\delta S}{S} \tag{1}$$

The variable D is a scalar that varies from 0 to 1. For undamaged material, $D = 0$ and for fully damaged material $D = 1$.

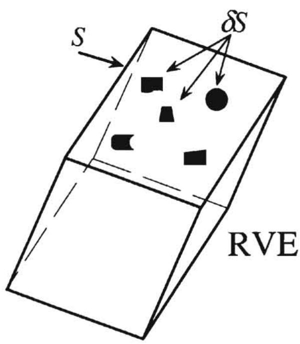

Figure 1 Micro-meso definition of damage in RVE

2.3 Representative volume element

An RVE must be small enough to avoid smoothing of high gradients but large enough to represent an average of the microprocesses. The following order of magnitude of the RVE for different materials are employed in experiments and numerical analysis:

Metals and ceramics:	0.1 mm^3
Polymers and most composites:	1 mm^3
Wood:	10 mm^3
Concrete:	100 mm^3

For damage analysis of composite, masonry, and concrete, the RVE contains multiple material phases and interfaces. In order to employ the continuous damage medium concept, a homogenisation process must be taken to homogenise the composites into effective materials. Nevertheless, at the mesoscale, the damage is highly localised within the RVE. It may occur in either material phase or along the interfaces. To present the damage using the continuous damage parameter, a nonlocalisation procedure should be used. There is a rich source of information available in the literature on nonlocal theories. As this topic is considered outside the scope of this paper, this matter is not further described here. However readers wishing to know more on nonlocalisation are directed to Benssousan et al. (1987), Bakhvalov and Panassenko (1989), Guedes and Kikuchi (1990), Paulo and Jan (1997), and Fish et al. (1999).

2.4 Measurement of damage

Damage can be measured directly from its definition or indirectly from its relation with other physical quantities. The main test methods are direct measurement, elasticity modulus method, ultrasonic wave method, and microhardness method.

Direct measurement can be applied using the definition (1). S here is an area of observing RVE section in a micrograph of damaged material. δS is the total crack area within the RVE section. The damage is $\delta S/S$.

As damage reduces material rigidity and stiffness, it has a relation with the elastic modulus as shown in (2)

$$D = 1 - \frac{\tilde{E}}{E} \tag{2}$$

where E is the elastic modulus of the undamaged material, and $\tilde{E}$ is the effective elasticity modulus of the damaged material. By testing the elasticity modulus of the materials with and without damage, the damage parameter D can be determined. This method is known as the elasticity modulus method.

Ultrasonic wave method is also based on the variation of the elasticity modulus. Longitudinal wave speed of the ultrasonic wave v_L has a relation with the elasticity modulus as shown in (3)

$$v_L^2 = \frac{E}{\rho} \frac{1-\upsilon}{(1+\upsilon)(1-2\upsilon)} \tag{3}$$

where υ and ρ are the Poisson's ratio and the density of the material, respectively. Damage does not affect υ if the elasticity is isotropic. υ remains constant when the material is damaged. As the damage consists mainly of microcracks or small cavities, ρ remains constant. Therefore, the longitudinal wave speed of the damaged material depends only on the effective modulus of elasticity.

$$\tilde{v}_L^2 = \frac{\tilde{E}}{\rho} \frac{1-\upsilon}{(1+\upsilon)(1-2\upsilon)} \tag{4}$$

Substituting (3) and (4) into (2), a relation between damage and longitudinal wave speed is derived as shown in (5)

$$D = 1 - \frac{\tilde{v}_L^2}{v_L^2} \tag{5}$$

By measuring the wave speeds of the materials with and without damage, the damage parameter can be calculated using (5).

Microhardness method measures damage based on the relation shown in (6)

$$D = 1 - \frac{\tilde{H}}{H} \tag{6}$$

where H and $\tilde{H}$ are microhardness of the undamaged material and that of damaged material respectively.

3. Basic concept of fracture

3.1 Stress intensity factors

High carbon high strength steels break in a brittle manner under the value of maximum stress be exhibiting only a marginal nonlinearity as shown in Fig.2.

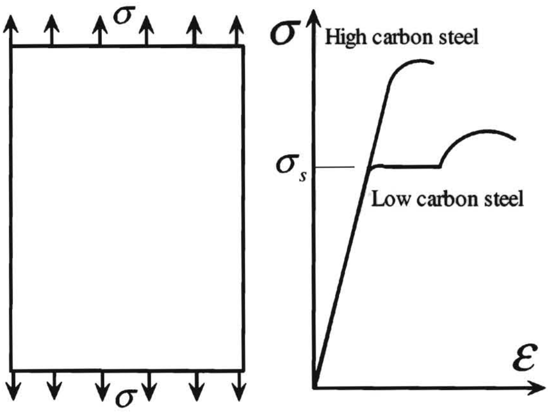

Figure 2 Steel plate without crack under tensile stress field

A classical low carbon ductile steel, also shown in Fig.2, deforms linearly until a yield stress of the material is reached. The yield stress is usually considered as its elastic limit stress. Beyond this stress level, the plate loses its stiffness.

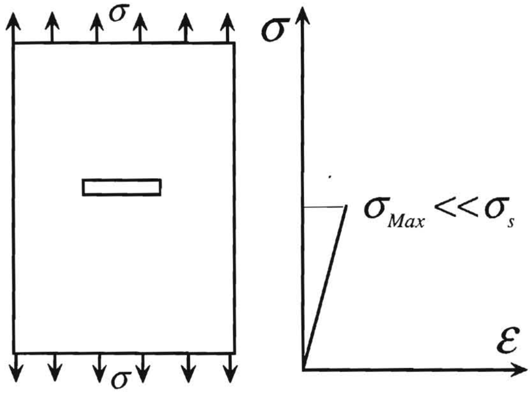

Figure 3 A plate with a single crack under tensile stress field

However, no matter whether the material is brittle or ductile, a plate with a crack behaves purely linearly. A maximum stress σ_{Max} as shown in Fig.3 leads to breakage of the plate at a stress level far lower than the yield stress of the virgin material. The reason

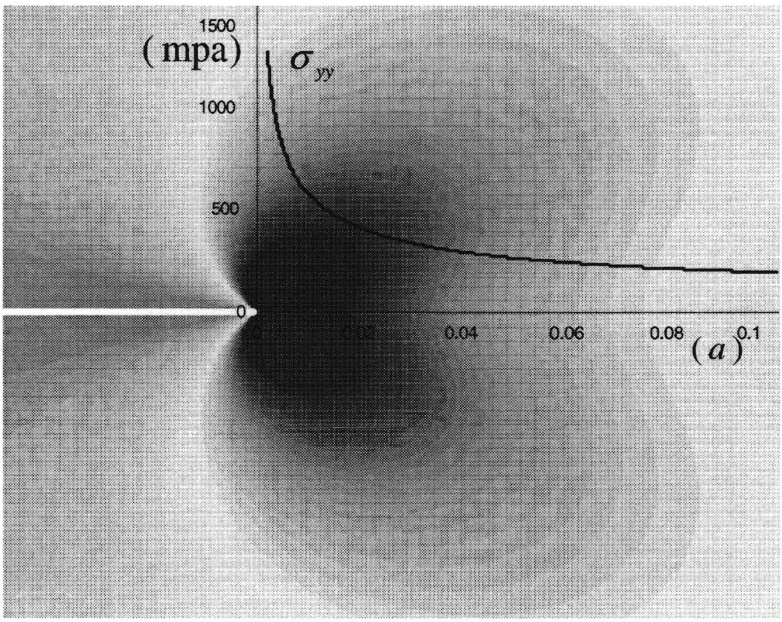

Figure 4 Stress file around the right crack tip

for this is the high stress concentration or singularity around the crack tip. Fig.4 shows the stress field near the crack tip.

It can be seen that the stress tends to infinity as it approaches the crack tip. As shown in (7) the singularity is defined in the form of square root, and the singularity intensity is characterised by the stress intensity factor (SIF denoted by K_I). Furthermore, crack propagation is controlled by the SIF. When the SIF reaches a critical value known as the fracture toughness (K_{IC}), the crack will propagate, otherwise, it remains static.

$$\sigma_{yy} = \frac{K_I}{\sqrt{2\pi}} r^{-\frac{1}{2}} \tag{7}$$

SIF has three modes, open mode, slide mode and tear mode. The above discussion and Fig.4 correspond to the open mode. For crack under more than one SIF, initiation and propagation of the cracking depends on combinations of the SIFs.

3.2 Stress singularities

Different cracks have different stress singularities. Initiation and propagation of a crack are controlled by stress singularity at the crack tip. It is necessary to make clear which factors would affect the singularity. As shown in Fig.5a, crack shape affects only the SIF, but not the stress singularity type as the singularity remains square root.

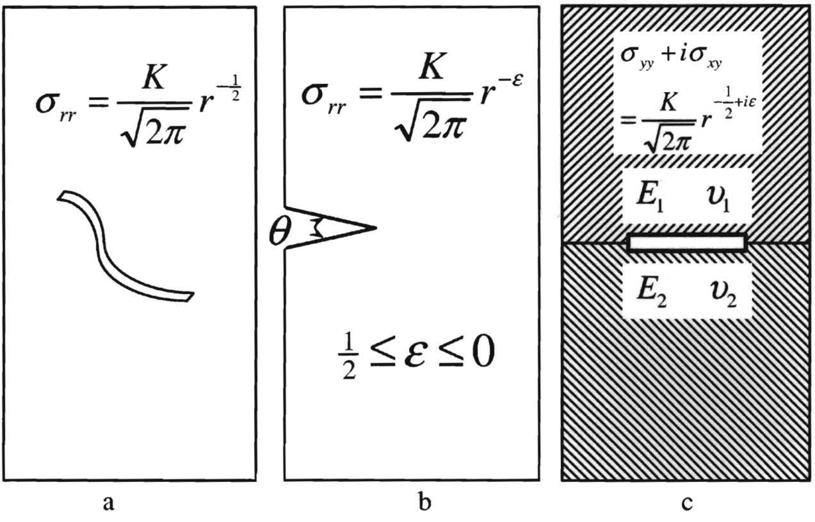

Figure 5 Different singularities

In the so-called notch problem shown in Fig.5b, the singularity depends on the notch angle θ with the singularity order varying between $-\frac{1}{2}$ and 0.

The interface crack problem shown in Fig.5c also has an effect on singularity order. The order becomes a complex number which makes the mode I and mode II SIFs coupling together.

If the material in the front of the crack tip varies gradually, the SIF may change relative to the homogeneous material case, but the singularity order will remain stationary at $-\frac{1}{2}$.

4. Beneficial effect of damage

Damage in the form of microdefects such as microcracks, microvoids, and microinclusions introduced into the brittle material during its production process play a role in enhancing the fracture toughness of the material.

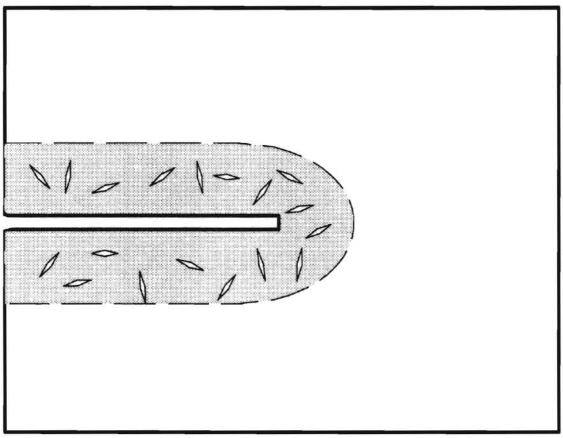

Figure 6 Macrocrack tip and microcracks around the tip

Under thermal or working loads, the microdefects already present in the material will grow until they reach a pre-design size. The material is designed so that, under these loadings, new microdefects would nucleate until all introduced microdefect sites are exhausted and a saturation stage of the microdefect is reached. When a macrocrack is developed in this kind of material as shown in Fig.6, the macrocrack tip is surrounded by microcrack clouds.

When the main crack propagates, microcracks nucleate and grow around the crack tip, and a process zone surrounding the main crack tip is developed. One factor of fracture toughness enhancement comes from the fact that the microcrack clouds in the process zone degrade the materials around the macrocrack tip. The tip is screened from remote loading by the degraded material. Another factor of fracture toughness enhancement is attributed to energy dissipation induced from the microcracks, because, as the macrocrack propagates, not the whole crack driving energy is used to open the macrocrack tip, but part of the energy is dissipated out in the processes of microcracking itself. These two factors provide the beneficial effects to fracture toughness of the materials. However, the fact is that fracture toughness enhancement at the macroscale is gained at the cost of damage at the microscale. Other damage styles like microinclusion and microvoid are also used for fracture toughness enhancement.

5. Review of previous studies

Beneficial damage, which is widely known as the shielding effect of microcracks has been investigated by three methods. The microcrack shielding effects were found by Hoagland et al., (1973), Claussen (1976) and Wu et al. (1978). Over the past 30 years, this kind of problem received considerable attention. Through experimental, computer simulation as well as analytical methods several solutions are presented in the literature. All these research works can be catalogued by three research models: continuum medium model, discrete interaction model, and finite element model.

5.1 Continuum medium model

The earlier continuum medium models assume the material around macrocrack tip as continuous material weakened by microcracks, and that interaction details among microcracks are ignored. Ortiz (1987) assumed that macrocrack tip is surround by a saturated zone in which the elasticity modulus is E_s. Outside the saturated zone is a transition zone in which the elasticity modulus varies from E_s to the normal value of undamaged material E_0 as shown in Fig.7a. A relation between SIF of the macrocrack tip and the effective elasticity modulus is derived as follows

$$\frac{K_t}{K_\infty} = \frac{1}{\sqrt{1+\beta(E_0/E_s-1)/(1-\upsilon_0^2)}} \tag{8}$$

where K_t is the SIF of the macrocrack tip; K_∞ is the SIF of the macrocrack tip when no microcrack is present; $\beta = 1.0942$; and υ_0 is the Poison's ratio of the undamaged

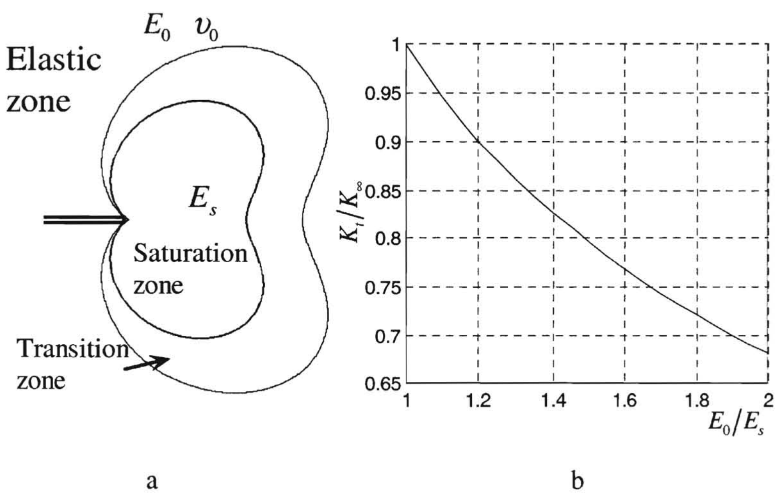

Figure 7 Continuous medium model

material.

The dependence of K_t/K_∞ on E_0/E_s is shown in Fig.7b. It can be seen that the normalised SIF of the macrocrack tip is less than one when the material in the front of the tip is weakened by the microcracks ($E_0/E_s > 1$). Thus the macrocrack tip is screened from the remote loading by the microcracks and the microcracks have beneficial effects on the fracture toughness of the material.

Later works considered interaction effects among the microcracks. The "self-consistent method" (Hill, 1965) was first developed for the microcrack-weakened solids by Budiansky and O'Connell (1976) with special attention directed to perfectly randomly distributed and weakly interacting microcracks.

The self-consistent method was further developed by Horii and Nemat-Nasser (1983) to take into account the effects of closed microcracks undergoing frictional sliding. Christensen and Lo (1979) proposed a three-phase "generalised self-consistent model." The "differential scheme" was investigated by Roscoe (1952, 1973), McLaughlin (1977), and Hashin (1988). Further, the "Mori-Tanaka method" was developed by Mori and Tanaka (1973), Benveniste (1986), and Zhao, Tandon, and Weng (1989). Some comparisons and assessments for the self-consistent method, the generalised self-consistent method, the Mori-Tanaka method, and/or the differential scheme were also presented by Horii and Sahasakmontri (1990), Laws and Dvorak (1987), Nemat-Nasser and Hori (1990), and Christensen (1990, for pure shear load only).

It is noted that the foregoing effective medium methods are only valid for low microcrack concentrations since they do not depend on the locations of microcracks. All of the aforementioned works can be categorised as stationary micromechanical models since all microcracks are assumed to be stationary; i.e., no microcracks are allowed to grow or nucleate during loading histories. For a constitutive theory to possess predictive capability, however, an evolutionary micromechanical damage model is warranted to account for pre-existing microcrack growth and/or new microcrack nucleation. In the current literature, there are indeed a number of micromechanical evolutionary damage models available. See, e.g., Krajcinovic and Fanella (1986), Fanella and Krajcinovic (1988), and Ju (1991b) by using the Taylor's model, as well as Sumarac and Krajcinovic (1987,1989), Krajcinovic and Sumarac (1989), Ju (1991a), Ju and Lee (1991), and Lee and Ju (1991) by using the self-consistent method. However, when microcrack concentration is larger and microcrack spacings are closer, strong microcrack interactions occur and hence the effective or continuum medium theories will no longer be appropriate.

5.2 Discrete interaction model

To estimate the strong interaction effect between the macrocrack tip and the near-tip microcracks, discrete interaction methods are developed. In this method, the macrocrack and microcracks are modelled as individual cracks that strongly interact with each other. The fracture toughness in such cases is strongly dependent on the geometry and spacing of microcracks.

Fig.8a shows a microcrack interacting with a macrocrack tip. Fig.8b shows the dependence of the SIF of the macrocrack tip on the location angle of the microcrack. The microcrack has a shielding effect on the macrocrack tip for angles $\varphi > 62^{\circ}$, and has an anti-shielding effect in the range $\varphi < 62^{\circ}$.

It should be noted that this problem corresponds to a body with macro and micro cracks subjected to opening mode loading. However, at the microscale, the problem is examined for a stress intensity factor vector $K = K_I + iK_{II}$ when the microcrack orientation (θ) varied being equal to the location angle of the microcrack. It should be also noted that the conclusion on the neutral angle between shielding and anti-shielding effect as above would not hold good for bodies subjected to loading other than opening mode loading.

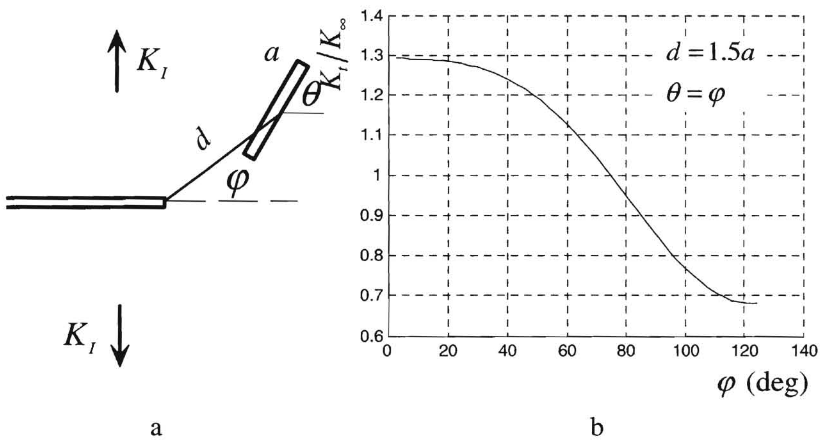

Figure 8 A microcrack interacting with a macrocrack tip

Strong microcrack interaction models can be found in the works represented by Gross (1982), Horii and Nemat-Nasser (1985), Hori and Nemat-Nasser (1987), Chen (1996), Zhao and Chen (1997), Han and Chen (1996, 1997, 1999a,b,c,d, 2000a,b,c) for two-dimensional deterministic microcracks, and by Kachanov and Montagut (1986), Kachanov (1987), Chudnovsky et al. (1987a,b), and Kachanov and Laures (1989) for two and three-dimensional deterministic arbitrary microcrack arrays.

After the interaction effect between one microcrack and the macrocrack tip is known, the interaction effect between all the microcracks and the macrocrack tip can be calculated by integrating the effect of one microcrack with the distribution function of the microcracks over the damaged zone.

Although the discrete interaction model can estimate the interaction effects among microcracks and macrocrack, it is difficult for this model to take into account the interaction effects among microcracks and the softening of materials around the macrocrack tip due to microcracks.

5.3 Finite element model

New finite elements are formulated for the beneficial damage problem. Based on a hybrid crack-tip element, the interaction problem as shown in Fig.9 is investigated in a finite plate with a macrocrack and a large number of microcracks.

The beneficial effects of the microcracks are calculated through the crack tip parameters. These parameters can be more easily obtained than that in the ordinary finite element

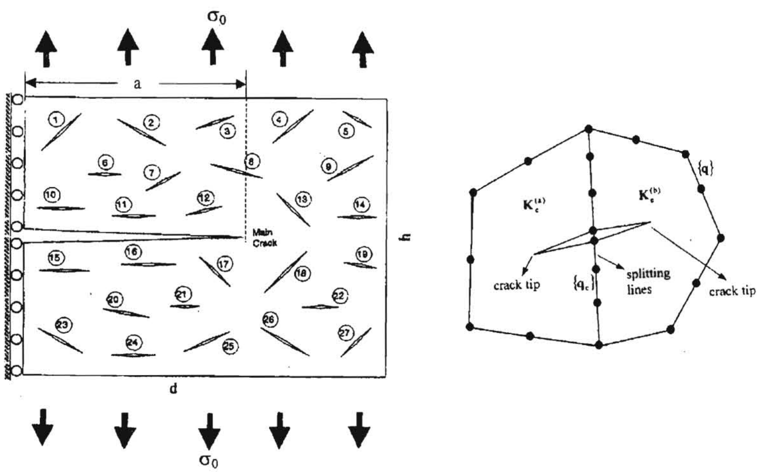

Figure 9 Finite element model and the hybrid crack element

method, because the hybrid crack-tip element has taken the crack tip singularity into the formulation, whilst the ordinary finite element method requires very detailed mesh to evaluate this parameter.

This method is easy to use in the examination of the effects of density, size, location, orientation, and growth rate of microcracks.

The works using this method can be found in Tong et al. (1973), Teixeira and Ji (1996a, 1996b), and Zeng et al. (2002).

6. Potential application of beneficial damage to railway engineering

Driven by fatigue of wheel/rail contact force, surface and internal cracks develop and propagate both in wheels and rails. Cracks in axles, bearings and bogie frames are often reported.

Cracks at various spots (head, web, foot) of the rail are not uncommon. Cracks in sleepers and loose fasteners are also reported. To ensure vehicle and passenger safety, periodical renewal and maintenance of the rail and wagon components must be taken. These undoubtedly increase the costs. The rail engineers seek crack growth control strategies to extend the working life of rail and other components and to decrease the frequency of maintenance. The beneficial damage method has the potential to be developed as a crack control strategy.

Fig.10 shows a vertical split head rail. In this case, the crack under the wheel/rail contact patch grows vertically within the railhead. A large split will result in complete failure of the rail.

If some damage can be introduced around the split tips, the damage will screen the crack tips from the fatigue force, thereby slowing down crack propagation rate. Furthermore, increase in the level of fracture toughness would also help.

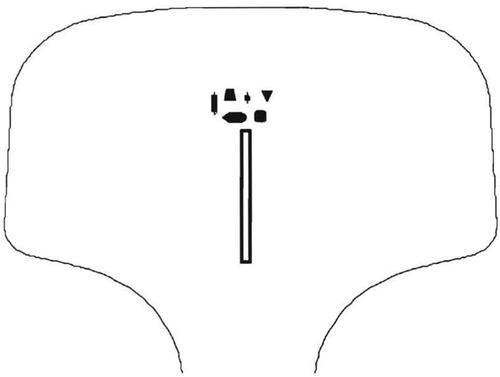

Figure 10 A vertical split tip surrounded by introduced damage

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