Progress in Mechanics of Structures and Materials – Moss & Dhakal (eds)
© 2007 Taylor & Francis Group, London, ISBN13 978-0-415-42692-3

An inverse approach for the evaluation of plastic constitutive parameters from structural response

M. Dhanasekar & S. Jia
Centre for Railway Engineering, Central Queensland University, Rockhampton, Australia

ABSTRACT: Nonlinear response of structures is commonly predicted from the experimentally evaluated elastoplastic material parameter dataset. This paper presents an inverse approach for the identification of plastic constitutive parameters from the response of structures obtained from the finite element method coupled with genetic algorithm. Behaviour of a slab containing centrally located fillet curved notch subjected to cyclic bending is used for the evaluation of the plastic properties and the hysteretic stress-strain curves obtained from the evaluation are in good agreement with the original curves. The approach could be used in developing constitutive relations for materials under complex 3D states of stress by judiciously designing an experimental setup involving complex shapes of structures with simple load application, which will significantly reduce the costs of generating complex 3D loading on simple shapes of specimens as traditionally followed. The material parameters obtained are more realistic as they account for the true status of the structures.

1 INTRODUCTION

Plastic constitutive parameters are conventionally measured from laboratory tests on standard specimens of simple shape (coupons), normally subjected to uniaxial loading (tension/ compression). The material data derived from such tests are traditionally used in the design of structures, a process that has generally been regarded as providing reliable results for structures that are assumed to have been subjected to static loading. However when fatigue life of structures under heavy loads (low cycle fatigue life) is required to be assessed, this process is increasingly being questioned because of two reasons. First the standard specimen testing does not account for complex stress states and geometry effects of the practical structures. Second the standard specimen testing normally does not represent the material of an aged structure especially in terms of damage level the structure is subjected to during its service, unless testing is carried out on specimens that are sampled from the real structure.

This paper presents an inverse approach for the identification of plastic constitutive parameters of engineering structures. For this purpose a finite element method (FEM) coupled with a Genetic Algorithm (GA) capable of dataset searching for material properties has been developed. The approach could be used in developing constitutive properties of materials under complex 3D states of stress by judiciously designing an experimental setup involving complex shapes of structures with simple load application, which will significantly reduce the costs of generating complex 3D loading on simple shapes of specimens as traditionally followed. The material parameters obtained from the inverse approach is expected to provide better prediction of fatigue lives of structures as they account for the true status of the geometry, the boundary and loading conditions as well as the stress states.

The mathematical formulation of the inverse method is based on converting the parameter identification problem as an optimisation problem. The objective function of the optimisation problem is defined as the sum of the squares of relative differences between experimental strains and FEA calculated strains over a region of interest. Due to the complexities of expressing the strains analytically, direct experimental or field determination is used as a matter of preference. In the absence of such experimental data, this paper has adopted numerical solutions as the prime objective of the paper is presenting the inverse approach that incorporates GA coupled with FEM rather than presenting practical outcomes.

The optimisation problems ought to deal with search techniques that work robustly and efficiently in the numerical paradigm. Gradient method and GA are two most popular search techniques used for such optimisation problems (Whitley, 2001). The gradient method is reported as efficient but not robust whilst the GA is found as robust but not as efficient as the gradient method. Therefore several researchers proposed a combined search technique to exploit the benefits of both methods. In this research the GA was adopted as the accuracy of the identified plastic constitutive data was considered more important than the time taken for the calculation.

Kirsch (1996) has detailed the mathematical theory for several kinds of inverse problems. In general every physical problem could be formulated either directly or inversely with the inverse formulation often being *ill-posed* with uncertainties of existence, uniqueness and stability of the solutions. As such, a robust solution technique would be of prime importance for the inverse formulations, with the efficiency or economy of the solution schemes dropped lower in preference (this might change if real time predictions are required).

Forestier et al (2001) proposed an inverse method coupled to a 3D finite element software and gradient optimisation technique for the determination of thermal and mechanical properties of metals. Complex forging operation was considered to generate complex stress states and plastic flow for better prediction of the plastic properties. Effect of potential noise (in experimental dataset) to the accuracy of the constants determined and the correlation coefficients for the four plastic constants are also presented.

Laurno et al. (2001) presented an experimental technique coupled with FE and error minimisation using classical optimisation methods for the determination of damage parameters for aluminum. Eight damage parameters for strain-rate dependent voided material were determined from the notched tensile test specimen dataset.

Araujo et al. (2002) have developed a finite element model coupled with a gradient optimisation method for the determination of elastic and piezoelectric properties from the experimental vibration dataset obtained from plates of composite materials. Six elastic properties and two piezoelectric properties were determined of which one of the transverse shear stiffness properties was determined poorly with large error.

Farukawa et al. (2002) presented an automated system for the simulation and parameter identification of inelastic constitutive models using a coupled experimental dataset and genetic algorithm (GA). Simplified Ramberg-Osgood model capable of defining plastic properties of materials subjected to uniaxial monotonic and cyclic loading cases is presented.

Zwick et al. (2002) presented a GA algorithm for the evaluation of two electric properties of the individual layers of the sandwiched composite from the reflection and transmission of electromagnetic waves. The paper provides detailed flow chart of the GA algorithm. The results were of very high accuracy.

Bucaille et al. (2003) proposed a finite element method for the determination of the plastic properties (yield stress and hardening exponent) of metals through instrumented indentation experiments using different sharp indenters to generate complex stress states and significant plastic flow.

Inverse Eigenvalue problems have extensively been reported in the literature for condition monitoring of engineering assets. Vibration data signature or noise levels are typically used in these studies to identify damages. Lin (2004) presented an inverse method for determining the location of a single crack in a simply supported single span beam.

Kang et al. (2004) have presented an inverse technique containing coupled FE and GA for the determination of the interfacial strength and stiffness properties of matrix composites. Four parameters were determined and the method was reported as being computationally expensive.

From the above review of recent literature base it could be observed that, although inverse method is increasingly being used for material parameter identification, there are relatively only few papers available which deal with the plastic properties determination. Even those few papers have focused on simple 1D models of plastic stress – strain relations. This paper reports an inverse method that includes a coupled FE and GA for the determination of the material constitutive parameters required for defining the kinematic and isotropic hardening of elastoplastic materials that form a structure of relatively complex geometry. A two step numerical simulation process was used to demonstrate the approach proposed in this paper. Firstly, a FEM analysis of a structural component subjected to low cycle fatigue loading was carried out using an *assumed dataset* of material properties to determine strain profiles at some key locations (considered as "experimental" dataset). Based on the strain profile, the inverse approach of the coupled FEA and GA was later employed with random initial parameter values to search for the assumed material parameters. The numerical simulation has shown that the combined GA and FEA can be an effective inverse method of material parameter identification for low cycle fatigue assessment of structures.

2 PROCEDURE OF INVERSE IDENTIFICATION OF MATERIAL PARAMETERS

The algorithm for the inverse approach of identifying material parameters is detailed in this section. The algorithm includes repeated execution of the FEM software ABAQUS and the search process of GA scripted in MATLAB.

Step 1: Obtain experimentally measured strain distribution at a region of interest from a structure that is subjected to loading sufficient to cause plastic deformation. Either monotonic or high-amplitude low cyclic loading history would suffice. The experiment could refer to testing carried out in the laboratory or in the field.

Step 2: Define the objective function Ov as in Eq. (1).

$$Ov = \sum [(\varepsilon_{ijC} - \varepsilon_{ijE}) / \varepsilon_{ijE}]^2 \quad (1)$$

in which ε_{ijE} are strain components at a region of interest obtained from the experiment and ε_{ijC} are strain components at the same regions calculated from the FEM.

Step 3: Using GA identify the plastic constitutive parameters:

(i) For each parameter, define a range (the upper and lower limits) that are dictated by the previous knowledge of material dataset for known materials; where such knowledge does not exist, much wider range could be provided. The wider the range, the larger will be the computational time; convergence is generally guaranteed as GA is robust.
(ii) Produce population (real floating point values, without encoding) of the parameters within the range specified in (i) above.
(iii) Calculate the objective function for the population with following steps
  (a) MATLAB calls ABAQUS to carry out the FEM calculations for each individual of the population,
  (b) Using ABAQUS post-process subroutine and MATLAB program, read the result in a binary file to obtain the strain distribution ε_{ijC},
  (c) Calculate the Objective Values from equation (1).
(iv) Calculate the fitness for the whole population.
(v) Select the individuals according to the fitness.
(vi) Recombine (crossover) individuals to produce offspring.
(vii) Mutate the offspring where necessary.
(viii) Calculate the objective function for the offspring.
(ix) Insert best offspring in the population replacing the worst parents to generate new population.
(x) Calculate the objective function for the new population using step (iii). Check if best objective value satisfies termination criterion, for example objective value $Ov < 5e-3$.
(xi) Repeat sub-steps (iv) to (x) until the specified termination criterion is satisfied.
(xii) Output the identified material property parameters.
(xiii) Using the parameters obtained in (xii), generate stress-strain curves.
(xiv) Compare the predicted stress-strain curves (step xiii as above) with the experimental curves (step 1).

3 A NUMERICAL SIMULATION

3.1 *Theory of material plasticity*

Material plastic theory comprises of three principal rules, namely the yield rule, the flow rule and the hardening rule. The rules used in the following rate independent numerical simulation are briefly summarised in this section (Chen et al. 1988; ABAQUS 6.5 Manual). Metallic materials are considered and cyclic loading is specifically addressed.

For most metals, the yield criterion is represented using Von Mises yield function,

$$f(\sigma - \alpha) = \sigma^0 \quad (2)$$

in which $f(\sigma - \alpha)$ is the equivalent Mises stress; α is backstress (for monotonic loading backstress will be zero); and σ^0 is the size of the initial yield surface (size of the elastic range).

The plastic flow is assumed to occur as per the associated plastic flow:

$$\dot{\varepsilon}_{pl} = \frac{\partial f(\sigma - \alpha)}{\partial \sigma} \dot{\bar{\varepsilon}}_{pl} \quad (3)$$

in which $\dot{\varepsilon}_{pl}$ represents the rate of plastic flow and $\dot{\bar{\varepsilon}}_{pl}$ is the equivalent plastic strain rate,

$$\dot{\bar{\varepsilon}}_{pl} = \sqrt{\frac{2}{3} \dot{\varepsilon}_{pl} : \dot{\varepsilon}_{pl}} \quad (4)$$

For cyclically loaded materials combined isotropic/kinematic hardening model is required. Kinematic hardening rule is represented as,

$$\dot{\alpha} = C \frac{1}{\sigma^0} (\sigma - \alpha) \dot{\bar{\varepsilon}}_{pl} - \gamma \alpha \dot{\bar{\varepsilon}}_{pl} \quad (5)$$

in which C and γ are the material parameters that define the initial hardening modulus and the rate at which the hardening modulus decreases with the increase in plastic strain, respectively.

The isotropic hardening behaviour of the material is modelled with the exponential law

$$\sigma^0 = \sigma|_0 + Q_\infty (1 - e^{-b\bar{\varepsilon}_{pl}}) \quad (6)$$

in which $\sigma|_0$ is initial yield stress, $\bar{\varepsilon}_{pl}$ is equivalent plastic strain, Q_∞ is the maximum increase in the

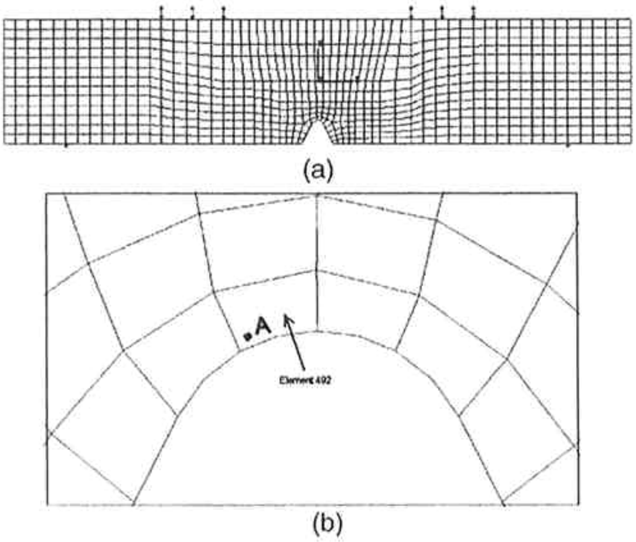

Figure 1. (a) The finite element model of the flexural slab. (b) An enlarged view of the notch tip area of the slab.

elastic range, and b defines the rate at which the maximum size is reached with the development of plastic straining.

This research has adopted the numerical simulation to identify the plastic material parameters, $\bar{\varepsilon}_{pl}\,|_0$, σ_0, α, γ, C, Q_∞ and b with the inverse model that couples the FEM with GA, where $\bar{\varepsilon}_{pl}\,|_0$ is the initial equivalent plastic strain; symbols for all other six plastic parameters have previously been defined in this section.

3.2 *FEA modelling of bending of a notched slab*

A simply supported slab containing fillet curved notch at its mid span subjected to four point bending is considered here as a numerical example to illustrate the method of identifying plastic constitutive parameters using the inverse approach presented in this paper. The length and thickness of the slab were kept as 200 mm × 40 mm respectively, with the width very large. The slab was assumed to be subjected to cyclic uniformly distributed line loadings.

The elastic constants were defined as $E = 2.1e+5$ MPa and $\mu = 0.3$. The initial yield stress was set as $\sigma\,|_0 = 200$ MPa.

The finite element calculation was implemented within the MATLAB7.0 script, which repeatedly called for the execution of ABAQUS6.5. The mesh used in the FEA is shown in Figure 1. Element type used was the 8-node plane strain, second-order element (ABAQUS element CPE8).

The following procedure was used to demonstrate the capability of the inverse approach for the plastic material parameter identification. Although the algorithm assumes that we require experimental strain profile (Step 1, Section 2), we do not have that information

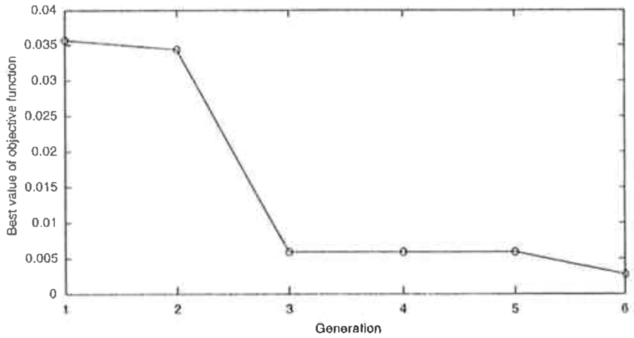

Figure 2. The evolution curve of best value of objective function as a function of generations.

at the time of writing this paper. Therefore a forward FEA was carried out as a first step to represent the "experimental" strain profile. For the purpose of calibration of the inverse approach presented in this paper, eight plastic parameters of the material of the slab have been assumed as a priori. The magnitude of these parameters b, Q_∞, $\bar{\varepsilon}_{pl}\,|_0$, C, γ, α_{11}, α_{22}, α_{33} are 0.26, 2000 MPa, 0.43, 25500 MPa, 81.00, 128.0 MPa, −181 MPa, and 53.0 MPa respectively (Doghri, 1993).

3.3 *Result*

Based on the strain distribution predicted by the FEM, the GA method searched for the "best" parameters within the range specified using random initial parameter values until the value of the objective function (Eq. 1) reached the termination criterion value of 5e-3. Several trials were used to check for the robustness of the GA. In the calculation presented here the population of each generation had six subpopulations and each subpopulation had 20 individuals. In the sixth generation, the objective function for one of the individuals reached 2.7e-3, satisfying the termination criterion. Figure 2 shows the evolution of the best value of objective function with the increase in the number of generations.

The assumed original material parameter values were extracted from the American Standard AISI1010 provided in Doghri (1993) and ABAQUS 6.5 Manual. GA identified material values and specified ranges are listed in Table 1. The maximum relative error of the GA identified values is only 2.72% compared to the assumed original values.

Figures 3 and 4 show the hysteretic stress-strain curves of the material of the slab; both the predictions and the assumed original parameters are shown the figures. In both of these diagrams, stress and strain directions refer to the x axis (longitudinal flexural direction) illustrated in Figure 1(a). Figures 3 and Figure 4 are curves of the longitudinal strain vs. longitudinal stress at the point A of the slab (Figure 1(b)), subjected to loading history that ranged from 0 kN to 6.8 kN and −6.8 kN to 6.8 kN respectively.

Table 1. The original and the GA identified parameter values.

Parameters	Original value	GA identified value	Specified range	
			Upper	Lower
b	0.26	0.26	0.3	0.2
Q_∞	2000 MPa	2044 MPa	2100 MPa	1900 MPa
$\bar{\varepsilon}_{pl}$	0.43	0.42	0.45	0.35
C	25500 MPa	25534 MPa	25600 MPa	25400 MPa
γ	81.00	81.40	85.00	75.00
α_{11}	128.0 MPa	128.5 MPa	135.0 MPa	120.0 MPa
α_{22}	-181 MPa	-180.3 MPa	-175.0 MPa	-185.0 MPa
α_{33}	53.0 MPa	51.6 MPa	55.0 MPa	45.0 MPa

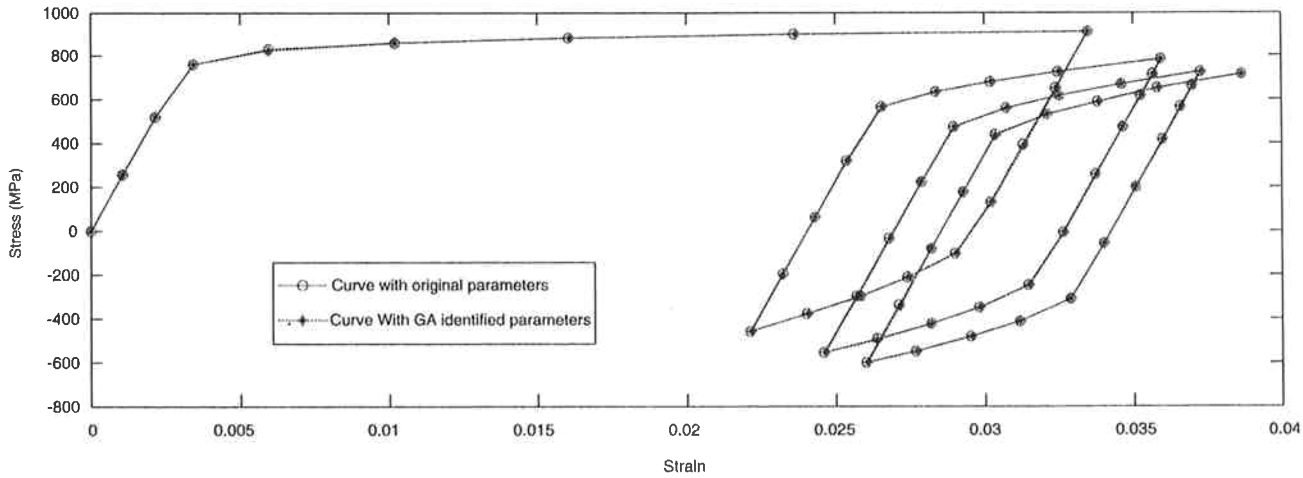

Figure 3. Stress vs. strain curves of the notch tip region for the loading range 0 to 6.8 kN.

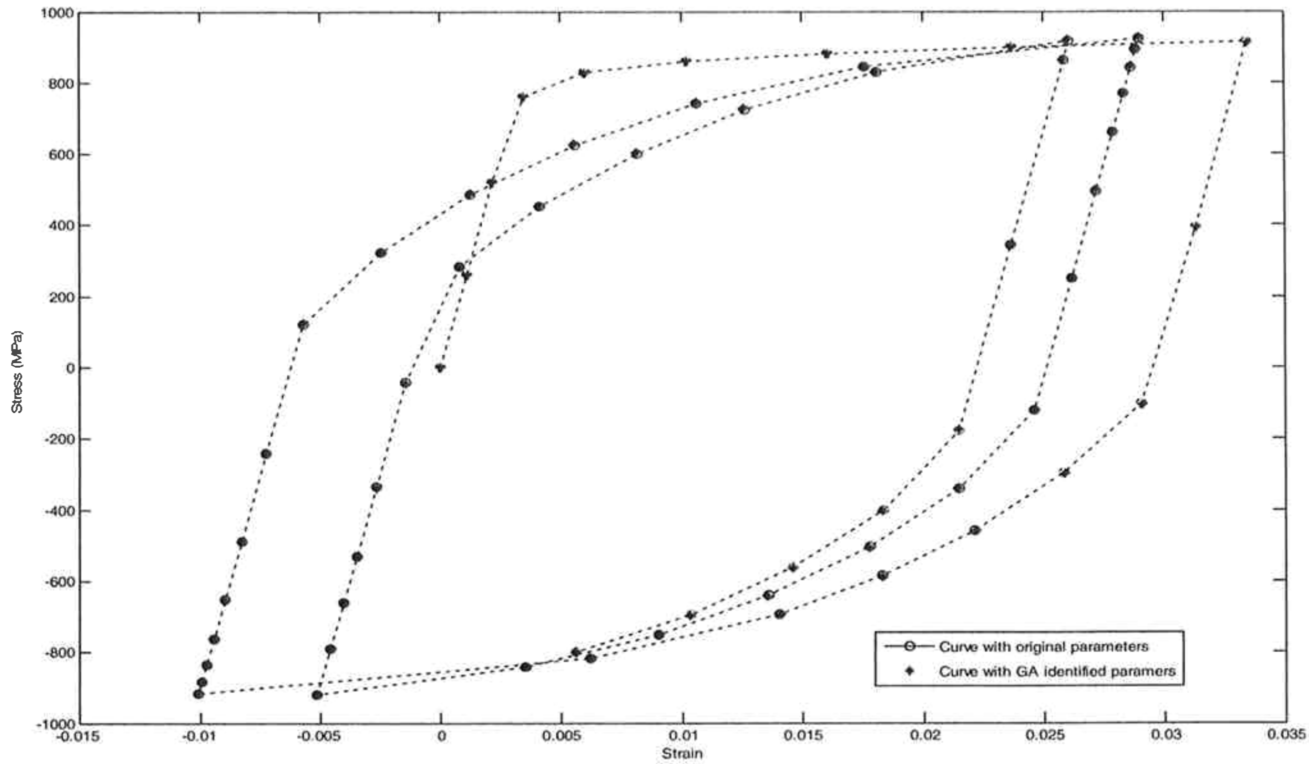

Figure 4. Stress vs. strain curves of the notch tip region for the loading range -6.8 kN to 6.8 kN.

In these figures, the FEM coupled GA identified stress-strain curves are very close to the curves corresponding to the assumed original parameters for each of the load cycles (loops) in both loading cases. Plots of vertical stress – vertical strain and shear stress – shear strain curves have also been drawn and examined for the goodness of the predicted values; all plots have reinforced the above finding.

Analyses for the same information as above, with the loading range from 0 kN to 8 kN and −8 kN to 8 kN have also been carried out to examine sensitivity to loading if any. The results of these analyses are not explicitly provided here due to space restriction.

This demonstrates that the inverse method that couples the GA and FEM can be used for material plastic parameter identification that is accurate enough for engineering applications.

4 DISCUSSION AND CONCLUSION

This paper has proposed an inverse method coupling the GA and the FEM as an approach for material plastic parameter identification from structural response. The inverse method is based on an objective function that requires minimisation of the difference between the state of strain at a region on the structure predicted experimentally and inversely. Strain data are normally small decimal numbers which range around 0.01. Therefore, the relative difference between the experimental data and the finite element coupled GA calculated data are more suitable than the absolute difference to be used for defining the objective function. The numerical calculation in this research showed that the equation 1 is an efficient objective function.

GA is generally regarded as not as efficient as other search techniques such as the gradient method. The method provided in this paper consumed approximately 8 hours to obtain the results of the example described in section 3 in a personal computer platform. Although the time consumed is considerable, it is still much cheaper than testing a specimen from a structure. Furthermore, by using super computer, parallel algorithm, etc. the efficiency of the inverse method presented in the paper (without modifying the GA technique) could be improved.

GA is a robust numerical searching method. Finite element method is a mature and broadly accepted engineering structure calculation method. Coupling these two methods together provides advantages of both methods for the material plastic parameter identification.

The example provided in the paper has demonstrated that the inverse approach is feasible for material plastic parameter identification of engineering structures, which is a convenient and accurate enough method. The experimental validation is being done in our laboratory to further investigate feasibility of this method.

REFERENCES

ABAQUS 6.5 2005 *Online Documentation*, Chapters 1 of Example Problems and Chapter 4 of Theory Manual.

Araujo, A.L., MotaSoares, C.M., Herskovits, J. & Pederson, P. 2002 Development of a finite element model for the identification of mechanical and piezoelectric properties through gradient optimisation and experimental vibration data, *Composite Structures* 58: 307–318.

Bucaille, J.L., Stauss, S. Felder, E. & Michler, J. 2003 Determination of plastic properties of metals by instrumented indentation using different sharp indenters,*Acta Materialia* 51(6): 1663–1678.

Chen W.F. & Han, D.J. 1988 *Plasticity for Structural Engineers*, Springer-Verlag New York Inc.

Doghri, I. 1993 Fully Implicit Integration & Consistent Tangent Modulus in Elasto-Plasticity, *International Journal for Numerical Methods in Engineering* 36: 3915–3932.

Forestier, R., Massoni E. & Chastel, Y. 2002 Estimation of constitutive parameters using an inverse method coupled to a 3D finite element software, *Journal of Materials Processing Technology* 125–126: 594–601.

Furukawa, T. Sugata, T. Yoshimura, S. & Hoffman, M. 2002 An automated system for simulation and parameter identification of inelastic constitutive models, *Computer Methods in Applied Mechanics and Engineering* 191(21–22): 2235–2260.

Kang, Y. L., Lin X. H. & Qin, Q. H. 2004 Inverse/genetic method and its application in identification of mechanical parameters of interface in composite, *Composite Structures* 66: 449–458.

Kirsch, A. 1996 *An introduction to the mathematical theory of inverse problems*, Applied Mathematical Sciences, Vol. 120, Springer.

Lauro, F., Bennani, B., Croix, P. & Oudin, J. 2001. Identification of the damage parameters for anisotropic materials by inverse technique: application to an aluminium, *Journal of Material Processing Technology* 118: 472–477.

Lin, H.P. 2004 Direct & inverse methods on free vibration analysis of simply supported beams with a crack, *Engineering Structures* 26; 427–436.

Whitley, D. 2001 An overview of evolutionary algorithms: practical issues & common pitfalls, *Information and Software Technology* 43(14): 817–831.

Zwick, T., Haala, J. & Weisbeck, W. 2002. A Genetic Algorithm for the evaluation of material parameters of compound multilayered structures, *IEEE Transactions on Microwave Theory and Techniques* 50(4); 1180–1187.