

# Network-Based Output Tracking Control for Continuous-Time Systems



Dawei Zhang

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Professor Qing-Long Han



# Declaration

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# Abstract

This dissertation is concerned with network-based output tracking control for both linear systems and nonlinear systems described by T-S fuzzy models. The systems are classified as Case I: the systems can be stabilized by non-delayed static output feedback controllers; and Case II: the systems can not be stabilized by non-delayed static output feedback controllers, but can be stabilized by delayed static output feedback controllers.

The use of a communication network in a tracking control system enables a remote execution of tracking control, reduces system complexity, and increases system flexibility with low cost. When exchanging data between a system and a tracking controller over a network, network-induced delays and/or packet dropouts are inevitable and occur in two different types: one is the unavoidable network-induced delays and/or packet dropouts which are the sources of poor performance and/or instability (deterioration effects: negative effects) for the systems described in Case I, and the other is the intentionally introduced network-induced delays and/or packet dropouts which can be used purposefully to improve the tracking control performance for the systems described in Case II. In this dissertation, both positive and negative effects of network-induced delays and/or packet dropouts on the system stability and tracking performance are investigated.

Case I: For a linear system, network-based output tracking control via an observer-based controller is considered. The update inputs of the system and an observer-based controller are asynchronous due to network-induced delays and/or packet

dropouts in the controller-to-actuator channel. Taking into consideration the asynchronous inputs, the network-based tracking control system is modeled as a system with two different interval time-varying delays. Notice that a separation principle can not be applied to design an observer gain and a control gain due to the asynchronous inputs of the plant and the controller. Instead, a novel design algorithm is proposed by applying a particle swarm optimization technique with the feasibility of the stability criterion to search for the minimum  $H_\infty$  tracking performance and the corresponding gains. The network-based output tracking control via a state feedback controller is also considered. Criteria for stability and tracking performance are also obtained.

For a nonlinear system described by a T-S fuzzy model, network-based output tracking control via a fuzzy state feedback controller is studied. The network-based tracking control system is represented by an asynchronous T-S fuzzy system with an interval time-varying sawtooth delay. Notice that a routine relaxation method for a traditional T-S fuzzy system can not be used for stability analysis and controller design of the asynchronous fuzzy system. Instead, a new relaxation method is proposed by utilizing asynchronous constraints on fuzzy membership functions to introduce some free-weighting matrices. Using the proposed relaxation method and a discontinuous simple Lyapunov-Krasovskii functional, some new delay-dependent criteria for  $H_\infty$  tracking performance analysis and existence of a fuzzy state feedback controller are formulated in terms of linear matrix inequalities.

Case II: For a linear system, by intentionally inserting a communication network between the system and a static output feedback controller, a network-induced delay is purposefully produced in the feedback control loop to achieve a stable and satisfactory tracking control. A new discontinuous complete Lyapunov-Krasovskii functional is constructed to derive a delay-dependent criterion for  $H_\infty$  tracking performance analysis. By applying a particle swarm optimization technique with feasi-

bility of the criterion, a novel tracking control design algorithm is proposed to search for the minimum  $H_\infty$  tracking performance and the corresponding control gain.

For a nonlinear system described by an asynchronous T-S fuzzy model, an  $H_\infty$  tracking performance criterion is derived by employing a new complete Lyapunov-Krasovskii functional and taking into consideration the asynchronous constraints. Then a particle swarm optimization algorithm is proposed to design the fuzzy static output feedback tracking controller.

Several examples are provided to demonstrate the effectiveness of the proposed methods for Case I and Case II, respectively.



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# Notations

$\triangleq$	equal by definition
$\subseteq$	contained in or equal to
$\subset$	contained but not equal to
$\in$	set membership
$\cup$	union
$\mathbb{Z}$	the set of non-negative integers
$\mathbb{N}$	the set of positive integers
$\mathbb{R}^n$	the $n$ -dimensional Euclidean space
$\mathbb{R}^{n \times m}$	the set of all the real $n \times m$ matrices
$\mathcal{L}_2[0, \infty)$	the space of square integrable functions on $[0, \infty)$
$\ x\ $	Euclidean norm of a vector $x$
$\ A\ $	induced 2-norm of a matrix $A$
$\text{diag}\{A_1, \dots, A_n\}$	diagonal matrix with $A_1, \dots, A_n$ on the diagonal
$A^T$	transpose of a matrix $A$
$A < B$	$A - B$ is negative definite for symmetric matrices $A$ and $B$
$A \leq B$	$A - B$ is negative semi-definite for symmetric matrices $A$ and $B$
$I$	an identity matrix
$\lambda_{\min}(P)$	the minimum eigenvalue of a symmetric matrix $P$
$\lambda_{\max}(P)$	the maximum eigenvalue of a symmetric matrix $P$
$\begin{bmatrix} A_{11} & * \\ A_{21} & A_{22} \end{bmatrix}$	$\begin{bmatrix} A_{11} & A_{21}^T \\ A_{21} & A_{22} \end{bmatrix}$



# Chapter 1

## Introduction

Output tracking control has wide practical applications in mobile robot control [2], [45], mechanical systems [5], [10], [16], DC motor control [17], [67], attitude tracking and path following in ocean [43], [118], satellite and spacecraft attitude control [51], [59], [115], [117], flight control [74], [119], unmanned vehicles [83], [112] and so on. These practical applications provide a major incentive for research on output tracking control. Generally speaking, the objective of output tracking control is to drive the output of a physical plant via a feedback controller to track the output of a reference model or a prescribed trajectory signal as close as possible. There are two classes of physical plants in practice. One class of plants assume that all the state variables are available for measurement, and the output tracking control can be implemented via a state feedback controller. The other class of plants have part of the states that can not be directly measured, and the output tracking control can be implemented via an output feedback controller or an observer-based controller. Throughout this dissertation, output tracking control via a state feedback controller, an output feedback controller or an observer-based controller is called state feedback tracking control, output feedback tracking control and observer-based tracking control, respectively.

There are some results available on output tracking control by using several control approaches such as adaptive control approach [13], [60], [127], variable structure

control approach [21], [43], [96], linear quadratic optimal control approach [45], [61], [71], [99], [107], [117], [121] and  $H_\infty$  output tracking control approach [10], [11], [12], [26], [30], [68], [75], [131]. So far these approaches have been applied to design tracking controllers of many systems encountered in engineering applications. For example, the  $H_\infty$  output tracking control approach has been used for tracking controller design of manufacturing systems [10], nervous systems [11], non-holonomic mechanical control systems [12] and pulse-width modulation systems [26]. The basic process of the  $H_\infty$  output tracking control approach can be described as follows: a physical plant and a reference model are first transformed into an augmented system (or a tracking error system) with an external disturbance, then a tracking controller depending on feedback outputs (or state errors) is designed such that the augmented system (or the tracking error system) is asymptotically stable with a prescribed  $H_\infty$  tracking performance. From the physical meaning point of view, the idea of this approach is that the effect of any unknown external disturbance inputs on the output tracking error is attenuated with a desired  $H_\infty$  performance. Using this approach, the disturbance input of the plant and the exogenous signal of the reference model are not required to be specified. In addition, the designed  $H_\infty$  tracking controller can be easily implemented without resorting to a feedback linearization technique or a complicated adaptive scheme.

It is worth pointing out that most aforementioned results on output tracking control are only suitable for a traditional tracking control system where a physical plant and a tracking controller are located close to each other and connected in a point-to-point wiring manner. However, as a result of the high complexity and wide geographical distribution of modern industrial systems, the traditional point-to-point architecture is no longer able to meet new requirements such as modularity, integrated diagnostics, the ability of remote control, easy maintenance and low cost. To meet the new requirements, a communication network is inserted between the

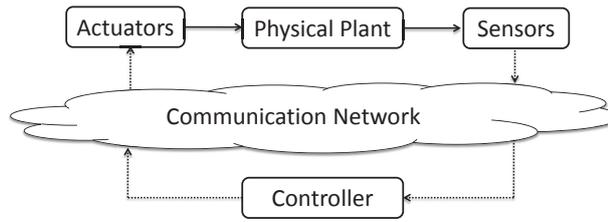


Figure 1.1: A typical configuration of a network-based control system

physical plant and the tracking controller. Compared with a traditional tracking control system, the insertion of the network in a tracking control system leads to network-induced delays and/or packet dropouts, which have an effect on system stability and tracking performance. In this dissertation, we will deal with modeling, stability analysis and controller design of a network-based tracking control system in the presence of network-induced delays and/or packet dropouts.

## 1.1 Network-based control systems

In a network-based control system (NCS), the controller is connected to the physical plant via a communication network that provides access to all the sensors and actuators, as depicted in Figure 1.1. Such a network-based controller can eliminate unnecessary wiring, efficiently share data, and easily fuse global information to make decisions over a wide operating range. In the past decades, there has been a rapidly growing interest in NCSs due to their practical requirements and theoretical significance, see survey papers [4], [46], [47], [129], [146], PhD theses [50], [73], [77], [86], [90], [94], [105], [143], [144], [152], books [48], [136] and the references therein. In the literature, emerging issues in NCSs can be summarized as modeling of an NCS, stability analysis, control synthesis, design of network scheduling protocols, network reliability and security, co-design of both network scheduling and network control, and applications of network-based control. Among these issues, the primary one is modeling of an NCS. In the following, we review the current research on an NCS from a modeling method perspective and list some typical NCS modeling methods.

### **Discrete-time Model Approach**

The process of this approach is that a continuous-time controlled plant in an NCS is first discretized into a discrete-time system with a fixed sampling [38], [104] or a time-varying sampling [19] over a sampling interval, then the closed-loop system is augmented to be a finite dimensional discrete-time model including the past values of the system output and the past values of its input. Using the discrete-time model approach, the stochastic optimal control problem of an NCS with network-induced delays which are shorter than a sampling period  $h$  in [95] or longer than  $h$  in [49] is addressed. The main difficulty of this approach is in the modeling and design complexity when the plant receives more than one control input in a sampling interval.

### **Switched System Approach**

Using this approach, an NCS is first formulated as a discrete-time switched system under some ideal assumptions, then the problems of stability analysis and controller design for an NCS can be reduced to the corresponding ones for the switched system. As pointed out in [50], the disadvantage of this approach is that the controller is required to work at a higher frequency than the sampling frequency. In [78] and [79], an NCS is transformed into a sampled-data model with an equally divided sampling period  $T = h/N$ , where  $h$  is the sampling period and  $N$  is the number of division; by choosing the network-induced delay  $\tau = dT$  with a changing integer  $d$ , an arbitrary switch model is established to describe the NCS. The stability and disturbance attenuation property of an NCS with both network-induced delays and packet dropouts in time average sense [78] or bounded absolutely [79] are analyzed. In [159], an NCS is modeled as a switch system with four subsystems which describe four cases that whether packet dropouts occur in the sensor-to-controller channel and the controller-to-actuator channel. Then some criteria for output feedback stabilization of the switch system are derived by using a piecewise Lyapunov functional approach and an average dwell time method.

### **Model-based System Approach**

In a model-based NCS, an explicit model of a controlled plant is employed to provide a state estimate of the plant between transmission times. The transmission time is assumed to be constant in [87] and Markov chain-driven or identically independently distributed in [88]. In [87] and [88], the controlled plant is a linear system and only a sensor-to-controller channel model-based NCS (that is, the controller and the actuator are cascaded together) is considered. When the plant is nonlinear and connected with a given dynamic controller via the sensor-to-controller channel and the controller-to-actuator channel, a network protocol is presented to search for the maximum allowable transfer interval (MATI) in consideration of time-varying communication delays and possibility of packet dropouts in [101]. However, it is difficult to design the controller for a two-channel model-based NCS because of the asynchronous inputs of the plant and the controller, which is caused by network-induced delays and packet dropouts in the controller-to-actuator channel.

### **Stochastic System Approach**

The most important feature of this approach is that network-induced delays or packet dropouts in an NCS are characterized by two types of stochastic models: a Markov statistic process model and a Bernoulli random binary distribution model. For example, network-induced delays in both the sensor-to-controller channel and the controller-to-actuator channel are modeled to be two homogeneous Markov chains in [154]. In [112], packet dropouts are modeled to be a simple two-state Markov process. Other Markov process modeling can be found in [50], [116], [143], [144] and so on. In [147], both the sensor-to-controller delays and the controller-to-actuator delays are described by linear functions of stochastic variables satisfying a Bernoulli random binary distribution. Similarly, packet dropouts in two channels are assumed to obey Bernoulli distribution in [140]. Although Markov statistic process and Bernoulli distribution are two attractive modeling methods, it is complex

to characterize network-induced delays and packet dropouts simultaneously. And a prior probability distribution is necessary in [50], [140], [143], [144] and [154].

### **Time-varying Delay System Approach**

The fundamental idea of this approach is to formulate an NCS as a system with an interval time-varying sawtooth delay induced by sample-and-hold behaviors, packet dropouts and network-induced delays. The sawtooth delay is on some time intervals between updating instants which consist of available sampling instants (packet dropouts can be considered as variations in the sampling intervals or delays) and network-induced delays, see details in [28], [29], [54], [56], [58], [91], [125], [137], [149] and [157]. This approach can easily capture many features in an NCS such as sampling, quantization, network-induced delays and packet dropouts. Moreover, this approach is able to incorporate network-induced delays which larger than the sampling interval without increasing modeling complexity. Furthermore, it is not difficult to use a Lyapunov-Krasovskii functional method to derive some delay-dependent criteria for stability analysis and controller design of an NCS. Notice that less conservative criteria can be used to achieve a better maximum allowable delay bound (MADB) of an NCS. So it is important to reduce the conservatism of the delay-dependent criteria [29], [46], [56]. Recently, in [90] and [91], a time-varying delay impulsive system is employed to describe an NCS and a modified Lyapunov-Krasovskii functional is proposed to derive some less conservative stability criteria.

Besides the above-mentioned modeling methods of an NCS, there are some other modeling methods such as a sampling NCS model [92], [103] and a master-to-slave NCS model [109], [111]. In these NCS modeling methods, network-induced delays and packet dropouts, which have an effect on the stability and system performance of an NCS, are two essential issues. Based on different NCS models, there are a large number of results available on stability and stabilization but a few results on network-based output tracking control.

## 1.2 Network-based output tracking control

Network-based output tracking control has gained an increasing interest in recent years, see for example, [15], [28], [37], [54], [133], [137], [139], [148] and [150]. The defining feature of a network-based tracking control system is that a physical plant and a tracking controller are distributed and they exchange data through a communication network. The use of the network in a tracking control system makes system modeling, stability analysis and controller design more complex. Due to the introduction of the network, network-induced delays are inevitable and some packets may be dropped. To achieve a stable and satisfactory tracking control, it is significant to investigate the effect of network-induced delays and/or packet dropouts on system stability and tracking performance.

Taking network-induced delays and/or packet dropouts into account, several tracking control approaches have been proposed to deal with the problems of network-based output tracking control for both linear and nonlinear systems. For example, in [15] and [133], an NCS with uncertain, time-varying samplings and network-induced delays is transformed into a tracking error system with an external input caused by a feedforward signal mismatch, then *the input-to-state stability property* of the tracking error system is analyzed to ensure with a prescribed tracking error; *the linear quadratic optimal approach* is applied to study the optimal tracking control for an NCS with across erasure communication links in [37] and the quantized tracking control for a discrete-time NCS in [150], respectively; in [148], a discrete-time wireless NCS is represented by a Markov jump system by using Markov chains to model communication delays and packet dropouts, then *the network-based  $H_2$  output tracking control* is considered; in [28], [54], [137] and [139], under the consideration of network-induced delays and packet dropouts, an NCS is modeled as a system with an interval time-varying delay and *the network-based  $H_\infty$  output tracking control* is investigated.

In the literature [15], [28], [37], [54], [133], [137], [139], [148], [150], it is usually thought that the presence of a network-induced delay in the feedback control loop can degrade tracking performance or even cause system instability. However, the positive effect of a network-induced delay on network-based output tracking control has not been investigated. Moreover, the existing works [15], [28], [37], [133], [137], [139], [148] and [150] have largely been focused on network-based state feedback tracking control for linear systems where all the states are measurable. But there has been little work on network-based output tracking control for a system with some unmeasurable states. Furthermore, only a state feedback tracking controller is considered in the aforementioned results. Using a state feedback controller, a two-channel NCS is equivalent to a sensor-to-actuator channel NCS without affecting the system stability and performance, where the two-channel NCS means the NCS where a controlled plant and a controller are interconnected via the sensor-to-controller channel and the controller-to-actuator channel. However, this is not the case for an observer-based controller. It is interesting to dig out the difference in modeling, stability analysis and controller design of a network-based tracking control system between state feedback tracking control and observer-based tracking control.

On the other hand, many practical systems are nonlinear and Takagi-Sugeno (T-S) fuzzy models can be employed to represent a class of nonlinear systems on a compact region, which demands study on network-based output tracking control for T-S fuzzy systems. Recently, without using the knowledge of membership functions, a network-based fuzzy output tracking controller has been designed in [54]; however, the control design method provides a fuzzy state feedback controller with a same control gain for different fuzzy control rules, which means that only a network-based linear controller is developed, see [93]. It is obviously conservative to employ a linear controller to perform the network-based output tracking control for a nonlinear system via a T-S fuzzy model. Notice that less conservative delay-dependent criteria for

$H_\infty$  tracking performance analysis and controller design can provide an appropriate tradeoff between the maximum allowable delay and the minimum  $H_\infty$  tracking performance. It is necessary to develop some less conservative results on network-based output tracking control for a system, especially for a T-S fuzzy system.

From the above discussion, one can conclude that the following research problems need to be solved.

- Does a network-induced delay have a positive effect on network-based output tracking control? If yes, how to use the network-induced delay to produce a stable and satisfactory tracking control?
- How to design a network-based output tracking controller for a system with some unmeasurable states?
- What kind of difference does there exist in modeling, stability analysis and controller design of a network-based tracking control system between state feedback tracking control and observer-based tracking control?
- How to design a network-based nonlinear fuzzy tracking controller for a T-S fuzzy system by using the knowledge of fuzzy membership functions?
- How to achieve a less conservative delay-dependent criterion for  $H_\infty$  tracking performance analysis and controller design?

We will solve the above problems in Chapter 2, 3, 4, 5 and 6 of this dissertation, respectively.

### 1.3 Significance of this research

From a practical point of view, network-based output tracking control has been applied in many industrial and military systems such as DC motors [17], spacecrafts [59] and unmanned vehicles [112], and it has extensive potential applications

in space and terrestrial exploration, access in hazardous environments, domestic robots, experimental facilities, manufacturing plants and so on. Compared with a traditional tracking control system, one important advantage of a network-based tracking control system is that the insertion of a communication network between a physical plant and a tracking controller enables a remote execution of tracking control. Meanwhile, the use of the network in the feedback control system can effectively reduce the system complexity, and increase the system flexibility and reliability with nominal economical investments. More specifically, control and feedback data are exchanged among the system's components (sensors, actuators, controllers, etc.) in the form of digital signals through a communication network. Digital signals can be easily integrated and have strong protection against a variety of noise sources. Since a digital cable can carry multiple signals on a signal channel, the amount of cabling can be reduced dramatically. On the other hand, the application of the network in the feedback control loop imposes communication delays or possibility of packet dropouts, which must be considered in system modeling, tracking performance analysis and tracking control design. To date there have been more NCS experimental platforms for simulation of network-based tracking control. For example, an experimental result on network-based tracking control of a unicycle-type mobile robot with the effect of network-induced delays is demonstrated in [2], where the tracking controller is implemented in such a way that the robot located at the Eindhoven University of Technology in The Netherlands is controlled from the Tokyo Metropolitan University in Japan and viceversa. To sum up, advances in the existing and potential industrial applications, the advantages arising from an NCS, and some challenging issues such as network-induced delays and packet dropouts provide some general motivations of the present study.

From a theoretical point of view, in this dissertation, we will propose some solutions to the problems mentioned at the end of Section 1.2.

*This dissertation will investigate the positive effect of a network-induced delay on network-based output tracking control for a system that can not be stabilized by a non-delayed static output feedback controller, but can be stabilized by a delayed static output feedback controller.* To the best of the author's knowledge, this is the first time to investigate the positive effect of network-induced delay on tracking control performance and the proposed idea can be considered a unique contribution to the field. It is a common view that the presence of a network-induced delay in the feedback control loop will cause system instability and performance degradation. For the system under consideration, *we take a different and novel view* and we investigate whether the network-induced delay has a positive effect on system stability and tracking control performance. Such a class of systems encountered in engineering are a damped harmonic oscillator, a structural system, an internal combustion engine and so on, see [1], [34], [84], [132]. For these systems, ignoring the network connection, and cascading the systems and static output feedback controllers together, it is impossible to ensure a stable tracking control; however, by intentionally inserting a network between the systems and static output feedback controllers, network-induced delays are purposefully produced in the feedback control loop to achieve a stable and satisfactory tracking control. Notice that a simple Lyapunov-Krasovskii functional can not be employed for  $H_\infty$  tracking performance analysis of the considered system since it requires that the system can be stabilized by a non-delayed controller. Instead, in this dissertation, a new discontinuous complete Lyapunov-Krasovskii functional will be constructed to derive some delay-dependent  $H_\infty$  tracking performance criteria.

*This dissertation will propose some network-based output tracking control strategies for a system with some unmeasurable states.* In most results on network-based output tracking control [15], [28], [37], [133], [137], [139], [148] and [150], the system states are assumed to be completely measurable. However, in many practical

situations, it is physically difficult to measure all state variables of a system due to economical or technical reasons. When not all the state variables are available for direct measurement, two feasible alternative ways to perform the network-based output tracking control are using an output feedback tracking controller and an observer-based tracking controller. Compared with a dynamic output feedback controller, a static output feedback controller can be implemented with low cost. Network-based static output feedback tracking control and observer-based tracking control will be considered in this dissertation.

*This dissertation will discuss the difference in modeling, stability analysis and controller design of a network-based tracking control system between state (or static output) feedback tracking control and observer-based tracking control.* Notice that for time-invariant controllers such as a state feedback controller and a static output feedback controller, a two-channel NCS is equivalent to a sensor-to-actuator channel NCS without affecting the system stability and tracking performance. However, this is not the case for an observer-based controller. When the network-based output tracking control is performed by an observer-based controller, as a result of network-induced delays and packet dropouts in the controller-to-actuator channel, the inputs of the controlled plant and the observer-based controller are updated asynchronously at different time instants and with different frequencies. Such an asynchronous characteristic will be considered in the network-based output tracking control via an observer-based controller.

*This dissertation will propose some design methods of a network-based nonlinear fuzzy tracking controller for T-S fuzzy systems by taking into consideration the knowledge of membership functions.* In the references [15], [28], [37], [133], [137], [139], [148] and [150], the controlled plant is a linear system. However, many physical plants in practice are inherently nonlinear and network-based tracking control for a nonlinear system is more difficult than the one of a linear system. Since a

T-S fuzzy model can efficiently represent a class of nonlinear dynamic systems on a compact region [100], [124], many researchers devote themselves to dealing with a network-based control problem for a T-S fuzzy system. For example, network-based guaranteed cost control [156], stabilization [57], [98],  $H_\infty$  control [128], [157] and output tracking control [54] for a T-S fuzzy system have recently been studied. However, there are some limitations in these references. In [57], [98] and [156], the network-based fuzzy controllers, which depend on available sampled-data measurement of feedback states and continuous measurement of premise variables, are clearly not able to applied in practice since the premise variables of the controllers can not be continuously measured due to sampling behaviors and data transmission. Using a network-based fuzzy controller associated with sampled-data measurement of both feedback states and premise variables, the resulting system is represented by an asynchronous T-S fuzzy system in [54], [128] and [157]. In [128] and [157], some routine relaxation methods in [66], [80] and [122] for a traditional T-S fuzzy system are used to analyze the  $H_\infty$  performance and design the fuzzy controller for an asynchronous T-S fuzzy system, which is technically wrong, see the comment paper [65]. Without using these relaxation methods, the network-based fuzzy tracking controller designed by using the method in [54] has a same control gain for different fuzzy control rules. In other words, only a linear controller for a T-S fuzzy system can be developed by using the design method in [54], which is conservative, see [93]. Consequently, designing a network-based nonlinear fuzzy tracking controller for a T-S fuzzy system is still an open problem to be solved. It should be noted that the knowledge of fuzzy membership functions is not considered in the literature [54], [57], [98], [128], [156] and [157]. In this dissertation, we will propose some new design methods of a network-based fuzzy tracking controller by using the knowledge of membership functions.

*Some less conservative delay-dependent criteria for  $H_\infty$  tracking performance*

*analysis and control design methods will be established in this dissertation.* Some useful additional information like the piecewise-linear time-varying delay information and the knowledge of membership functions is fully utilized in the derivation of a delay-dependent criterion. Notice that a delay-dependent criterion for the existence of a controller for an NCS are expressed in terms of nonlinear matrix inequalities (both in output feedback and state feedback cases). To establish an LMI-based design result, it is inevitable to introduce some linearization techniques, such as equality or inequality constraints and iterative algorithms, which bring some conservatism. Instead of using the routine linearization techniques, in this dissertation, a particle swarm optimization algorithm will be proposed to search for the minimum  $H_\infty$  tracking performance and the corresponding control gain.

## 1.4 Organization of this dissertation

This dissertation is concerned with the network-based output tracking control for continuous-time systems. The systems are classified as Case I: the systems can be stabilized by non-delayed static output feedback controllers; and Case II: the systems can not be stabilized by non-delayed static output feedback controllers, but can be stabilized by delayed static output feedback controllers. More specifically, Case I: For linear systems, network-based output tracking control via state feedback controllers and observer-based controllers, respectively, is considered; for nonlinear systems described by T-S fuzzy models, network-based output tracking control via fuzzy state feedback controllers is studied. Case II: For both linear systems and nonlinear systems described by T-S fuzzy models, network-based output tracking control via static output feedback controllers are considered. This dissertation is organized as follows:

- **Chapter 2:** This chapter deals with network-based state feedback tracking control for a linear system. Some less conservative delay-dependent criteria for

$H_\infty$  tracking performance analysis and tracking controller design are derived by using a new discontinuous simple Lyapunov-Krasovskii functional and a generalized Jensen integral inequality. Two examples are provided to compare the conservatism between the obtained criteria and some existing ones.

- **Chapter 3:** This chapter considers network-based output tracking control for a linear system via an observer-based controller. The update inputs of the linear system and the observer-based tracking controller are asynchronous due to network-induced delays and packet dropouts in the controller-to-actuator channel. Taking into consideration the asynchronous inputs, the network-based tracking control system is modeled as a system with two different interval time-varying delays. By using a simple Lyapunov-Krasovskii functional including both the lower and upper bounds of two interval delays, a new delay-dependent criterion is derived such that the resulting system is exponentially stable with a prescribed  $H_\infty$  tracking performance. The observer-based tracking controller design is converted into a particle swarm optimization algorithm.
- **Chapter 4:** This chapter considers network-based fuzzy state feedback tracking control for a nonlinear system in a T-S fuzzy model. Using a network-based fuzzy controller that depends on available sampled-data of both premise variables and feedback states, the closed-loop system is described by an asynchronous T-S fuzzy system with an interval time-varying delay. Since a routine relaxation method for stability analysis and controller design of a traditional T-S fuzzy system does not work for the asynchronous fuzzy system, a new relaxation method is proposed by utilizing asynchronous constraints on membership functions to introduce some free-weighting matrices. Using the proposed relaxation method and a simple discontinuous Lyapunov-Krasovskii functional, some new delay-dependent criteria for  $H_\infty$  tracking performance analysis and controller design are established in terms of linear matrix inequalities.

- **Chapter 5:** This chapter investigates the positive effect of a network-induced delay on network-based output tracking control for a linear system that can not be stabilized by a non-delayed static output feedback controller, but can be stabilized by a delayed static output feedback controller. A delay-dependent  $H_\infty$  tracking performance criterion is derived by using a new discontinuous complete Lyapunov-Krasovskii functional. By applying a particle swarm optimization technique with the feasibility of the criterion, a novel tracking control design algorithm is proposed to search for the minimum  $H_\infty$  tracking performance and the corresponding control gain.
- **Chapter 6:** This chapter investigates the positive effect of a network-induced delay on network-based output tracking control for a nonlinear system that can not be stabilized by a non-delayed fuzzy static output feedback controller, but can be stabilized by a delayed fuzzy static output feedback controller. For such a system, we purposefully introduce a network-induced delay in the feedback control loop to produce a stable and satisfactory tracking control. New delay-dependent criteria on  $H_\infty$  tracking performance analysis are derived by using a discontinuous complete Lyapunov-Krasovskii functional and the asynchronous constraints on fuzzy membership functions. A particle swarm optimization algorithm is proposed to search for the minimum  $H_\infty$  tracking performance and corresponding output feedback gains.
- **Chapter 7:** In this chapter, we give some concluding remarks and directions for future research work.

## 1.5 Contributions of this dissertation

The contributions of this dissertation are that

- **The positive effect of a network-induced delay on network-based output tracking control** is investigated for the system that can not be stabilized by a non-delayed static output feedback controller, but can be stabilized by a delayed static output feedback controller. By intentionally inserting a communication network between such a system and a static output feedback controller, a network-induced delay is purposefully produced in the feedback control loop to achieve a stable and satisfactory tracking control.
- **A new discontinuous complete Lyapunov-Krasovskii functional**, which makes use of the lower bound of the network-induced delay, the sawtooth time-varying delay and its upper bound, is constructed to derive a delay-dependent criterion for  $H_\infty$  tracking performance analysis. The obtained criterion is of less conservatism since the derivation involves the coupling property between the present state and the past state, the division of a delay interval and the inherent piecewise-linear time-varying delay information.
- **A novel design algorithm of static output feedback controller** is proposed by applying a particle swarm optimization technique with the feasibility of an LMI-based criterion on  $H_\infty$  tracking performance analysis, which can effectively avoid regular linearization techniques such as equality or inequality constraints and iterative algorithms in delay-dependent control design.
- **A new NCS model of observer-based tracking control** is established by a system with two different interval time-varying sawtooth delays, which differs from some existing NCS model of state feedback tracking control described by a system with an interval time-varying delay. The different effect of network-induced delays and packet dropouts in the sensor-to-controller channel and the controller-to-actuator channel on modeling, stability analysis and controller design of a network-based tracking control system is dug out.

- **A new relaxation method of an asynchronous T-S fuzzy system** is proposed by utilizing asynchronous constraints on fuzzy membership functions to introduce some free-weighting matrices. Using a network-based fuzzy controller that depends on available sampled-data measurement of both premise variable and feedback state (or output), the network-based T-S fuzzy system is equivalent to an asynchronous T-S fuzzy system with an interval time-varying sawtooth delay. Since a routine relaxation method for a traditional fuzzy system can not be used to analyze  $H_\infty$  tracking performance for the asynchronous fuzzy system, a new relaxation method is proposed by using asynchronous constraints on membership functions. The proposed method can not only reduce the conservatism of the delay-dependent criteria for  $H_\infty$  tracking performance analysis, but also allow the existence of a network-based nonlinear fuzzy controller with different control gains for different fuzzy control rules, which can ensure a better  $H_\infty$  tracking performance.

## Chapter 2

# Network-based state feedback tracking control for linear systems

### 2.1 Introduction

Network-based output tracking control is to drive the output of a physical plant, via a network-based feedback controller, to track the output of a reference model as close as possible. A typical configuration of a network-based tracking control system is shown in Figure 2.1. Consider the following physical plant

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) + E\omega(t) \\ y(t) = Cx(t) + Du(t) \\ x(t_0) = x_0 \end{cases} \quad (2.1)$$

where  $x(t) \in \mathbb{R}^n$ ,  $u(t) \in \mathbb{R}^m$  and  $y(t) \in \mathbb{R}^l$  are the state, the control input and the output, respectively;  $\omega(t) \in \mathbb{R}^v$  is the external disturbance and  $\omega(t) \in \mathcal{L}_2[t_0, \infty)$ ;  $x(t_0) = x_0$  is the initial state; and  $A$ ,  $B$ ,  $C$ ,  $D$  and  $E$  are some constant matrices of appropriate dimensions.

The reference model is described by

$$\begin{cases} \dot{x}_r(t) = A_r x_r(t) + B_r r(t) \\ y_r(t) = C_r x_r(t) \\ x_r(t_0) = x_{r0} \end{cases} \quad (2.2)$$

where  $x_r(t) \in \mathbb{R}^{\bar{n}}$ ,  $r(t) \in \mathbb{R}^{\bar{v}}$  and  $y_r(t) \in \mathbb{R}^l$  are the state, the energy bounded input and the output, respectively;  $x_r(t_0) = x_{r0}$  is the initial state;  $A_r$ ,  $B_r$  and  $C_r$  are given constant matrices of appropriate dimensions, and  $A_r$  is a Hurwitz matrix.

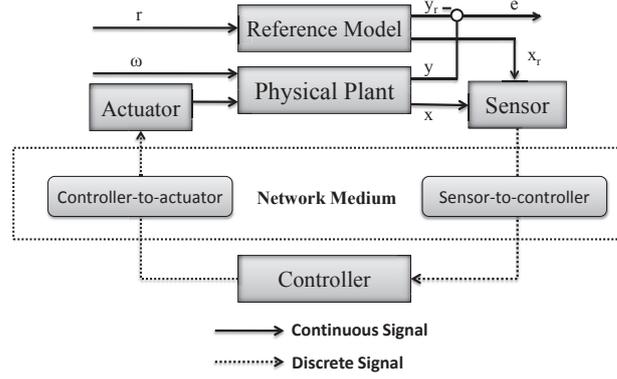


Figure 2.1: A two-channel network-based tracking control system

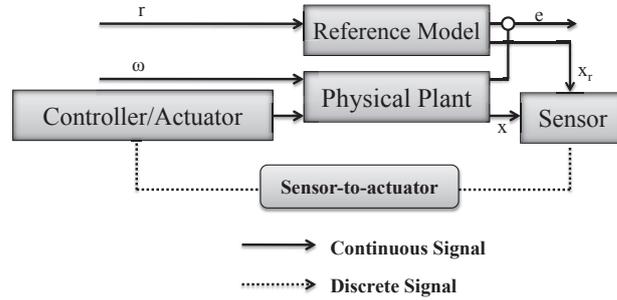


Figure 2.2: A one-channel network-based tracking control system

In this chapter, the network-based output tracking control is performed by a state feedback controller of the form  $u(t) = F_1 x(t) + F_2 x_r(t)$ , where  $F_1$  and  $F_2$  are the control gains to be determined. For a time-invariant state feedback controller, the two-channel network-based control system (NCS) in Figure 2.1 is equivalent to a one-channel NCS in Figure 2.2 without affecting the stability of the corresponding closed-loop system, see [46] and [153]. The sampled-data of feedback states  $x(jh)$  and  $x_r(jh)$  ( $\forall j \in \mathbb{N}$ ) are transmitted to the controller via a communication network, where  $h$  is the sampling period. After receiving available state measurement, the controller computes and sends a control signal to the actuator through the network. The control signal  $u(i_k h)$  is received by the actuator at the time instant  $i_k h + \tau_k$ , where  $i_k$  ( $\forall k \in \mathbb{N}$ ) are some nonnegative integers which indicate the control signals that successfully update the actuator,  $\{i_1, i_2, i_3, \dots\} \subset \{0, 1, 2, \dots\}$ ,  $k$  is the serial number of the updating instants at the actuator,  $\tau_k = \tau_k^{sc} + \tau_k^{ca}$ ,  $\tau_k^{sc}$  ( $\forall k \in \mathbb{N}$ ) are

the sensor-to-controller delays associated with the sampled-data of feedback states  $x(i_k h)$  and  $x_r(i_k h)$  ( $\forall k \in \mathbb{N}$ ), and  $\tau_k^{ca}$  ( $\forall k \in \mathbb{N}$ ) are the controller-to-actuator delays associated with the control signals  $u(i_k h)$  ( $\forall k \in \mathbb{N}$ ). The actuator holds the control signal  $u(i_k h)$  to input the plant (2.1) until an updated control signal is received. Then we obtain the following augmented system

$$\begin{cases} \dot{\xi}(t) = \bar{A}\xi(t) + \bar{B}\bar{F}\xi(t - \tau(t)) + \bar{E}\bar{\omega}(t) \\ e(t) = \bar{C}\xi(t) + D\bar{F}\xi(t - \tau(t)) \\ \tau(t) = t - i_k h, t \in [i_k h + \tau_k, i_{k+1} h + \tau_{k+1}), \forall k \in \mathbb{N} \end{cases} \quad (2.3)$$

where

$$\begin{aligned} \bar{A} &= \begin{bmatrix} A & 0 \\ 0 & A_r \end{bmatrix}, \bar{B} = \begin{bmatrix} B \\ 0 \end{bmatrix}, \bar{E} = \begin{bmatrix} E & 0 \\ 0 & B_r \end{bmatrix}, \bar{C} = [C \quad -C_r], \bar{F} = [F_1 \quad F_2], \\ e(t) &= y(t) - y_r(t), \xi(t) = [x^T(t) \quad x_r^T(t)]^T, \bar{\omega}(t) = [\omega^T(t) \quad r^T(t)]^T. \end{aligned}$$

Defining  $\tau_m = \min_{k \in \mathbb{N}} \{\tau_k\}$  and  $\tau_M = \max_{k \in \mathbb{N}} \{(i_{k+1} - i_k)h + \tau_{k+1}\}$ , we have

$$0 < \tau_m \leq \tau(t) \leq \tau_M, t \in [i_k h + \tau_k, i_{k+1} h + \tau_{k+1}). \quad (2.4)$$

In particular, choosing  $\tau_k = \eta_k$  and  $i_k h + \tau_k = t_k$ , the system (2.3) reduces to the augmented system in [28]. For the system (2.3), the initial condition is supplemented as  $\xi(t) = \phi(t)$ ,  $t \in [t_0 - \tau_M, t_0]$ , where  $\phi(t)$  is a continuous function on  $[t_0 - \tau_M, t_0]$  and  $\phi(t_0) = [x_0^T \quad x_{r0}^T]^T$ ; and the following  $H_\infty$  tracking performance is chosen

$$\int_{t_0}^{t_f} e^T(t) U e(t) dt \leq V(t_0) + \gamma^2 \int_{t_0}^{t_f} \bar{\omega}^T(t) \bar{\omega}(t) dt \quad (2.5)$$

where  $t_f$  is the terminal time,  $\gamma > 0$  is the desired tracking performance level,  $U > 0$  is the weighting matrix, and  $V(t_0)$  is the energy function of initial states.

To achieve the objective of network-based output tracking control, a network-based state feedback tracking controller is designed such that the system (2.3) is asymptotically stable with a prescribed  $H_\infty$  tracking performance. There are some available delay-dependent criteria which can judge whether the system (2.3) has a prescribed  $H_\infty$  tracking performance, see [28] and [139]. It is shown that less conservative criteria for  $H_\infty$  tracking performance analysis and controller design can

provide an appropriate tradeoff between the maximum allowable delay and the minimum  $H_\infty$  tracking performance for an NCS. Usually, one chooses the  $H_\infty$  tracking performance index  $\gamma$  (or the upper delay bound  $\tau_M$ ) to show the conservatism of the derived the delay-dependent criteria. For given values of  $\tau_m$  and  $\tau_M$  (or  $\gamma$ ), the smaller the  $H_\infty$  tracking performance  $\gamma$  (or the larger the delay bound  $\tau_M$ ) is, the less conservative the criteria are. In [28] and [139], the procedure of estimating the integral terms in deriving the criteria by using a Lyapunov-Krasovskii functional (LKF) brings some conservatism. More specifically, in [28], the term  $-\int_{t-\tau_M}^{t-\tau_m} \dot{\eta}^T(s)R\dot{\eta}(s)ds$  is enlarged to  $-\int_{t-\tau(t)}^{t-\tau_m} \dot{\eta}^T(s)R\dot{\eta}(s)ds$ ; in [139], some useful terms  $-(\tau_M - \tau(t)) \int_{t-\tau(t)}^{t-\tau_m} \dot{\eta}^T(s)R\dot{\eta}(s)ds$  and  $-(\tau(t) - \tau_m) \int_{t-\tau_M}^{t-\tau(t)} \dot{\eta}^T(s)R\dot{\eta}(s)ds$  are omitted. On the other hand, in [28] and [139], the inherent piecewise-linear time-varying delay information  $\dot{\tau}(t)=1$  on  $[i_k h + \tau_k, i_{k+1} h + \tau_{k+1})$  ( $\forall k \in \mathbb{N}$ ) is not considered in the derivation. Recently, a generalized Jensen integral inequality combining the convex delay analysis approach is proposed in [158] to estimate some related integral terms appearing in the derivative of the chosen LKF. The generalized Jensen integral inequality facilitates to fully use the integral terms and introduce some free-weighting matrices to reduce the conservatism. Moreover, the choice of an appropriate LKF is crucial for deriving less conservative delay-dependent  $H_\infty$  performance criteria. In this chapter, we will construct the following discontinuous LKF, which makes use of the sawtooth delay  $\tau(t)$  and its lower and upper bounds,

$$\begin{aligned}
V(t) = & \xi^T(t)P\xi(t) + \int_{-\tau_m}^0 \xi^T(t+s)Q_1\xi(t+s)ds + \int_{-\tau_M}^0 \xi^T(t+s)Q_2\xi(t+s)ds \\
& + \tau_m \int_{-\tau_m}^0 \int_s^0 \dot{\xi}^T(t+\theta)R_1\dot{\xi}(t+\theta)d\theta ds + \int_{-\tau_M}^{-\tau_m} \int_s^0 \dot{\xi}^T(t+\theta)R_2\dot{\xi}(t+\theta)d\theta ds \\
& + (\tau_M - \tau(t)) \int_{-\bar{\tau}(t)}^0 \dot{\xi}^T(t+\theta)R_3\dot{\xi}(t+\theta)d\theta + (\tau_M - \tau(t))\chi^T(t)R_4\chi(t) \quad (2.6)
\end{aligned}$$

where  $P > 0$ ,  $Q_i > 0$  ( $i = 1, 2$ ) and  $R_i > 0$  ( $i = 1, 2, 3, 4$ ),  $\chi(t) = x(t) - x(t - \bar{\tau}(t))$ ,  $\bar{\tau}(t) = \tau(t) - \tau_k$  and  $\tau(t) = t - i_k h$  for  $t \in [i_k h + \tau_k, i_{k+1} h + \tau_{k+1})$ ,  $\forall k \in \mathbb{N}$ .

Using the LKF candidate (2.6) and the generalized Jensen integral inequality

in [158], we will derive some less conservative criteria for  $H_\infty$  tracking performance analysis and controller design. Two examples will be given to show that the derived criteria are less conservative than the existing ones.

To end this section, we introduce two lemmas.

**Lemma 2.1.** *There exist some positive scalars  $\varepsilon_i > 0$  ( $i = 1, 2$ ) and a functional  $V(t, \xi_t(\theta), \dot{\xi}_t(\theta)) : \mathbb{R} \times \mathbb{R}^n \times \mathbb{R}^n \rightarrow \mathbb{R}$  such that*

$$\varepsilon_1 \|\xi(t)\|^2 \leq V(t, \xi_t(\theta), \dot{\xi}_t(\theta)) \leq \varepsilon_2 \|\xi_t\|_{\mathbb{W}}^2 \quad (2.7)$$

where  $\xi_t(\theta) = \xi(t+\theta)$  and  $\dot{\xi}_t(\theta) = \dot{\xi}(t+\theta)$ ,  $\forall \theta \in [-\tau_M, 0]$ , and the space of functions  $\xi_t(\theta)$  and  $\dot{\xi}_t(\theta)$  is denoted by  $\mathbb{W}$  with the norm

$$\|\xi_t\|_{\mathbb{W}} = \sup_{\theta \in [-\tau_M, 0]} \{\|\xi_t(\theta)\|, \|\dot{\xi}_t(\theta)\|\}.$$

Let the functional  $V(t) = V(t, \xi_t(\theta), \dot{\xi}_t(\theta))$  be absolutely continuous for  $t \neq i_k h + \tau_k$  and satisfy

$$V(i_k h + \tau_k) \leq \lim_{t \rightarrow (i_k h + \tau_k)^-} V(t), \quad \forall k \in \mathbb{N} \quad (2.8)$$

where  $\lim_{t \rightarrow (i_k h + \tau_k)^-} V(t)$  is a limit taken from the left,  $t \in [i_{k-1} h + \tau_{k-1}, i_k h + \tau_k)$ .

Define  $\dot{V}(t) = \limsup_{\delta \rightarrow 0} \frac{1}{\delta} [V(t+\delta) - V(t)]$ , where  $V(t+\delta) = V(t+\delta, \xi_{t+\delta}(\theta), \dot{\xi}_{t+\delta}(\theta))$ .

Then

- (i) the system (2.3) with  $\bar{\omega}(t) = 0$  is asymptotically stable if there exists an  $\varepsilon_3 > 0$  such that the derivative of  $V(t)$  along (2.3) with  $\bar{\omega}(t) = 0$  satisfies

$$\dot{V}(t) \leq -\varepsilon_3 \|\xi(t)\|^2 \quad (2.9)$$

for  $t \in [i_k h + \tau_k, i_{k+1} h + \tau_{k+1})$ ,  $\forall k \in \mathbb{N}$ .

- (ii) the  $H_\infty$  tracking performance (2.5) can be ensured for all nonzero  $\bar{\omega}(t) \in \mathcal{L}_2[t_0, \infty)$  if along (2.3), the following inequality holds

$$\dot{V}(t) + e^T(t) U e(t) - \gamma^2 \bar{\omega}^T(t) \bar{\omega}(t) < 0 \quad (2.10)$$

for  $t \in [i_k h + \tau_k, i_{k+1} h + \tau_{k+1})$ ,  $\forall k \in \mathbb{N}$ .

*Proof:* (i) Considering that  $V(t)$  is absolutely continuous for  $t \neq i_k h + \tau_k$  and its derivative satisfies (2.9) for  $t \in [i_k h + \tau_k, i_{k+1} h + \tau_{k+1})$  ( $\forall k \in \mathbb{N}$ ), we have  $V(t) \leq V(i_k h + \tau_k)$  for  $t \in [i_k h + \tau_k, i_{k+1} h + \tau_{k+1})$ ,  $\forall k \in \mathbb{N}$ . Using the conditions (2.7)-(2.8), we obtain

$$\varepsilon_1 \|\xi(t)\|^2 \leq V(t) \leq \lim_{t \rightarrow (i_k h + \tau_k)^-} V(t) \leq \dots \leq \lim_{t \rightarrow (i_0 h + \tau_0)^-} V(t) \leq V(t_0) \leq \varepsilon_2 \|\xi_{t_0}\|_{\mathbb{W}}^2, \quad \forall k \in \mathbb{N}.$$

Then, following the proof of Lyapunov-Krasovskii Stability Theorem in [34], we conclude the asymptotic stability of the system (2.3) with  $\bar{\omega}(t) = 0$  for  $t \in [t_0, \infty)$ .

(ii) For  $t \in [i_k h + \tau_k, i_{k+1} h + \tau_{k+1})$  ( $\forall k \in \mathbb{N}$ ), we have

$$\begin{aligned} \int_{i_k h + \tau_k}^t e^T(s) U e(s) ds &= \int_{i_k h + \tau_k}^t \left[ \dot{V}(s) + e^T(s) U e(s) - \gamma^2 \bar{\omega}^T(s) \bar{\omega}(s) \right] ds \\ &\quad + \gamma^2 \int_{i_k h + \tau_k}^t \bar{\omega}^T(s) \bar{\omega}(s) ds + V(i_k h + \tau_k) - V(t). \end{aligned} \quad (2.11)$$

If the inequality (2.10) holds, it follows from (2.11) that

$$\int_{i_k h + \tau_k}^t e^T(s) U e(s) ds \leq \gamma^2 \int_{i_k h + \tau_k}^t \bar{\omega}^T(s) \bar{\omega}(s) ds + V(i_k h + \tau_k) - V(t) \quad (2.12)$$

for  $t \in [i_k h + \tau_k, i_{k+1} h + \tau_{k+1})$  ( $\forall k \in \mathbb{N}$ ). Define  $t_f = i_{T+1} h + \tau_{T+1}$ , where  $i_{T+1}$  is the integer which indicates the last control signal received by the actuator. Notice that  $\bigcup_{k=1}^{k=T} [i_k h + \tau_k, i_{k+1} h + \tau_{k+1}) = [t_0, t_f)$  and  $V(t) \geq \varepsilon_1 \|\xi(t)\|^2$ . Then it can be seen from (2.12) that the  $H_\infty$  tracking performance (2.5) can be ensured, which completes the proof.

The generalized Jensen integral inequality is given by the following lemma.

**Lemma 2.2.** [158] For any constant matrix  $R \in \mathbb{R}^{n \times n}$ ,  $R = R^T > 0$ ,  $Z \in \mathbb{R}^{m \times k}$ , a scalar  $\tau > 0$ , a vector function  $\dot{\xi} : [0, \tau] \rightarrow \mathbb{R}^m$  such that the integration concerned is well defined, let

$$\int_0^\tau \dot{\xi}(\beta) d\beta = \mathcal{E} \psi$$

where  $\mathcal{E} \in \mathbb{R}^{m \times k}$  and  $\psi \in \mathbb{R}^k$ . Then

$$\int_0^\tau \dot{\xi}^T(\beta) R \dot{\xi}(\beta) d\beta \geq \psi^T (\mathcal{E}^T Z + Z^T \mathcal{E} - \tau Z^T R^{-1} Z) \psi.$$

## 2.2 A simple LKF method for tracking performance analysis

In this section, using a simple LKF method and Lemma 2.1, we will derive a new delay-dependent criterion such that the system (2.3) is asymptotically stable with a prescribed  $H_\infty$  tracking performance. For simplicity of presentation, let

$$\begin{aligned}\eta^T(t) &= [\eta_1^T(t) \ \eta_2^T(t)], \\ \eta_1^T(t) &= [\xi^T(t) \ \dot{\xi}^T(t) \ \xi^T(t - \tau(t))], \\ \eta_2^T(t) &= [\xi^T(t - \bar{\tau}(t)) \ \xi^T(t - \tau_m) \ \xi^T(t - \tau_M)], \\ e_1 &= [I \quad 0 \quad 0 \quad 0 \quad 0 \quad 0]_{p \times 6p}, \\ e_2 &= [0 \quad I \quad 0 \quad 0 \quad 0 \quad 0]_{p \times 6p}, \\ e_3 &= [0 \quad 0 \quad I \quad 0 \quad 0 \quad 0]_{p \times 6p}, \\ e_4 &= [0 \quad 0 \quad 0 \quad I \quad 0 \quad 0]_{p \times 6p}, \\ e_5 &= [0 \quad 0 \quad 0 \quad 0 \quad I \quad 0]_{p \times 6p}, \\ e_6 &= [0 \quad 0 \quad 0 \quad 0 \quad 0 \quad I]_{p \times 6p}\end{aligned}$$

where  $e_i$  ( $i = 1, 2, \dots, 6$ ) are  $p \times 6p$  matrices;  $I$  denotes a  $p \times p$  identity matrix, the others in  $e_i$  ( $i = 1, 2, \dots, 6$ ) are  $p \times p$  zero matrices;  $p$  is the dimension of  $\xi(t)$  and  $p = n + \bar{n}$ . Then the delay-dependent criterion is given by

**Proposition 2.1.** *Given positive scalars  $\gamma$ ,  $\tau_m$  and  $\tau_M$ , gain matrices  $F_1$ ,  $F_2$  and a weighting matrix  $U > 0$ , the system (2.3) is asymptotically stable with a prescribed  $H_\infty$  tracking performance  $\gamma$  if there exist symmetric matrices  $P > 0$ ,  $Q_i > 0$  ( $i = 1, 2$ ),  $R_i > 0$  ( $i = 1, 2, 3, 4$ ) and matrices  $S_i$  ( $i = 1, 2, 3$ ),  $X_i$  ( $i = 1, 2$ ) such that*

$$\begin{bmatrix} \Omega^0 & * & * & * \\ \Gamma & -\gamma^2 I & 0 & * \\ \delta S_1 & 0 & -\delta R_2 & * \\ \delta S_3 & 0 & 0 & -\delta R_3 \end{bmatrix} < 0 \quad (2.13)$$

$$\begin{bmatrix} \Omega^0 + \delta \Omega^1 & * & * \\ \Gamma & -\gamma^2 I & * \\ \delta S_2 & 0 & -\delta R_2 \end{bmatrix} < 0 \quad (2.14)$$

where

$$\begin{aligned}
\Omega^0 &= e_1^T (X_1^T \bar{A} + \bar{A}^T X_1 + Q_1 + Q_2 - R_1 - R_4 + \bar{C}^T U \bar{C}) e_1 + e_1^T (P + \bar{A}^T X_2 - X_1^T) e_2 \\
&\quad + e_1^T X_1^T \bar{B} \bar{F} e_3 + e_1^T R_4 e_4 + e_1^T R_1 e_5 + e_2^T (P - X_1 + X_2^T \bar{A}) e_1 + e_3^T \bar{F}^T \bar{B}^T X_1 e_1 \\
&\quad + e_4^T R_4 e_1 + e_5^T R_1 e_1 + e_2^T (\tau_m^2 R_1 + \delta R_2 - X_2 - X_2^T) e_2 + e_2^T X_2^T \bar{B} \bar{F} e_3 \\
&\quad + e_3^T \bar{F}^T \bar{B}^T X_2 e_2 - e_4^T R_4 e_4 - e_5^T (Q_1 + R_1) e_5 - e_6^T Q_2 e_6 + (e_5 - e_3)^T S_1 \\
&\quad + S_1^T (e_5 - e_3) + S_2^T (e_3 - e_6) + (e_3 - e_6)^T S_2 + S_3^T (e_1 - e_4) + (e_1 - e_4)^T S_3, \\
\Omega^1 &= e_1^T R_4 e_2 + e_2^T R_4 e_1 + e_2^T R_3 e_2 - e_2^T R_4 e_4 - e_4^T R_4 e_2, \\
\Gamma &= \bar{E}^T X_1 e_1 + \bar{E}^T X_2 e_2, \quad \delta = \tau_M - \tau_m.
\end{aligned}$$

*Proof:* First, we show that the LKF (2.6) satisfies the condition (2.7) for  $P > 0$ ,  $Q_i > 0$  ( $i = 1, 2$ ) and  $R_i > 0$  ( $i = 1, 2, 3, 4$ ). It is clear to see that  $V(t) \geq \lambda_{i_k} \|\xi(t)\|^2$  for  $t \in [i_k h + \tau_k, i_{k+1} h + \tau_{k+1})$ , where  $\lambda_{i_k} > 0$  ( $\forall k \in \mathbb{N}$ ). Notice that  $\bigcup_{k=0}^{\infty} [i_k h + \tau_k, i_{k+1} h + \tau_{k+1}) = [t_0, \infty)$ . Then we have  $V(t) \geq \varepsilon_1 \|\xi(t)\|^2$  for  $t \in [i_T h + \tau_T, \infty)$ , where  $\varepsilon_1 = \sum_{k=1}^{k=T} \{\lambda_{i_k}\} > 0$ . For  $P > 0$ ,  $Q_i > 0$  ( $i = 1, 2$ ) and  $R_i > 0$  ( $i = 1, 2, 3, 4$ ), we have

$$\begin{aligned}
V(t) &\leq \|\xi(t)\|^2 \lambda_{\max}(P) + \int_{-\tau_m}^0 \|\xi_t(s)\|^2 ds \lambda_{\max}(Q_1) + \int_{-\tau_M}^0 \|\xi_t(s)\|^2 ds \lambda_{\max}(Q_2) \\
&\quad + \tau_m^2 \int_{-\tau_m}^0 \|\dot{\xi}_t(\theta)\|^2 d\theta \lambda_{\max}(R_1) + (\tau_M - \tau_m) \int_{-\tau_M}^0 \|\dot{\xi}_t(\theta)\|^2 d\theta \lambda_{\max}(R_2) \\
&\quad + (\tau_M - \tau_m)^2 \int_{-\tau_M}^0 \|\dot{\xi}_t(\theta)\|^2 d\theta \lambda_{\max}(R_3) + (\tau_M - \tau_m)^2 \int_{-\tau_M}^0 \|\dot{\xi}_t(\theta)\|^2 d\theta \lambda_{\max}(R_4) \\
&\leq \kappa_1 \max_{\theta \in [-\tau_M, 0]} \|\xi_t(\theta)\|^2 + \kappa_2 \max_{\theta \in [-\tau_M, 0]} \|\dot{\xi}_t(\theta)\|^2 \tag{2.15}
\end{aligned}$$

where

$$\begin{aligned}
\kappa_1 &= \lambda_{\max}(P) + \tau_m \lambda_{\max}(Q_1) + \tau_M \lambda_{\max}(Q_2), \\
\kappa_2 &= \tau_m^3 \lambda_{\max}(R_1) + \tau_M (\tau_M - \tau_m) \lambda_{\max}(R_2) + (\tau_M - \tau_m)^3 [\lambda_{\max}(R_3) + \lambda_{\max}(R_4)].
\end{aligned}$$

Second, we show that the LKF (2.6) satisfies the condition (2.8) for  $P > 0$ ,  $Q_i > 0$  ( $i = 1, 2$ ) and  $R_i > 0$  ( $i = 1, 2, 3, 4$ ). In the LKF (2.6),  $R_3$ ,  $R_4$ -dependent terms are discontinuous at the updating instant  $i_k h + \tau_k$  and do not increase along  $i_k h + \tau_k$  since

they are non-negative before  $i_k h + \tau_k$  and become zero just after  $i_k h + \tau_k$  ( $\forall k \in \mathbb{N}$ ); the other terms are continuous on  $[t_0, \infty)$ . Thus, we obtain the condition (2.8).

Third, we consider the asymptotic stability for the system (2.3) with  $\bar{\omega}(t) = 0$ . Taking the derivative of the LKF (2.6) along the trajectory of the system (2.3) with  $\bar{\omega}(t) = 0$ , we have

$$\begin{aligned}
\dot{V}(t) = & \eta^T(t)[e_1^T(Q_1 + Q_2 - R_1 - R_4)e_1 + e_1^T P e_2 + (\tau_M - \tau(t))(e_1^T R_4 e_2 + e_2^T R_4 e_1)]\eta(t) \\
& + \eta^T(t)[e_1^T R_4 e_4 + e_1^T R_1 e_5 + e_2^T P e_1 + e_4^T R_4 e_1 + e_5^T R_1 e_1 + e_2^T(\tau_m^2 R_1 + \delta R_2)e_2]\eta(t) \\
& - \eta^T(t)[e_4^T R_4 e_4 + e_5^T(R_1 + Q_1)e_5 + e_6^T Q_2 e_6]\eta(t) - (\tau_M - \tau(t))e_2^T R_3 e_2]\eta(t) \\
& - \eta^T(t)[(\tau_M - \tau(t))(e_2^T R_4 e_4 + e_4^T R_4 e_2)]\eta(t) - \tau_m \int_{t-\tau_m}^t \dot{\xi}^T(s) R_1 \dot{\xi}(s) ds \\
& - \int_{t-\tau(t)}^{t-\tau_m} \dot{\xi}^T(s) R_2 \dot{\xi}(s) ds - \int_{t-\tau_M}^{t-\tau(t)} \dot{\xi}^T(s) R_2 \dot{\xi}(s) ds - \int_{t-\bar{\tau}(t)}^t \dot{\xi}^T(s) R_3 \dot{\xi}(s) ds \\
& + 2\eta^T(t)(e_1^T X_1^T + e_2^T X_2^T)(\bar{A}e_1 + \bar{B}\bar{F}e_3 - e_2)\eta(t)
\end{aligned} \tag{2.16}$$

for  $t \in [i_k h + \tau_k, i_{k+1} h + \tau_{k+1})$ ,  $\forall k \in \mathbb{N}$ .

Using Jensen integral inequality, we obtain

$$-\tau_m \int_{t-\tau_m}^t \dot{\xi}^T(s) R_1 \dot{\xi}(s) ds \leq -\eta^T(t)(e_1 - e_5)^T R_1 (e_1 - e_5)\eta(t). \tag{2.17}$$

Applying Lemma 2.2 with  $\mathcal{E} = e_5 - e_3$ ,  $\psi = \eta(t)$  and  $Z = S_1$  to  $-\int_{t-\tau(t)}^{t-\tau_m} \dot{\xi}^T(s) R_2 \dot{\xi}(s) ds$  yields

$$\begin{aligned}
& - \int_{t-\tau(t)}^{t-\tau_m} \dot{\xi}^T(s) R_2 \dot{\xi}(s) ds \\
& \leq \eta^T(t) [(e_5 - e_3)^T S_1 + S_1^T (e_5 - e_3)] \eta(t) + (\tau(t) - \tau_m)\eta^T(t) S_1^T R_2^{-1} S_1 \eta(t).
\end{aligned} \tag{2.18}$$

Similarly, the following inequalities hold

$$\begin{aligned}
& - \int_{t-\tau_M}^{t-\tau(t)} \dot{\xi}^T(s) R_2 \dot{\xi}(s) ds \\
& \leq \eta^T(t) [(e_3 - e_6)^T S_2 + S_2^T (e_3 - e_6)] \eta(t) + (\tau_M - \tau(t))\eta^T(t) S_2^T R_2^{-1} S_2 \eta(t)
\end{aligned} \tag{2.19}$$

$$\begin{aligned}
& - \int_{t-\bar{\tau}(t)}^t \dot{\xi}^T(s) R_3 \dot{\xi}(s) ds \\
& \leq \eta^T(t) [(e_1 - e_4)^T S_3 + S_3^T (e_1 - e_4)] \eta(t) + (\tau(t) - \tau_m)\eta^T(t) S_3^T R_3^{-1} S_3 \eta(t).
\end{aligned} \tag{2.20}$$

Then it follows from (2.16)-(2.20) that

$$\dot{V}(t) \leq \eta^T(t)\Omega^0(t)\eta(t) \quad (2.21)$$

for  $t \in [i_k h + \tau_k, i_{k+1} h + \tau_{k+1})$ ,  $\forall k \in \mathbb{N}$ , where

$$\begin{aligned} \Omega^0(t) &= \Omega^1(t) + \Omega^2(t) + \Omega^0, \\ \Omega^1(t) &= (\tau_M - \tau(t))(S_2^T R_2^{-1} S_2 + \Omega^1), \\ \Omega^2(t) &= (\tau(t) - \tau_m)(S_1^T R_1^{-1} S_1 + S_3^T R_3^{-1} S_3). \end{aligned}$$

Considering that  $\Omega^0(t)$  is a convex combination of  $\Omega^1(t)$  and  $\Omega^2(t)$  on  $\tau(t) \in [\tau_m, \tau_M]$ , we obtain (2.9) if  $\Phi^i < 0$  ( $i = 1, 2$ ) for  $t \in [i_k h + \tau_k, i_{k+1} h + \tau_{k+1})$ ,  $\forall k \in \mathbb{N}$ , where

$$\begin{aligned} \Phi^1 &= \Omega^0 + \Omega^1(t)|_{\tau(t)=\tau_m}, \\ \Phi^2 &= \Omega^0 + \Omega^2(t)|_{\tau(t)=\tau_M}. \end{aligned}$$

Using Schur complement to the linear matrix inequalities (LMIs) (2.13)-(2.14), we can obtain  $\Phi^i < 0$  ( $i = 1, 2$ ). It follows that the condition (2.9) is satisfied, which means that the system (2.3) with  $\bar{\omega}(t) = 0$  is asymptotically stable.

Lastly, we consider the  $H_\infty$  tracking performance (2.5) for the system (2.3). Taking the derivative of the LKF (2.6) along (2.3), we have

$$\dot{V}(t) \leq \begin{bmatrix} \eta(t) \\ \omega(t) \end{bmatrix}^T \begin{bmatrix} \tilde{\Phi}^0(t) & * \\ \Gamma & -\gamma^2 I \end{bmatrix} \begin{bmatrix} \eta(t) \\ \omega(t) \end{bmatrix} - e^T(t) U e(t) + \gamma^2 \omega^T(t) \omega(t) \quad (2.22)$$

for  $t \in [i_k h + \tau_k, i_{k+1} h + \tau_{k+1})$ ,  $\forall k \in \mathbb{N}$ , where

$$\tilde{\Phi}^0(t) = \Omega^0(t) + e_1^T \bar{C}^T U \bar{C} e_1.$$

Using the convex combination technique and Schur complement to the LMIs (2.13)-(2.14), we have

$$\begin{bmatrix} \tilde{\Phi}^0(t) & * \\ \Gamma & -\gamma^2 I \end{bmatrix} \leq 0. \quad (2.23)$$

Then it follows from (2.22) and (2.23) that the condition (2.10) is obtained. From Lemma 2.1, we can see that the  $H_\infty$  tracking performance (2.5) is ensured for the system (2.3), which completes the proof.

**Remark 2.1.** In the proof of Proposition 2.1, we use a generalized Jensen integral inequality to estimate integral terms  $-\int_{t-\tau(t)}^{t-\tau_m} \dot{\xi}^T(s) R_2 \dot{\xi}(s) ds$ ,  $-\int_{t-\tau_M}^{t-\tau(t)} \dot{\xi}^T(s) R_2 \dot{\xi}(s) ds$  and  $-\int_{t-\bar{\tau}(t)}^t \dot{\xi}^T(s) R_3 \dot{\xi}(s) ds$ , which yield some quadratic terms with inversely weight coefficients. Then the convex delay analysis approach can be employed to establish the delay-dependent criterion. Moreover, the inherent piecewise-linear time-varying delay information  $\dot{\tau}(t) = 1$  on  $[i_k h + \tau_k, i_{k+1} h + \tau_{k+1})$  ( $\forall k \in \mathbb{N}$ ) is fully used in the derivation. Therefore, it is expected that these criteria are of less conservatism.

For comparison purpose, if the lower bound of the network-induced delay is assumed to zero, i.e.,  $\tau_m = 0$ , we can similarly obtain the following proposition.

**Proposition 2.2.** *Given two positive scalars  $\gamma$  and  $\tau_M$ , gain matrices  $F_1, F_2$ , and a weighting matrix  $U > 0$ , the system (2.3) is asymptotically stable with a prescribed  $H_\infty$  tracking performance  $\gamma$  if there exist symmetric matrices  $P > 0, Q_2 > 0, R_i > 0$  ( $i = 2, 3, 4$ ) and matrices  $S_i$  ( $i = 1, 2, 3$ ),  $X_i$  ( $i = 1, 2$ ) such that*

$$\begin{bmatrix} \Omega^0 & * & * & * \\ \Gamma & -\gamma^2 I & * & * \\ \tau_M S_1 & 0 & -\tau_M R_2 & * \\ \tau_M S_3 & 0 & 0 & -\tau_M R_3 \end{bmatrix} < 0 \quad (2.24)$$

$$\begin{bmatrix} \Omega^0 + \tau_M \Omega^1 & * & * \\ \Gamma & -\gamma^2 I & * \\ \tau_M S_2 & 0 & -\tau_M R_2 \end{bmatrix} < 0 \quad (2.25)$$

where

$$\begin{aligned} \Omega^0 = & e_1^T (X_1^T \bar{A} + \bar{A}^T X_1 + Q_2 - R_4 + \bar{C}^T U \bar{C}) e_1 \\ & + e_1^T (P - X_1^T + \bar{A}^T X_2) e_2 + e_2^T (P - X_1 + X_2^T \bar{A}) e_1 \\ & + e_1^T X_1^T \bar{B} \bar{F} e_3 + e_1^T R_4 e_4 + e_2^T (\tau_M R_2 - X_2 - X_2^T) e_2 \\ & + e_2^T X_2^T \bar{B} \bar{F} e_3 + e_3^T \bar{F}^T \bar{B}^T X_1 e_1 + e_4^T R_4 e_1 \\ & + e_3^T \bar{F}^T \bar{B}^T X_2 e_2 - e_4^T R_4 e_4 - e_5^T Q_2 e_5 \\ & + (e_1 - e_3)^T S_1 + S_1^T (e_1 - e_3) + S_2^T (e_3 - e_5) \\ & + (e_3 - e_5)^T S_2 + S_3^T (e_1 - e_4) + (e_1 - e_4)^T S_3, \end{aligned}$$

$$\begin{aligned}
e_1 &= [I \quad 0 \quad 0 \quad 0 \quad 0]_{p \times 5p}, \\
e_2 &= [0 \quad I \quad 0 \quad 0 \quad 0]_{p \times 5p}, \\
e_3 &= [0 \quad 0 \quad I \quad 0 \quad 0]_{p \times 5p}, \\
e_4 &= [0 \quad 0 \quad 0 \quad I \quad 0]_{p \times 5p}, \\
e_5 &= [0 \quad 0 \quad 0 \quad 0 \quad I]_{p \times 5p}.
\end{aligned}$$

*Proof:* The proof is similar to that of Proposition 2.1 and omitted.

## 2.3 State feedback tracking control design

We now establish a sufficient criterion on the existence of a network-based state feedback tracking controller for the system (2.1)-(2.2). The criterion is given by

**Proposition 2.3.** *Given some positive scalars  $\gamma$ ,  $\tau_m$  and  $\tau_M$ , a tuning parameter  $\sigma$ , and a weighting matrix  $U > 0$ , the system (2.3) is asymptotically stable with an  $H_\infty$  tracking performance if there exist symmetric matrices  $\bar{P} > 0$ ,  $\bar{Q}_i > 0$  ( $i=1,2$ ),  $\bar{R}_i > 0$  ( $i=1,2,3,4$ ), and some matrices  $X$ ,  $\bar{S}_i$  ( $i=1,2,3$ ),  $Y$  such that*

$$\begin{bmatrix} \bar{\Omega}^0 & * & * & * & * \\ \bar{\Gamma} & -\gamma^2 I & * & * & * \\ \delta \bar{S}_1 & 0 & -\delta \bar{R}_2 & * & * \\ \delta \bar{S}_3 & 0 & 0 & -\delta \bar{R}_3 & * \\ \bar{C}X e_1 & 0 & 0 & 0 & -U^{-1} \end{bmatrix} < 0 \quad (2.26)$$

$$\begin{bmatrix} \bar{\Omega}^0 + \delta \bar{\Omega}^1 & * & * & * \\ \bar{\Gamma} & -\gamma^2 I & * & * \\ \delta \bar{S}_2 & 0 & -\delta \bar{R}_2 & * \\ \bar{C}X e_1 & 0 & 0 & -U^{-1} \end{bmatrix} < 0 \quad (2.27)$$

where  $e_i$  ( $i=1,2,\dots,6$ ) are given in Proposition 2.1 and

$$\begin{aligned}
\bar{\Omega}^0 &= e_1^T (\bar{A}X + X^T \bar{A}^T + \bar{Q}_1 + \bar{Q}_2 - \bar{R}_1 - \bar{R}_4) e_1 + e_1^T (\bar{P} - X + \sigma X^T \bar{A}^T) e_2 \\
&+ e_2^T (\bar{P} - X^T + \sigma \bar{A}X) e_1 + e_1^T \bar{B}Y e_3 + e_3^T Y^T \bar{B}^T e_1 + e_1^T \bar{R}_4 e_4 + e_4^T \bar{R}_4 e_1 \\
&+ e_1^T \bar{R}_1 e_5 + e_5^T \bar{R}_1 e_1 + e_2^T (\tau_m^2 \bar{R}_1 - \sigma X - \sigma X^T) e_2 + e_2^T \delta \bar{R}_2 e_2 + e_2^T \sigma \bar{B}Y e_3 \\
&+ e_3^T \sigma Y^T \bar{B}^T e_2 - e_4^T \bar{R}_4 e_4 - e_5^T (\bar{Q}_1 + \bar{R}_1) e_5 - e_6^T \bar{Q}_2 e_6 + (e_5 - e_3)^T \bar{S}_1 \\
&+ \bar{S}_1^T (e_5 - e_3) + \bar{S}_2^T (e_3 - e_6) + (e_3 - e_6)^T \bar{S}_2 + \bar{S}_3^T (e_1 - e_4) + (e_1 - e_4)^T \bar{S}_3,
\end{aligned}$$

$$\bar{\Omega}^1 = e_1^T \bar{R}_4 e_2 + e_2^T \bar{R}_4 e_1 + e_2^T \bar{R}_3 e_2 - e_2^T \bar{R}_4 e_4 - e_4^T \bar{R}_4 e_2, \quad \bar{\Gamma} = \bar{E}^T e_1 + \sigma \bar{E}^T e_2.$$

Moreover, the control gains  $\bar{F}$  are given by  $\bar{F} = YX^{-1}$ .

*Proof:* From Proposition 2.1, we can see that the system (2.3) is asymptotically stable with a prescribed  $H_\infty$  tracking performance  $\gamma$  if the LMIs (2.13)-(2.14) are satisfied. Pre- and post-multiplying both sides of the inequality (2.13) with  $\text{diag}\{\Delta, \Delta, I, X, X\}^T$  and its transpose, the inequality (2.14) with  $\text{diag}\{\Delta, \Delta, I, X\}^T$  and its transpose, and introducing  $\Delta = \text{diag}\{X, X, X\}$ ,  $X = X_1^{-1} = \sigma X_2^{-1}$ ,  $\bar{P} = X^T P X$ ,  $\bar{Q}_i = X^T Q_i X$  ( $i=1, 2$ ),  $\bar{R}_i = X^T R_i X$  ( $i=1, 2, 3, 4$ ),  $\bar{S}_i^T = \text{diag}\{\Delta, \Delta\}^T S_i^T X$  ( $i=1, 2, 3$ ),  $Y = \bar{F}X$ , then we can obtain (2.26)-(2.27) by using Schur complement.

In the case  $\tau_m = 0$ , we have the following proposition.

**Proposition 2.4.** *Given positive scalars  $\gamma$  and  $\tau_M$ , a tuning parameter  $\sigma$ , and a weighting matrix  $U > 0$ , the system (2.3) is asymptotically stable with an  $H_\infty$  tracking performance if there exist symmetric matrices  $\bar{P} > 0$ ,  $\bar{Q}_2 > 0$ ,  $\bar{R}_i > 0$  ( $i=2, 3, 4$ ), and matrices  $X$ ,  $\bar{S}_i$  ( $i=1, 2, 3$ ),  $Y$  such that*

$$\begin{bmatrix} \bar{\Omega}^0 & * & * & * & * \\ \bar{\Gamma} & -\gamma^2 I & * & * & * \\ \tau_M \bar{S}_1 & 0 & -\tau_M \bar{R}_2 & * & * \\ \tau_M \bar{S}_3 & 0 & 0 & -\tau_M \bar{R}_3 & * \\ \bar{C}X e_1 & 0 & 0 & 0 & -U^{-1} \end{bmatrix} < 0 \quad (2.28)$$

$$\begin{bmatrix} \bar{\Omega}^0 + \tau_M \bar{\Omega}^1 & * & * & * \\ \bar{\Gamma} & -\gamma^2 I & * & * \\ \tau_M \bar{S}_2 & 0 & -\tau_M \bar{R}_2 & * \\ \bar{C}X e_1 & 0 & 0 & -U^{-1} \end{bmatrix} < 0 \quad (2.29)$$

where  $e_i$  ( $i=1, 2, \dots, 5$ ) are given in Proposition 2.2 and

$$\begin{aligned} \bar{\Omega}^0 &= e_1^T (\bar{A}X + X^T \bar{A}^T + \bar{Q}_2 - \bar{R}_4) e_1 + e_1^T (\sigma X^T \bar{A}^T - X + \bar{P}) e_2 + e_1^T \bar{B}Y e_3 + e_1^T \bar{R}_4 e_4 \\ &+ e_2^T (\sigma \bar{A}X - X^T + \bar{P}) e_1 + e_3^T Y^T \bar{B}^T e_1 + e_4^T \bar{R}_4 e_1 + e_2^T (\tau_M \bar{R}_2 - \sigma X - \sigma X^T) e_2 \\ &+ e_2^T \sigma \bar{B}Y e_3 + e_3^T \sigma Y^T \bar{B}^T e_2 - e_4^T \bar{R}_4 e_4 + (e_1 - e_3)^T \bar{S}_1 + \bar{S}_1^T (e_1 - e_3) \\ &+ \bar{S}_2^T (e_3 - e_5) - e_5^T \bar{Q}_2 e_5 + (e_3 - e_5)^T \bar{S}_2 + \bar{S}_3^T (e_1 - e_4) + (e_1 - e_4)^T \bar{S}_3. \end{aligned}$$

Moreover, the control gains  $\bar{F}$  are given by  $\bar{F} = YX^{-1}$ .

Table 2.1: Comparison of  $\gamma_{min}$  between the proposed method and the one in [28]

$\tau_m(s)$	0	0.05	0.10	0.15	0.20
$\gamma_{min}$ [28]	3.9018	3.1017	2.5700	2.1922	1.9103
$\gamma_{min}$ the proposed method	1.6427	1.5904	1.5369	1.4828	1.4289

## 2.4 Numerical examples

In this section, two examples are provided to illustrate the advantages of the derived criteria. The first one compares the conservatism of the proposed  $H_\infty$  tracking performance criteria (Proposition 2.1 and Proposition 2.2) and those criteria in [28]. The second one shows that the network-based state feedback controller designed by Proposition 2.3 achieves a better  $H_\infty$  tracking performance than that in [28].

*Example 1.1:* Consider the following system matrices  $A, B, C, D, E, A_r, B_r, C_r$  and the control gains  $F_1$  and  $F_2$ , which are borrowed from [28]

$$\begin{aligned}
 A &= \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad E = \begin{bmatrix} 0.2 \\ 0.1 \end{bmatrix}, \\
 C &= [1 \quad 0], \quad D = 0.5, \quad A_r = -1, \quad B_r = 1, \quad C_r = 0.5, \\
 F_1 &= [-1 \quad 1], \quad F_2 = 1.
 \end{aligned}$$

Choose  $\tau_M = 0.43s$ , which implies that the maximum allowable consecutive dropout bound is 2, the maximum allowable delay bound is  $0.40s$  and the sampling period is  $0.01s$  [28]. Given  $\tau_m = 0$ , using Proposition 2.2, we can determine the minimum  $H_\infty$  tracking performance  $\gamma_{min} = 1.6427$ . For different  $\tau_m > 0$ , using Proposition 2.1, one can obtain the minimum  $H_\infty$  tracking performance  $\gamma_{min}$ , which is shown in Table 2.1.

From Table 2.1, it is clear to see that the minimum  $H_\infty$  tracking performance  $\gamma$  obtained by Proposition 2.1 and Proposition 2.2 is much smaller than that provided by Theorem 1 and Corollary 1 in [28], which means that the proposed performance criteria are less conservative than that in [28].

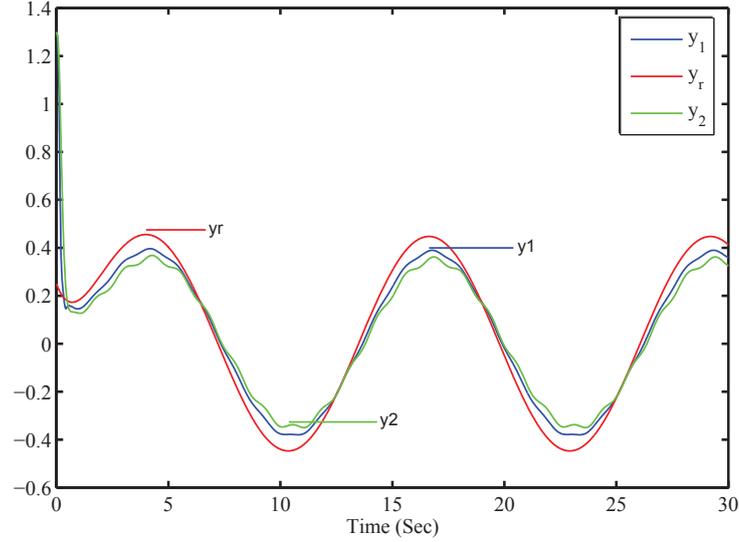


Figure 2.3: The outputs of the system (2.30)-(2.31) with  $\omega(t)$  and  $r(t)$

*Example 2.2:* In this example, the controlled plant is a satellite system that consists of two rigid bodies joined by a flexible link [8], [28]. The dynamic equations of the satellite system are given by

$$\begin{aligned} J_1 \ddot{\theta}_1(t) + f(\dot{\theta}_1(t) - \dot{\theta}_2(t)) + k(\theta_1(t) - \theta_2(t)) &= u(t), \\ J_2 \ddot{\theta}_2(t) + f(\dot{\theta}_1(t) - \dot{\theta}_2(t)) + k(\theta_1(t) - \theta_2(t)) &= \omega(t). \end{aligned}$$

When the output is  $\theta_2(t)$  and the parameters are  $J_1 = J_2 = 1$ ,  $k = 0.09$ ,  $f = 0.04$ , the state-space representation of the satellite system is given by

$$\begin{cases} \dot{x}(t) = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -0.09 & 0.09 & -0.04 & 0.04 \\ 0.09 & -0.09 & 0.04 & -0.04 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} u(t) + \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} \omega(t) \\ y(t) = [0 \ 1 \ 0 \ 0] x(t) \end{cases} \quad (2.30)$$

where the external disturbance input is assumed to be  $\omega(t) = 0.5 \sin 5t$ .

Consider the following reference model [28]

$$\begin{cases} \dot{x}_r(t) = -x_r(t) + r(t) \\ y_r(t) = 0.5x_r(t) \end{cases} \quad (2.31)$$

where  $r(t) = \sin 0.5t$ .

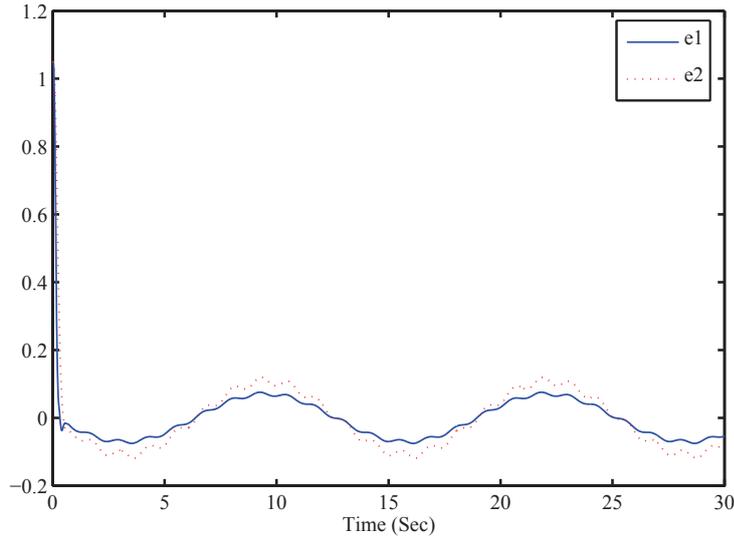


Figure 2.4: The tracking errors of the system (2.30)-(2.31) with  $\omega(t)$  and  $r(t)$

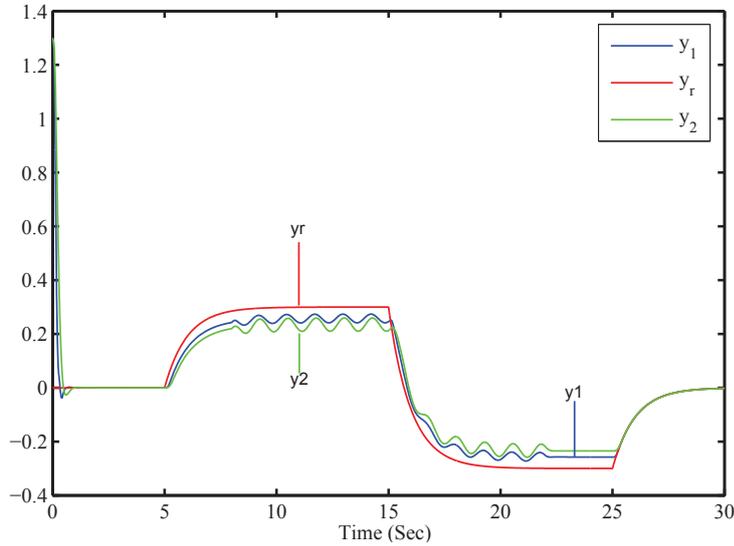


Figure 2.5: The outputs of the system (2.30)-(2.31) with  $\tilde{\omega}(t)$  and  $\tilde{r}(t)$

Suppose that a communication network is used to connect the system (2.30) with a state feedback tracking controller  $u(t) = F_1 x(t) + F_2 x_r(t)$ . Given the sampling period  $h = 10ms$ , the delay bounds  $\tau_m = 5ms$  and  $\tau_M = 25ms$ . Using Theorem 2 in [28], the minimum  $H_\infty$  tracking performance is  $\gamma_{min} = 0.1267$  and the control gains are  $F_1 = [-41.56 \quad -17630.50 \quad -20.92 \quad -4256.35]$  and  $F_2 = 6917.26$  (denoted by  $GM_1$ ). Then applying Proposition 2.3, we obtain the  $\gamma_{min} = 0.0779$ ,  $F_1 = [-58.094 \quad -58571 \quad -28.812 \quad -8881.7]$  and  $F_2 = 25173$  (denoted by  $GM_2$ ). It is clear

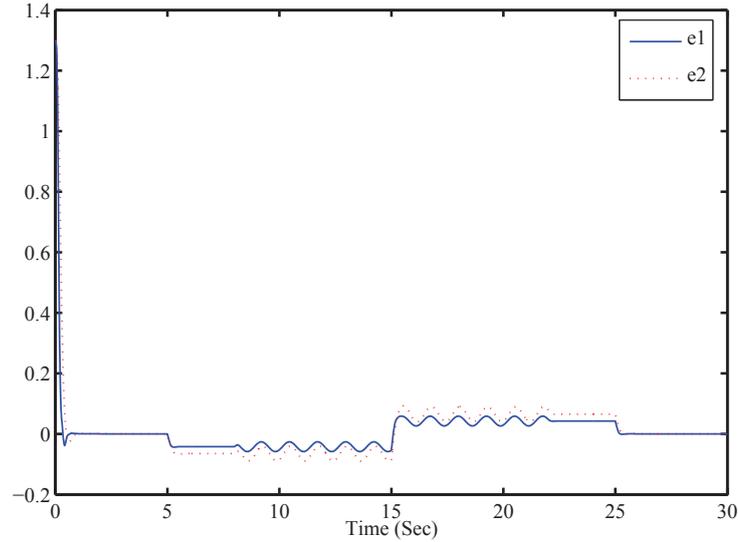


Figure 2.6: The tracking errors of the system (2.30)-(2.31) with  $\tilde{\omega}(t)$  and  $\tilde{r}(t)$

to see that a smaller  $H_\infty$  tracking performance can be achieved by Proposition 2.2 than that in [28].

In simulation, we choose  $t_f = 30s$  and the initial states  $x(0) = [-0.5 \ 1.3 \ 0.3 \ -0.3]^T$  and  $x_r(0) = 0.5$ . Then we depict the outputs of the system (2.30)-(2.31) by Figure 2.3. In Figure 2.3,  $y_r$  is the output of the reference model (2.30),  $y_1$  and  $y_2$  are the outputs of the system (2.30) controlled by the network-based tracking controller with  $GM_1$  and  $GM_2$ , respectively. Correspondingly, the output tracking errors are compared in Figure 2.4.

For comparison purpose, we borrow another set of input signals from [28]

$$\tilde{\omega}(t) = \begin{cases} 0.1\sin(5t), & 8s \leq t \leq 22s \\ 0, & \text{otherwise,} \end{cases}$$

$$\tilde{r}(t) = \begin{cases} 0.6, & 5s \leq t \leq 15s \\ -0.6, & 15s < t \leq 25s \\ 0, & \text{otherwise.} \end{cases}$$

In Figure 2.5,  $y_i$  ( $i = 1, 2$ ) are the outputs of the system (2.30) via the proposed controller and the controller designed in [28], respectively. The corresponding output tracking errors are compared in Figure 2.6. Clearly, we can see from Figure 2.3-Figure 2.6 that a better tracking effect for the system (2.30) is ensured by using the controller designed by Proposition 2.3 than that designed by Theorem 2 in [28].

## 2.5 Summary

This chapter has dealt with network-based state feedback tracking control for linear systems. The network-based tracking control system has been represented by a system with an interval time-varying sawtooth delay. A new discontinuous Lyapunov-Krasovskii functional, which makes use of the sawtooth delay and its lower and upper bounds, has been proposed to derive some delay-dependent criteria for  $H_\infty$  tracking performance analysis and controller design. Since (1) the generalized Jensen integral inequality combining the convex delay analysis method is adopted to estimate to integral terms and (2) the inherent piecewise-linear time-varying delay information is fully used in the derivation, it is expected that these criteria are of less conservatism. Two examples has been given to illustrate that the derived criteria are less conservative than the existing ones.

## Chapter 3

# Network-based output tracking control for linear systems via an observer-based controller

### 3.1 Introduction

There are some results available to deal with network-based output tracking control [28], [54], [133] and [137]. In the literature [28], [54], [133] and [137], all the states of a controlled plant are assumed to be *completely measurable* and the network-based output tracking control is performed by *a state feedback controller*. In fact, in some practical situations, it is physically difficult to measure all the process variables of a controlled plant. When not all the state variables of a controlled plant are measurable, a feasible alternative way to implement the network-based output tracking control is to use an observer-based tracking controller.

Compared with network-based state feedback tracking control, there has been little work on observer-based tracking control for a network-based control system (NCS). In this chapter, we will focus on network-based output tracking control for a linear system controlled by an observer-based controller. The introduction of the network leads to network-induced delays and packet dropouts in the sensor-to-controller channel and the controller-to-actuator channel. The effects of network-induced delays and packet dropouts in the two channels on modeling, stability and

observer-based tracking control will be investigated. Notice that for network-based state feedback tracking control in [28], [54], [133], and [137], a two-channel NCS is equivalent to a sensor-to-actuator channel NCS without affecting the stability and performance of the corresponding closed-loop system because a state feedback controller is time-invariant and it depends only on specific state information that successfully drives the actuator ([46], [153]), where the sensor-to-actuator channel means the channel lumping both sensor-to-controller channel and controller-to-actuator channel together. *However, this is not the case for observer-based tracking control via a network since the stability and tracking performance of the closed-loop system are affected by network-induced delays and packet dropouts in the sensor-to-actuator channel and the sensor-to-controller channel.* More specifically, the updated inputs of the controlled plant and the observer-based tracking controller are subjected to network-induced delays and packet dropouts in the *sensor-to-actuator* channel and the *sensor-to-controller* channel, respectively. Due to the effect of network-induced delays and packet dropouts in the *controller-to-actuator* channel, the inputs of the controlled plant and the observer-based controller are *updated in an asynchronous way*. Consequently, we model the closed-loop system as a system with *two different* interval time-varying delays which include information about network-induced delays and packet dropouts in the *sensor-to-actuator* channel and the *sensor-to-controller* channel, respectively. A Lyapunov-Krasovskii functional, which makes use of the lower and upper bounds of the two interval delays, is constructed to derive an LMI-based delay-dependent criterion such that the closed-loop system has a prescribed  $H_\infty$  tracking performance. Owing to the two different interval delays, a separation principle *can not be employed* to design the observer-based tracking controller in the network environment. Accordingly, we will develop a new control design method, which can determine an observer gain and a control gain by solving an optimization problem of a desired  $H_\infty$  tracking performance.

On the other hand, it is noted that some heuristic search methods play a key role in solving complex design optimization problems. A typical method is the particle swarm optimization (PSO) technique, which is a population-based global optimization algorithm inspired by social behaviors of animals such as fish schooling and birds flocking ([18], [62]). This technique has been well studied due to its easy implementation, stable convergence characteristic and computational efficiency [130]. Recently, the PSO technique has been applied to design a proportional-integral-derivative controller that minimizes an  $H_\infty$  performance index for traditional point-to-point systems in the frequency domain ([64], [151]). However, the potential of the PSO technique in finding the solution of an observer-based  $H_\infty$  tracking controller for an NCS *has not been explored*. Therefore, we will present a new design algorithm of the observer-based tracking controller by applying the PSO technique with feasibility of an LMI-based stability criterion. This design algorithm can be used to search the minimum  $H_\infty$  tracking performance and the observer gain and the control gain for an NCS. *Unlike* the frequency domain method in [64] and [151], in the proposed PSO algorithm, an LMI-based stability criterion in the time domain is employed to judge whether the closed-loop system is stable with a prescribed  $H_\infty$  tracking performance. By using the feasibility of the LMI-based criterion, the proposed method facilitates to indicate the potential of the evolutionary process in the PSO technique. In addition, using the proposed design algorithm, it is not required to specify the transfer function of the closed-loop system.

In summary, in this chapter, we will consider network-based output tracking control for a linear system via an observer-based controller. By taking into consideration asynchronous inputs of the linear system and the controller, the network-based tracking control system will be modeled as a system with two different interval time-varying delays. A delay-dependent criterion for  $H_\infty$  tracking performance analysis will be established in terms of linear matrix inequalities. Since a separation principle

can not be employed to design the observer-based controller in the network environment, a novel design algorithm will be proposed by applying the PSO technique with the feasibility of the LMI-based criterion to solve the minimum  $H_\infty$  tracking performance and the corresponding observer gain and control gain.

## 3.2 System modeling and problem statement

Consider the following controlled plant

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) + D\omega(t) \\ y(t) = Cx(t) \\ x(t_0) = x_0 \end{cases} \quad (3.1)$$

where  $x(t) \in \mathbb{R}^n$ ,  $u(t) \in \mathbb{R}^m$ ,  $y(t) \in \mathbb{R}^v$  and  $\omega(t) \in \mathcal{L}_2[t_0, \infty)$  are the state, the control input, the output, and the external disturbance, respectively;  $x(t_0) = x_0$  is the initial state;  $A$ ,  $B$ ,  $C$  and  $D$  are constant matrices with appropriate dimensions.

The reference signal  $y_r(t)$  is generated by

$$\begin{cases} \dot{x}_r(t) = A_r x_r(t) + B_r r(t) \\ y_r(t) = C_r x_r(t) \\ x_r(t_0) = x_{r0} \end{cases} \quad (3.2)$$

where  $x_r(t) \in \mathbb{R}^n$  is the state vector,  $r(t) \in \mathbb{R}^r$  is the energy bounded input vector and  $y_r(t) \in \mathbb{R}^v$  is the output vector, respectively;  $x_r(t_0) = x_{r0}$  is the initial state;  $A_r$ ,  $B_r$  and  $C_r$  are constant matrices. It is assumed that  $A_r$  is Hurwitz and  $x_r(t)$  is measurable to be used for control signals.

In this paper, we assume that the states of the physical plant (3.1) are not completely measurable. In this case, an observer-based controller can be constructed to estimate the states and perform output tracking control task. The observer-based controller tracking is given by

$$\begin{cases} \dot{\hat{x}}(t) = A\hat{x}(t) + B\hat{u}(t) + L(y(t) - \hat{y}(t)) \\ \hat{y}(t) = C\hat{x}(t) \\ \hat{u}(t) = F(\hat{x}(t) - x_r(t)) \\ \hat{x}(t) = 0, t \leq t_0 \end{cases} \quad (3.3)$$

where  $\hat{x}(t) \in \mathbb{R}^n$  is the state estimate vector,  $\hat{u}(t) \in \mathbb{R}^m$  is the input vector, and  $\hat{y}(t) \in \mathbb{R}^v$  is the output vector;  $L$  and  $F$  are an observer gain and a control gain,

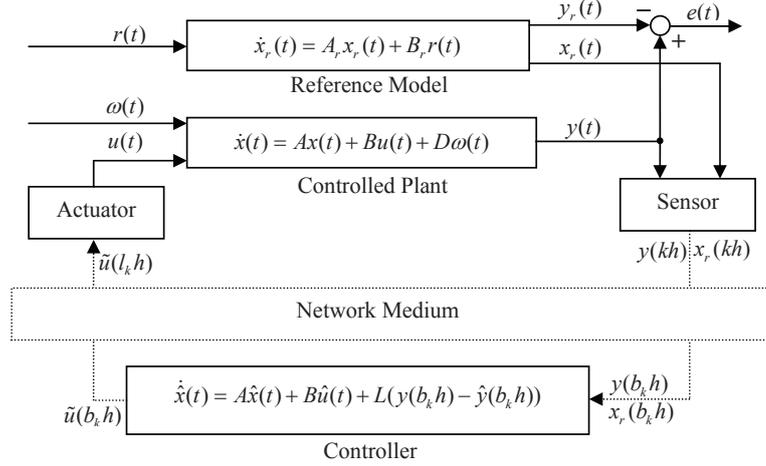


Figure 3.1: A typical configuration of a network-based tracking control system

respectively. As shown in Figure 3.1, a communication network is inserted between the plant (3.1) and the controller (3.3). The timeline of a transmitted signal goes as follows. The measurement  $y(kh)$  and  $x_r(kh)$  ( $k \in \mathbb{Z}$ ), where  $h$  is a sampling period, is augmented as a single packet with a time stamp and transmitted to the controller via the sensor-to-controller channel. The controller is equipped with a computational hardware which can implement some scheduling protocol such as priority scheduling (high priority to new packets) to actively drop outdated packets by using time stamped information. After the sensor-to-controller delay  $\tau_{b_i}^{sc}$ ,  $y(b_i h)$  and  $x_r(b_i h)$  ( $i \in \mathbb{N}$ ) are available to update the following observer-based controller on  $[b_i h + \tau_{b_i}^{sc}, b_{i+1} h + \tau_{b_{i+1}}^{sc})$  ( $\forall i \in \mathbb{N}$ )

$$\begin{cases} \dot{\hat{x}}(t) = A\hat{x}(t) + B\hat{u}(t) + L(y(b_i h) - \hat{y}(t)) \\ \hat{y}(t) = C\hat{x}(b_i h) \\ \hat{u}(t) = \tilde{u}(b_i h) = F(\hat{x}(b_i h) - x_r(b_i h)) \end{cases} \quad (3.4)$$

where  $b_i$  ( $\forall i \in \mathbb{N}$ ) are some nonnegative integers which indicate the packets that successfully update the controller,  $\{b_1, b_2, b_3, \dots\} \subseteq \mathbb{Z}$  and the sequence  $\{b_i\}$  is strictly increasing. The controller operates in four steps: (i) storing sampled state estimate  $\hat{x}(kh)$ ; (ii) receiving transmitted packets; (iii) processing packets which consists of actively dropping outdated packets, extracting the current data  $\hat{x}(b_i h)$ , deleting the previous data  $\hat{x}(b_{i-1} h), \dots, \hat{x}((b_i-1)h)$ , and updating the controller by  $\hat{x}(b_i h)$ ,  $y(b_i h)$

and  $x_r(b_i h)$  ( $\forall i \in \mathbb{N}$ ); and (iv) outputting control signals  $\tilde{u}(b_i h)$  ( $\forall i \in \mathbb{N}$ ). The control signal  $\tilde{u}(b_i h)$  ( $i \in \mathbb{N}$ ) is transmitted in a single packet with a time stamp to the actuator. Similar to the controller, the actuator has a hardware that can actively drop outdated packets. In consequence,  $\tilde{u}(l_k h)$  ( $k \in \mathbb{N}$ ) is available to update the actuator after the controller-to-actuator delay  $\tau_{l_k}^{ca}$ . The actuator holds the signal until next update. Then the control input of the plant (3.1) is described by

$$u(t) = \tilde{u}(l_k h) = F(\hat{x}(l_k h) - x_r(l_k h)), t \in [l_k h + \tau_{l_k}, l_{k+1} h + \tau_{l_{k+1}}), \forall k \in \mathbb{N} \quad (3.5)$$

where  $\tau_{l_k} = \tau_{l_k}^{sc} + \tau_{l_k}^{ca}$ ,  $l_k$  ( $\forall k \in \mathbb{N}$ ) are some nonnegative integers which indicate the control signals that successfully update the actuator,  $\{l_1, l_2, l_3, \dots\} \subseteq \{b_1, b_2, b_3, \dots\}$  and  $\{l_k\}$  is strictly increasing.

**Remark 3.1.** In this paper, we consider two kinds of packet dropouts: network-induced packet dropouts and active packet dropouts, see Fig. 3.2. The network-induced packet dropouts are caused by a number of factors such as signal degradation and packet corruption in transmission. The active packet dropouts are outdated packets which are intentionally dropped for timing reconstruction by using time stamped information. In what follows, packet dropouts in the sensor-to-controller channel consist of network-induced packet dropouts in the sensor-to-controller channel and active packet dropouts in the controller; similarly, packet dropouts in the controller-to-actuator channel consist of network-induced packet dropouts in the controller-to-actuator channel and active packet dropouts in the actuator. Packet dropouts in the sensor-to-actuator channel includes both packet dropouts in the sensor-to-controller channel and the sensor-to-actuator channel.

**Remark 3.2.** The observer-based controller (3.4) is driven by available sampled-data  $y(b_i h)$  and  $x_r(b_i h)$ , and uses the difference between  $\hat{x}(b_i h)$  and  $x_r(b_i h)$  to produce control signals  $\tilde{u}(b_i h)$ ,  $\forall i \in \mathbb{N}$ . The computational delay in the controller (3.4) is assumed to be neglected. Different from the observer-based stabilizing controller that outputs control signals with a new sampling period ([89], [111]), the

observer-based tracking controller (3.4) outputs control signals  $\tilde{u}(b_i h)$  ( $\forall i \in \mathbb{N}$ ) in the updating instants  $\{b_i h + \tau_{b_i}^{sc}\}_{i=1}^{\infty}$  because  $\tilde{u}(b_i h)$  ( $\forall i \in \mathbb{N}$ ) involve the information about  $x_r(b_i h)$  ( $\forall i \in \mathbb{N}$ ). In addition, the control inputs of an observer-based stabilizing controller and a controlled plant in [89] and [111] are assumed to be synchronous, while the control inputs of the tracking controller (3.4) and the controlled plant (3.1) are updated *asynchronously* at different time instants and with different frequencies due to the effect of network-induced delays and packet dropouts in the controller-to-actuator channel.

Let  $\tau_1(t) = t - b_i h$  for  $t \in [b_i h + \tau_{b_i}^{sc}, b_{i+1} h + \tau_{b_{i+1}}^{sc})$  ( $i \in \mathbb{N}$ ) and  $\tau_2(t) = t - l_k h$  for  $t \in [l_k h + \tau_{l_k}, l_{k+1} h + \tau_{l_{k+1}})$  ( $k \in \mathbb{N}$ ). It is clear to see

$$\begin{cases} \tau_{b_i}^{sc} \leq \tau_1(t) \leq (b_{i+1} - b_i)h + \tau_{b_{i+1}}^{sc}, & t \in [b_i h + \tau_{b_i}^{sc}, b_{i+1} h + \tau_{b_{i+1}}^{sc}), \forall i \in \mathbb{N} \\ \tau_{l_k} \leq \tau_2(t) \leq (l_{k+1} - l_k)h + \tau_{l_{k+1}}, & t \in [l_k h + \tau_{l_k}, l_{k+1} h + \tau_{l_{k+1}}), \forall k \in \mathbb{N}. \end{cases} \quad (3.6)$$

Define  $\tau_{1M} = \max_{i \in \mathbb{N}} \{(b_{i+1} - b_i)h + \tau_{b_{i+1}}^{sc}\}$ ,  $\tau_{1m} = \min_{i \in \mathbb{N}} \{\tau_{b_i}^{sc}\}$ ,  $\tau_{2M} = \max_{k \in \mathbb{N}} \{(l_{k+1} - l_k)h + \tau_{l_{k+1}}\}$  and  $\tau_{2m} = \min_{k \in \mathbb{N}} \{\tau_{l_k}\}$ . Then we have

$$\begin{cases} 0 < \tau_{1m} \leq \tau_1(t) \leq \tau_{1M}, & t \in [b_i h + \tau_{b_i}^{sc}, b_{i+1} h + \tau_{b_{i+1}}^{sc}), \forall i \in \mathbb{N} \\ 0 < \tau_{2m} \leq \tau_2(t) \leq \tau_{2M}, & t \in [l_k h + \tau_{l_k}, l_{k+1} h + \tau_{l_{k+1}}), \forall k \in \mathbb{N} \end{cases} \quad (3.7)$$

where  $\tau_{1m} > 0$  and  $\tau_{2m} > 0$  are lower bounds of network-induced delays in the sensor-to-controller channel and the sensor-to-actuator channel, respectively;  $\tau_{1M}$  and  $\tau_{2M}$  can be viewed as synthetical indexes involving information about network-induced delays and packet dropouts in the sensor-to-controller channel and the sensor-to-actuator channel, respectively.

Notice that on  $[l_k h + \tau_{l_k}, l_{k+1} h + \tau_{l_{k+1}})$ , the actuator (3.5) holds the control signal  $\tilde{u}(l_k h)$ , while the observer-based controller (3.4) may witness not less than one update, see Figure 3.2. More specifically, the controller (3.4) holds the signals  $y(b_k^1 h)$ ,  $x_r(b_k^1 h)$  and  $\hat{x}(b_k^1 h)$  on  $[l_k h + \tau_{l_k}, b_k^2 h + \tau_{b_k^2}^{sc})$ , the signals  $y(b_k^2 h)$ ,  $x_r(b_k^2 h)$  and  $\hat{x}(b_k^2 h)$  on  $[b_k^2 h + \tau_{b_k^2}^{sc}, b_k^3 h + \tau_{b_k^3}^{sc})$ ,  $\dots$ , and the signals  $y(b_k^m h)$ ,  $x_r(b_k^m h)$  and  $\hat{x}(b_k^m h)$  on  $[b_k^m h + \tau_{b_k^m}^{sc}, l_{k+1} h + \tau_{l_{k+1}})$ , where  $m = d_k + 1$ ,  $m, d_k, k \in \mathbb{N}$ ,  $d_k$  is the time of

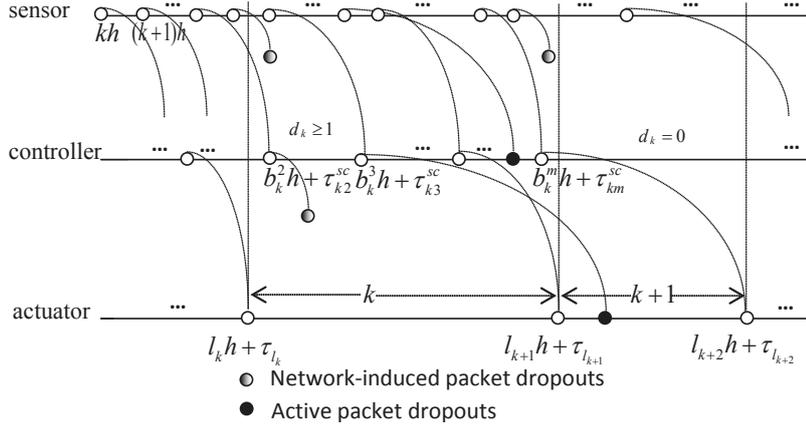


Figure 3.2: Time diagram of effective packets in the network transmission

update of the controller (3.4) and  $\{b_k^i\}$  ( $i = 1, 2, \dots, m$ ) denotes the sequence of available sampled-data in the controller (3.4) on  $[l_k h + \tau_{l_k}, l_{k+1} h + \tau_{l_{k+1}})$ , respectively. Obviously,  $\{b_k^1, b_k^2, \dots, b_k^m\}_{k=1}^\infty = \{b_i\}_{i=1}^\infty$  and  $l_k \leq b_k^1$  ( $k \in \mathbb{N}$ ). Then we define

$$\tau_1(t) = \begin{cases} t - b_k^1 h, & t \in \Omega_1 = [l_k h + \tau_{l_k}, b_k^2 h + \tau_{k2}^{sc}) \\ t - b_k^2 h, & t \in \Omega_2 = [b_k^2 h + \tau_{k2}^{sc}, b_k^3 h + \tau_{k3}^{sc}) \\ \dots & \dots \\ t - b_k^m h, & t \in \Omega_{d_k+1} = [b_k^m h + \tau_{km}^{sc}, l_{k+1} h + \tau_{l_{k+1}}) \end{cases} \quad (3.8)$$

where  $m = d_k + 1$  and  $d_k, k \in \mathbb{N}$ . In particular, when there is no update at the controller (3.4) on  $[l_k h + \tau_{l_k}, l_{k+1} h + \tau_{l_{k+1}})$ , i.e.,  $d_k = 0$  (see Figure 3.2), the controller (3.4) holds the signals  $y(b_k^1 h)$ ,  $x_r(b_k^1 h)$  and  $\hat{x}(b_k^1 h)$  on  $[l_k h + \tau_{l_k}, l_{k+1} h + \tau_{l_{k+1}})$  and  $\tau_1(t) = t - b_k^1 h$  for  $t \in [l_k h + \tau_{l_k}, l_{k+1} h + \tau_{l_{k+1}})$ , which can be regarded as a special case of (3.8), where  $l_k < b_k^1$  ( $k \in \mathbb{N}$ ).

Following the above description and considering (3.1), (3.2), (3.4), (3.5) and (3.8), the closed-loop system is represented by

$$\dot{\xi}(t) = \begin{cases} \bar{A}\xi(t) + \bar{B}_1 \xi(b_k^1 h) + \bar{B}_2 \xi(l_k h) + \bar{D}\bar{\omega}(t), & t \in \Omega_1, \quad \forall k \in \mathbb{N} \\ \bar{A}\xi(t) + \bar{B}_1 \xi(b_k^2 h) + \bar{B}_2 \xi(l_k h) + \bar{D}\bar{\omega}(t), & t \in \Omega_2, \quad \forall k \in \mathbb{N} \\ \dots & \dots \\ \bar{A}\xi(t) + \bar{B}_1 \xi(b_k^m h) + \bar{B}_2 \xi(l_k h) + \bar{D}\bar{\omega}(t), & t \in \Omega_{d_k+1}, \quad \forall k \in \mathbb{N} \end{cases} \quad (3.9)$$

$$e(t) = y(t) - y_r(t) = \bar{C}\xi(t) \quad (3.10)$$

where

$$\begin{aligned}\bar{A} &= \begin{bmatrix} A & 0 & 0 \\ 0 & A & 0 \\ 0 & 0 & A_r \end{bmatrix}, \bar{B}_1 = \begin{bmatrix} BF-LC & -BF & BF \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \\ \bar{B}_2 &= \begin{bmatrix} -BF & BF & -BF \\ -BF & BF & -BF \\ 0 & 0 & 0 \end{bmatrix}, \bar{D} = \begin{bmatrix} D & 0 \\ D & 0 \\ 0 & B_r \end{bmatrix}, \bar{C} = [0 \ C \ -C_r], \\ \xi(t) &= [x^T(t) - \hat{x}^T(t) \ x^T(t) \ x_r^T(t)]^T, \bar{\omega}(t) = [\omega^T(t) \ r^T(t)]^T.\end{aligned}$$

For  $t \in \Omega_i$  ( $i = 1, 2, \dots, d_k+1$ ),  $\forall k \in \mathbb{N}$ , rewrite (3.9)-(3.10) as

$$\begin{cases} \dot{\xi}(t) = \bar{A}\xi(t) + \bar{B}_1\xi(t-\tau_1(t)) + \bar{B}_2\xi(t-\tau_2(t)) + \bar{D}\bar{\omega}(t) \\ e(t) = \bar{C}\xi(t) \end{cases} \quad (3.11)$$

where  $\tau_1(t) = t - b_k^i h$  and  $\tau_{1m} \leq \tau_1(t) \leq \tau_{1M}$  for  $t \in \Omega_i$  ( $i = 1, 2, \dots, d_k+1$ ),  $\tau_2(t) = t - l_k h$  and  $\tau_{2m} \leq \tau_2(t) \leq \tau_{2M}$  for  $t \in [l_k h + \tau_{l_k}, l_{k+1} h + \tau_{l_{k+1}})$ , which can be seen from (3.7)-(3.8). Let  $\tau_M = \max\{\tau_{1M}, \tau_{2M}\}$ . Then the initial condition of the system (3.11) is supplemented by  $\xi(t) = \phi(t) = [\psi_1^T(t) \ 0 \ \psi_2^T(t)]^T$  for  $t \in [t_0 - \tau_M, t_0]$ , where  $\phi(t_0) = \xi_{t_0} = [x_0^T \ 0 \ x_{r0}^T]^T$ .

The following  $H_\infty$  tracking performance is chosen

$$\int_{t_0}^{t_f} e^T(t) U e(t) dt \leq V(\xi_{t_0}) + \gamma^2 \int_{t_0}^{t_f} \bar{\omega}^T(t) \bar{\omega}(t) dt \quad (3.12)$$

where  $t_0$  is the initial time that the actuator starts to work,  $t_f$  is the terminal time,  $\gamma$  is the desired tracking level,  $U > 0$  is the weighting matrix, and  $V(\xi_{t_0})$  is the energy function of initial states.

The purpose of this chapter is to determine the observer gain  $L$  and the control gain  $F$  such that the augmented system described by (3.9)-(3.10) is exponentially stable with a desired  $H_\infty$  tracking performance, which means that

- 1) the system (3.9)-(3.10) with  $\bar{\omega}(t) = 0$  is exponentially stable, that is, there exist constants  $\beta > 0$ ,  $\sigma > 0$  such that  $\|\xi(t)\|^2 \leq \beta \sup_{t_0 - \tau_M \leq s \leq t_0} \|\phi(s)\|^2 e^{-\sigma(t-t_0)}$  for  $t \geq t_0$ ;
- 2) the output tracking error  $e(t)$  satisfies the  $H_\infty$  tracking performance (3.12), for all nonzero  $\bar{\omega}(t) \in \mathcal{L}_2[t_0, \infty)$ .

### 3.3 A new criterion for $H_\infty$ tracking performance analysis

In this section, we will derive a new delay-dependent criterion for  $H_\infty$  tracking performance analysis, which can ensure that the system (3.9)-(3.10) has a prescribed  $H_\infty$  tracking performance  $\gamma$ . For simplicity of presentation, let

$$\begin{aligned}
 \eta_i^T(t) &= [\eta_{1i}^T(t) \ \eta_{2i}^T(t)], \\
 \eta_{1i}^T(t) &= [\xi^T(t) \ \xi^T(b_k^i h) \ \xi^T(l_k h) \ \xi^T(t - \tau_{1m})], \\
 \eta_{2i}^T(t) &= [\xi^T(t - \tau_{2m}) \ \xi^T(t - \tau_{1M}) \ \xi^T(t - \tau_{2M}) \ \bar{\omega}^T(t)], \\
 e_1 &= [I_{p \times p} \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0]_{p \times (7p+q)}, \\
 e_2 &= [0 \ I_{p \times p} \ 0 \ 0 \ 0 \ 0 \ 0 \ 0]_{p \times (7p+q)}, \\
 e_3 &= [0 \ 0 \ I_{p \times p} \ 0 \ 0 \ 0 \ 0 \ 0]_{p \times (7p+q)}, \\
 e_4 &= [0 \ 0 \ 0 \ I_{p \times p} \ 0 \ 0 \ 0 \ 0]_{p \times (7p+q)}, \\
 e_5 &= [0 \ 0 \ 0 \ 0 \ I_{p \times p} \ 0 \ 0 \ 0]_{p \times (7p+q)}, \\
 e_6 &= [0 \ 0 \ 0 \ 0 \ 0 \ I_{p \times p} \ 0 \ 0]_{p \times (7p+q)}, \\
 e_7 &= [0 \ 0 \ 0 \ 0 \ 0 \ 0 \ I_{p \times p} \ 0]_{p \times (7p+q)}, \\
 e_8 &= [0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ I_{q \times q}]_{q \times (7p+q)}
 \end{aligned}$$

where  $i = 1, 2, \dots, d_k+1$ ,  $d_k, k \in \mathbb{N}$ ;  $e_j$  ( $j=1, 2, \dots, 7$ ) are  $p \times (7p+q)$  matrices and  $e_8$  is a  $q \times (7p+q)$  matrix;  $I_{p \times p}$  and  $I_{q \times q}$  are  $p \times p$  and  $q \times q$  identity matrices, respectively; the others in  $e_j$  ( $j=1, 2, \dots, 8$ ) are zero matrices with appropriate dimensions;  $p$  and  $q$  are dimensions of  $\xi(t)$  and  $\bar{\omega}(t)$ , respectively. Then  $\xi(t) = e_1 \eta_i(t)$ ,  $\xi(b_k^i h) = e_2 \eta_i(t)$ ,  $\xi(l_k h) = e_3 \eta_i(t)$ , and so on, for  $t \in \Omega_i$  ( $i = 1, 2, \dots, d_k+1$ ), where  $d_k, k \in \mathbb{N}$ . Rewrite the closed-loop system (3.9)-(3.10) as

$$\dot{\xi}(t) = \begin{cases} (\bar{A}e_1 + \bar{B}_1 e_2 + \bar{B}_2 e_3 + \bar{D}e_8) \eta_1(t), & t \in \Omega_1, \quad \forall k \in \mathbb{N} \\ (\bar{A}e_1 + \bar{B}_1 e_2 + \bar{B}_2 e_3 + \bar{D}e_8) \eta_2(t), & t \in \Omega_2, \quad \forall k \in \mathbb{N} \\ \dots & \dots \\ (\bar{A}e_1 + \bar{B}_1 e_2 + \bar{B}_2 e_3 + \bar{D}e_8) \eta_{d_k+1}(t), & t \in \Omega_{d_k+1}, \quad \forall k \in \mathbb{N} \end{cases} \quad (3.13)$$

$$e(t) = \begin{cases} \bar{C}e_1\eta_1(t), & t \in \Omega_1, & \forall k \in \mathbb{N} \\ \bar{C}e_1\eta_2(t), & t \in \Omega_2, & \forall k \in \mathbb{N} \\ \dots & \dots & \dots \\ \bar{C}e_1\eta_{d_k+1}(t), & t \in \Omega_{d_k+1}, & \forall k \in \mathbb{N}. \end{cases} \quad (3.14)$$

We now state and establish the following delay-dependent criterion for  $H_\infty$  tracking performance analysis of the system (3.9)-(3.10)

**Proposition 3.1.** *For given a weighting matrix  $U > 0$ , positive scalars  $\tau_{1m}$ ,  $\tau_{2m}$ ,  $\tau_{1M}$ ,  $\tau_{2M}$  and  $\gamma$ , an observer gain  $L$  and a control gain  $F$ , the system described by (3.9)-(3.10) is exponentially stable with the  $H_\infty$  tracking performance (3.12) if there exist symmetric positive definite matrices  $P > 0$ ,  $Q_i > 0$ ,  $R_i > 0$  ( $i = 1, 2, 3, 4$ ) and matrices  $M_i$  ( $i = 1, 2, 3, 4$ ) such that*

$$\begin{bmatrix} \Psi & * & * \\ (\tau_{1M} - \tau_{1m})M_1 & -(\tau_{1M} - \tau_{1m})R_3 & * \\ (\tau_{2M} - \tau_{2m})M_3 & 0 & -(\tau_{2M} - \tau_{2m})R_4 \end{bmatrix} < 0 \quad (3.15)$$

$$\begin{bmatrix} \Psi & * & * \\ (\tau_{1M} - \tau_{1m})M_1 & -(\tau_{1M} - \tau_{1m})R_3 & * \\ (\tau_{2M} - \tau_{2m})M_4 & 0 & -(\tau_{2M} - \tau_{2m})R_4 \end{bmatrix} < 0 \quad (3.16)$$

$$\begin{bmatrix} \Psi & * & * \\ (\tau_{1M} - \tau_{1m})M_2 & -(\tau_{1M} - \tau_{1m})R_3 & * \\ (\tau_{2M} - \tau_{2m})M_3 & 0 & -(\tau_{2M} - \tau_{2m})R_4 \end{bmatrix} < 0 \quad (3.17)$$

$$\begin{bmatrix} \Psi & * & * \\ (\tau_{1M} - \tau_{1m})M_2 & -(\tau_{1M} - \tau_{1m})R_3 & * \\ (\tau_{2M} - \tau_{2m})M_4 & 0 & -(\tau_{2M} - \tau_{2m})R_4 \end{bmatrix} < 0 \quad (3.18)$$

where

$$\begin{aligned} \Psi = & e_1^T (P\bar{A} + \bar{A}^T P + Q_1 + \bar{C}^T U \bar{C} - R_1) e_1 + e_1^T P \bar{B}_1 e_2 + e_2^T \bar{B}_1^T P e_1 \\ & + e_1^T P \bar{B}_2 e_3 + e_3^T \bar{B}_2^T P e_1 + e_1^T R_1 e_4 + e_4^T R_1 e_1 + e_1^T P \bar{D} e_8 \\ & + e_8^T \bar{D}^T P e_1 + e_4^T R_2 e_5 + e_4^T (Q_2 + Q_3 - Q_1 - R_1 - R_2) e_4 \\ & + e_5^T R_2 e_4 + e_5^T (Q_4 - Q_2 - R_2) e_5 - e_6^T Q_3 e_6 - e_7^T Q_4 e_7 - e_8^T \gamma^2 I e_8 \\ & + (e_2 - e_6)^T M_1 + M_1^T (e_2 - e_6) + (e_4 - e_2)^T M_2 + M_2^T (e_4 - e_2) \\ & + (e_3 - e_7)^T M_3 + M_3^T (e_3 - e_7) + (e_5 - e_3)^T M_4 + M_4^T (e_5 - e_3) \\ & + (\bar{A}e_1 + \bar{B}_1 e_2 + \bar{B}_2 e_3 + \bar{D}e_8)^T \Lambda (\bar{A}e_1 + \bar{B}_1 e_2 + \bar{B}_2 e_3 + \bar{D}e_8), \\ \Lambda = & \tau_{1m}^2 R_1 + (\tau_{2m} - \tau_{1m})^2 R_2 + (\tau_{1M} - \tau_{1m}) R_3 + (\tau_{2M} - \tau_{2m}) R_4. \end{aligned}$$

*Proof:* Choose the following Lyapunov-Krasovskii functional

$$\begin{aligned}
V(\xi_t) = & \xi^T(t)P\xi(t) + \int_{t-\tau_{1m}}^t \xi^T(s)Q_1\xi(s)ds + \int_{t-\tau_{2m}}^{t-\tau_{1m}} \xi^T(s)Q_2\xi(s)ds \\
& + \int_{t-\tau_{1M}}^{t-\tau_{1m}} \xi^T(s)Q_3\xi(s)ds + \int_{t-\tau_{2M}}^{t-\tau_{2m}} \xi^T(s)Q_4\xi(s)ds \\
& + \tau_{1m} \int_{t-\tau_{1m}}^t \int_s^t \dot{\xi}^T(\theta)R_1\dot{\xi}(\theta)d\theta ds + (\tau_{2m} - \tau_{1m}) \int_{t-\tau_{2m}}^{t-\tau_{1m}} \int_s^t \dot{\xi}^T(\theta)R_2\dot{\xi}(\theta)d\theta ds \\
& + \int_{t-\tau_{1M}}^{t-\tau_{1m}} \int_s^t \dot{\xi}^T(\theta)R_3\dot{\xi}(\theta)d\theta ds + \int_{t-\tau_{2M}}^{t-\tau_{2m}} \int_s^t \dot{\xi}^T(\theta)R_4\dot{\xi}(\theta)d\theta ds \quad (3.19)
\end{aligned}$$

where  $P > 0$ ,  $Q_i > 0$ ,  $R_i > 0$  ( $i = 1, 2, 3, 4$ );  $\xi_t = \xi(t + \theta)$ ,  $\forall \theta \in [-\tau_M, 0]$  and  $\tau_M = \max\{\tau_{1M}, \tau_{2M}\}$ .

Taking the time-derivative of  $V(\xi_t)$  with respect to  $t$  on  $\Omega_1 = [l_k h + \tau_{l_k}, b_k^1 h + \tau_{k1}^{sc})$  ( $\forall k \in \mathbb{N}$ ) along the trajectory of the system (3.13)-(3.14), we have

$$\begin{aligned}
\dot{V}(\xi_t) = & 2\eta_1^T(t)[e_1^T P(\bar{A}e_1 + \bar{B}_1 e_2 + \bar{B}_2 e_3 + \bar{D}e_8) + e_1^T Q_1 e_1 + e_4^T (Q_2 + Q_3 - Q_1)e_4] \eta_1(t) \\
& + \eta_1^T(t)[e_5^T (Q_4 - Q_2)e_5 - e_6^T Q_3 e_6 - e_7^T Q_4 e_7] \eta_1(t) - \tau_{1m} \int_{t-\tau_{1m}}^t \dot{\xi}^T(s)R_1\dot{\xi}(s)ds \\
& - (\tau_{2m} - \tau_{1m}) \int_{t-\tau_{2m}}^{t-\tau_{1m}} \dot{\xi}^T(s)R_2\dot{\xi}(s)ds - \int_{t-\tau_{1M}}^{t-\tau_{1m}} \dot{\xi}^T(s)R_3\dot{\xi}(s)ds \\
& - \int_{t-\tau_{2M}}^{t-\tau_{2m}} \dot{\xi}^T(s)R_4\dot{\xi}(s)ds + \dot{\xi}^T(t)\Lambda\dot{\xi}(t). \quad (3.20)
\end{aligned}$$

Using Jensen integral inequality, for  $t \in \Omega_1$  ( $k \in \mathbb{N}$ ), we have

$$- \tau_{1m} \int_{t-\tau_{1m}}^t \dot{\xi}^T(s)R_1\dot{\xi}(s)ds \leq -\eta_1^T(t)(e_1 - e_4)^T R_1(e_1 - e_4)\eta_1(t) \quad (3.21)$$

$$- (\tau_{2m} - \tau_{1m}) \int_{t-\tau_{2m}}^{t-\tau_{1m}} \dot{\xi}^T(s)R_2\dot{\xi}(s)ds \leq -\eta_1^T(t)(e_4 - e_5)^T R_2(e_4 - e_5)\eta_1(t). \quad (3.22)$$

Notice that

$$- \int_{t-\tau_{1M}}^{t-\tau_{1m}} \dot{\xi}^T(s)R_3\dot{\xi}(s)ds = - \int_{t-\tau_{1M}}^{b_k^1 h} \dot{\xi}^T(s)R_3\dot{\xi}(s)ds - \int_{b_k^1 h}^{t-\tau_{1m}} \dot{\xi}^T(s)R_3\dot{\xi}(s)ds \quad (3.23)$$

$$- \int_{t-\tau_{2M}}^{t-\tau_{2m}} \dot{\xi}^T(s)R_4\dot{\xi}(s)ds = - \int_{t-\tau_{2M}}^{l_k h} \dot{\xi}^T(s)R_4\dot{\xi}(s)ds - \int_{l_k h}^{t-\tau_{2m}} \dot{\xi}^T(s)R_4\dot{\xi}(s)ds. \quad (3.24)$$

Applying Lemma 2.2 with  $\mathcal{E} = e_2 - e_6$ ,  $\psi = \eta_1(t)$  and  $Z = M_1$  yields

$$\begin{aligned}
- \int_{t-\tau_{1M}}^{b_k^1 h} \dot{\xi}^T(s)R_3\dot{\xi}(s)ds \leq & \eta_1^T(t)[(e_2 - e_6)^T M_1 + M_1^T (e_2 - e_6)] \eta_1(t) \\
& + \eta_1^T(t)[\tau_{1M} - (t - b_k^1 h)] M_1^T R_3^{-1} M_1 \eta_1(t). \quad (3.25)
\end{aligned}$$

Similarly, for other terms in (3.23) and (3.24), the following inequalities hold for  $t \in \Omega_1$  ( $\forall k \in \mathbb{N}$ ) and matrices  $M_2, M_3, M_4$  with appropriate dimensions

$$\begin{aligned} - \int_{b_k^1 h}^{t-\tau_{1m}} \dot{\xi}^T(s) R_3 \dot{\xi}(s) ds &\leq \eta_1^T(t) [(e_4 - e_2)^T M_2 + M_2^T (e_4 - e_2)] \eta_1(t) \\ &\quad + \eta_1^T(t) [(t - b_k^1) - \tau_{1m}] M_2^T R_3^{-1} M_2 \eta_1(t) \end{aligned} \quad (3.26)$$

$$\begin{aligned} - \int_{t-\tau_{2M}}^{l_k h} \dot{\xi}^T(s) R_4 \dot{\xi}(s) ds &\leq \eta_1^T(t) [(e_3 - e_7)^T M_3 + M_3^T (e_3 - e_7)] \eta_1(t) \\ &\quad + \eta_1^T(t) [\tau_{2M} - (t - l_k h)] M_3^T R_4^{-1} M_3 \eta_1(t) \end{aligned} \quad (3.27)$$

$$\begin{aligned} - \int_{l_k h}^{t-\tau_{2m}} \dot{\xi}^T(s) R_4 \dot{\xi}(s) ds &\leq \eta_1^T(t) [(e_5 - e_3)^T M_4 + M_4^T (e_5 - e_3)] \eta_1(t) \\ &\quad + \eta_1^T(t) [(t - l_k h) - \tau_{2m}] M_4^T R_4^{-1} M_4 \eta_1(t). \end{aligned} \quad (3.28)$$

From (3.20)-(3.26), we obtain

$$\dot{V}(\xi_t) \leq \eta_1^T(t) (\Xi(t) + \gamma^2 e_8^T e_8 - e_1^T \bar{C}^T U \bar{C} e_1) \eta_1(t), \quad t \in \Omega_1, \quad \forall k \in \mathbb{N} \quad (3.29)$$

where

$$\Xi(t) = \Psi + \Upsilon_1(t) + \Upsilon_2(t),$$

$$\Upsilon_1(t) = (\tau_{1M} - \tau_1(t)) M_1^T R_3^{-1} M_1 + (\tau_1(t) - \tau_{1m}) M_2^T R_3^{-1} M_2, \quad \tau_1(t) = t - b_k^m, \quad \forall k \in \mathbb{N},$$

$$\Upsilon_2(t) = (\tau_{2M} - \tau_2(t)) M_3^T R_4^{-1} M_3 + (\tau_2(t) - \tau_{2m}) M_4^T R_4^{-1} M_4, \quad \tau_2(t) = t - l_k h, \quad \forall k \in \mathbb{N}.$$

Notice that  $\Upsilon_1(t)$  and  $\Upsilon_2(t)$  are two convex combinations of  $M_1^T R_3^{-1} M_1$  and  $M_2^T R_3^{-1} M_2$  on  $\tau_1(t) \in [\tau_{1m}, \tau_{1M}]$ , and  $M_3^T R_4^{-1} M_3$  and  $M_4^T R_4^{-1} M_4$  on  $\tau_2(t) \in [\tau_{2m}, \tau_{2M}]$ , respectively. From (3.7)-(3.8), it can be seen that  $\tau_{1m} \leq \tau_1(t) \leq \tau_{1M}$  for  $t \in \Omega_i$  ( $i = 1, 2, \dots, d_k + 1, \forall k \in \mathbb{N}$ );  $\tau_{2m} \leq \tau_2(t) < \tau_{2M}$  for  $t \in \Omega_1$ ,  $\tau_{2m} < \tau_2(t) \leq \tau_{2M}$  for  $t \in \Omega_{d_k+1}$ , and  $\tau_{2m} < \tau_2(t) < \tau_{2M}$  for  $t \in \Omega_i$  ( $i = 2, 3, \dots, d_k, \forall k \in \mathbb{N}$ ). Thus, if the LMIs (3.15)-(3.18) are satisfied, using schur complement yields  $\Xi(t) < 0$  for  $t \in \Omega_1, \forall k \in \mathbb{N}$ .

First, we consider the exponential stability of the closed-loop system (3.9)-(3.10) with  $\bar{\omega}(t) = 0$  for  $t \in \Omega_1, \forall k \in \mathbb{N}$ . It can be seen from the LMIs (3.15)-(3.18) that  $\Xi(t) < 0$ , which implies that there exist a constant  $\varepsilon_1 > 0$  such that

$$\dot{V}(\xi_t) \leq -\varepsilon_1 \xi^T(t) \xi(t) - \varepsilon_1 \xi^T(b_k^1 h) \xi(b_k^1 h) - \varepsilon_1 \xi^T(l_k h) \xi(l_k h), \quad t \in \Omega_1, \quad \forall k \in \mathbb{N}. \quad (3.30)$$

Similarly, taking time-derivative of  $V(\xi_t)$  on  $\Omega_i$  ( $i = 2, 3, \dots, d_k + 1$ ), we have

$$\dot{V}(\xi_t) \leq \begin{cases} -\varepsilon_2[\xi^T(t)\xi(t) + \xi^T(b_k^2 h)\xi(b_k^2 h) + \xi^T(l_k h)\xi(l_k h)], & t \in \Omega_2 \\ -\varepsilon_3[\xi^T(t)\xi(t) + \xi^T(b_k^3 h)\xi(b_k^3 h) + \xi^T(l_k h)\xi(l_k h)], & t \in \Omega_3 \\ \dots \\ -\varepsilon_{d_k+1}[\xi^T(t)\xi(t) + \xi^T(b_k^m h)\xi(b_k^m h) + \xi^T(l_k h)\xi(l_k h)], & t \in \Omega_{d_k+1} \end{cases} \quad (3.31)$$

where  $\varepsilon_i > 0$  ( $i = 2, 3, \dots, d_k + 1$ ) and  $d_k \in \mathbb{N}$ ,  $\forall k \in \mathbb{N}$ .

Define a new Lyapunov-Krasovskii functional  $\tilde{V}(\xi_t) = e^{\sigma t}V(\xi_t)$  for  $t \in [l_k h + \tau_{l_k}, l_{k+1}h + \tau_{l_{k+1}})$ ,  $\forall k \in \mathbb{N}$ , where  $\sigma > 0$  is a constant to be determined. Considering (3.30)-(3.31) and integrating both sides of  $\dot{V}(\xi_t)$  from  $l_k h + \tau_{l_k}$  to  $t$ , we have

$$\begin{aligned} & \int_{l_k h + \tau_{l_k}}^t \dot{\tilde{V}}(\xi_s) ds = \tilde{V}(\xi_t) - \tilde{V}(\xi_{l_k h + \tau_{l_k}}) \\ & \leq \int_{l_k h + \tau_{l_k}}^t \sigma e^{\sigma s} V(\xi_s) ds - \varepsilon \int_{l_k h + \tau_{l_k}}^t e^{\sigma s} \{ \|\xi(s)\|^2 + \|\xi(l_k h)\|^2 \} ds \\ & - \varepsilon \int_{l_k h + \tau_{l_k}}^{b_k^2 h + \tau_{k2}^{sc}} e^{\sigma s} \|\xi(b_k^1 h)\|^2 ds - \varepsilon \sum_{i=1}^{v_k-1} \int_{b_k^{i+1} h + \tau_{k(i+1)}^{sc}}^{b_k^{i+2} h + \tau_{k(i+2)}^{sc}} e^{\sigma s} \|\xi(b_k^{i+1} h)\|^2 ds \\ & - \varepsilon \int_{b_k^{v_k+1} h + \tau_{k(v_k+1)}^{sc}}^t e^{\sigma s} \|\xi(b_k^{v_k+1} h)\|^2 ds \end{aligned} \quad (3.32)$$

where  $\varepsilon = \min\{\varepsilon_1, \varepsilon_2, \dots, \varepsilon_{d_k+1}\}$ ,  $v_k \leq d_k$  and  $v_k, d_k \in \mathbb{N}$ ,  $\forall k \in \mathbb{N}$ .

Since  $\bigcup_{k=1}^{\infty} [l_k h + \tau_{l_k}, l_{k+1} h + \tau_{l_{k+1}}) = [t_0, \infty)$  and  $V(\xi_t)$  is continuous on  $[t_0, \infty)$ , we have

$$\begin{aligned} & \tilde{V}(\xi_t) - \tilde{V}(\xi_{t_0}) \\ & \leq (\sigma \lambda_{\max}(P) - \varepsilon) \int_{t_0}^t e^{\sigma s} \|\xi(s)\|^2 ds + \varepsilon \varphi_1(t) + \varepsilon \varphi_2(t) \\ & + \sigma \tau_{1m} e^{\sigma \tau_{1m}} \lambda_{\max}(Q_1) \int_{t_0 - \tau_{1m}}^t e^{\sigma s} \|\xi(s)\|^2 ds \\ & + \sigma e^{\sigma \tau_{2m}} \lambda_{\max}(Q_2) \tau_{2m} \int_{t_0 - \tau_{2m}}^{t_0 - \tau_{1m}} e^{\sigma s} \|\xi(s)\|^2 ds \\ & + \sigma e^{\sigma \tau_{2m}} \lambda_{\max}(Q_2) (\tau_{2m} - \tau_{1m}) \int_{t_0 - \tau_{1m}}^t e^{\sigma s} \|\xi(s)\|^2 ds \\ & + \sigma e^{\sigma \tau_{1M}} \lambda_{\max}(Q_3) \tau_{1M} \int_{t_0 - \tau_{1M}}^{t_0 - \tau_{1m}} e^{\sigma s} \|\xi(s)\|^2 ds \\ & + \sigma e^{\sigma \tau_{1M}} \lambda_{\max}(Q_3) (\tau_{1M} - \tau_{1m}) \int_{t_0 - \tau_{1m}}^t e^{\sigma s} \|\xi(s)\|^2 ds \end{aligned}$$

$$\begin{aligned}
& + \sigma e^{\sigma\tau_{2M}} \lambda_{\max}(Q_4) \tau_{2M} \int_{t_0-\tau_{2M}}^{t_0-\tau_{2m}} e^{\sigma s} \|\xi(s)\|^2 ds \\
& + \sigma e^{\sigma\tau_{2M}} \lambda_{\max}(Q_4) (\tau_{2M} - \tau_{2m}) \int_{t_0-\tau_{2m}}^t e^{\sigma s} \|\xi(s)\|^2 ds \\
& + \sigma \tau_{1m}^3 e^{\sigma\tau_{1m}} \lambda_{\max}(R_1) \int_{t_0-\tau_{1m}}^t e^{\sigma s} \|\dot{\xi}(s)\|^2 ds \\
& + \sigma \tau_{2m} (\tau_{2m} - \tau_{1m})^2 e^{\sigma\tau_{2m}} \lambda_{\max}(R_2) \int_{t_0-\tau_{2m}}^t e^{\sigma s} \|\dot{\xi}(s)\|^2 ds \\
& + \sigma \tau_{1M} (\tau_{1M} - \tau_{1m})^2 e^{\sigma\tau_{1M}} \lambda_{\max}(R_3) \int_{t_0-\tau_{1M}}^t e^{\sigma s} \|\dot{\xi}(s)\|^2 ds \\
& + \sigma \tau_{2M} (\tau_{2M} - \tau_{2m})^2 e^{\sigma\tau_{2M}} \lambda_{\max}(R_4) \int_{t_0-\tau_{2M}}^t e^{\sigma s} \|\dot{\xi}(s)\|^2 ds \\
\leq & \zeta_1 \int_{t_0-\tau_{1m}}^t e^{\sigma s} \|\xi(s)\|^2 ds + \zeta_2 \varphi_1(t) + \zeta_3 \varphi_2(t) \\
& + \rho_1 \int_{t_0-\tau_{1m}}^{t_0} e^{\sigma s} \|\phi(s)\|^2 ds + \rho_2 \int_{t_0-\tau_{2m}}^{t_0} e^{\sigma s} \|\phi(s)\|^2 ds \\
& + \sigma \tau_{1M} (\tau_{1M} - \tau_{1m})^2 e^{\sigma\tau_{1M}} \lambda_{\max}(R_3) \|\bar{A}\|^2 \int_{t_0-\tau_{1M}}^{t_0} e^{\sigma s} \|\phi(s)\|^2 ds \\
& + \sigma \tau_{2M} (\tau_{2M} - \tau_{2m})^2 e^{\sigma\tau_{2M}} \lambda_{\max}(R_4) \|\bar{A}\|^2 \int_{t_0-\tau_{2M}}^{t_0} e^{\sigma s} \|\phi(s)\|^2 ds \\
& + \sigma \tau_{2m} e^{\sigma\tau_{2m}} \lambda_{\max}(Q_2) \int_{t_0-\tau_{2m}}^{t_0-\tau_{1m}} e^{\sigma s} \|\phi(s)\|^2 ds \\
& + \sigma \tau_{1M} e^{\sigma\tau_{1M}} \lambda_{\max}(Q_3) \int_{t_0-\tau_{1M}}^{t_0-\tau_{1m}} e^{\sigma s} \|\phi(s)\|^2 ds \\
& + \sigma \tau_{2M} e^{\sigma\tau_{2M}} \lambda_{\max}(Q_4) \int_{t_0-\tau_{2M}}^{t_0-\tau_{2m}} e^{\sigma s} \|\phi(s)\|^2 ds
\end{aligned} \tag{3.33}$$

where

$$\begin{aligned}
\varphi_1(t) = & \sum_{k=1}^{N-1} \left[ \int_{l_k h + \tau_{l_k}}^{b_k^2 h + \tau_{k1}^{sc}} e^{\sigma s} \|\xi(b_k^1 h)\|^2 ds + \sum_{i=1}^{d_k-1} \int_{b_k^{i+1} h + \tau_{k(i+1)}^{sc}}^{b_k^{i+2} h + \tau_{k(i+2)}^{sc}} e^{\sigma s} \|\xi(b_k^{i+1} h)\|^2 ds \right. \\
& \left. + \int_{b_k^m h + \tau_{km}^{sc}}^{l_{k+1} h + \tau_{l_{k+1}}} e^{\sigma s} \|\xi(b_k^m h)\|^2 ds \right] + \int_{l_N h + \tau_{l_N}}^{b_N^2 h + \tau_{N2}^{sc}} e^{\sigma s} \|\xi(b_N^1 h)\|^2 ds \\
& + \sum_{i=1}^{v_{N-1}} \int_{b_N^{i+1} h + \tau_{N(i+1)}^{sc}}^{b_N^{i+2} h + \tau_{N(i+2)}^{sc}} e^{\sigma s} \|\xi(b_N^{i+1} h)\|^2 ds + \int_{b_N^{v_{N-1}+1} h + \tau_{N(v_{N-1}+1)}^{sc}}^t e^{\sigma s} \|\xi(b_N^{v_{N-1}+1} h)\|^2 ds, \\
\varphi_2(t) = & \sum_{k=1}^{N-1} \int_{l_k h + \tau_{l_k}}^{l_{k+1} h + \tau_{l_{k+1}}} e^{\sigma s} \|\xi(l_k h)\|^2 ds + \int_{l_N h + \tau_{l_N}}^t e^{\sigma s} \|\xi(l_N h)\|^2 ds
\end{aligned}$$

$$\begin{aligned}
\zeta_1 &= -\varepsilon + \sigma \lambda_{\max}(P) + \rho_1 + \rho_2 + 2\sigma\tau_{1m}^3 e^{\sigma\tau_{1m}} \lambda_{\max}(R_1) \|\bar{A}\|^2 + 2\sigma\tau_{2m}(\tau_{2m} - \tau_{1m})^2 \\
&\quad \times e^{\sigma\tau_{2m}} \lambda_{\max}(R_2) \|\bar{A}\|^2 + 3\sigma\tau_{1M}(\tau_{1M} - \tau_{1m})^2 e^{\sigma\tau_{1M}} \lambda_{\max}(R_3) \|\bar{A}\|^2 \\
&\quad + 3\sigma\tau_{2M}(\tau_{2M} - \tau_{2m})^2 e^{\sigma\tau_{2M}} \lambda_{\max}(R_4) \|\bar{A}\|^2, \\
\zeta_2 &= -\varepsilon + 3\kappa \|\bar{B}_1\|^2, \\
\zeta_3 &= -\varepsilon + 3\kappa \|\bar{B}_2\|^2, \\
\rho_1 &= \sigma\tau_{1m} e^{\sigma\tau_{1m}} \lambda_{\max}(Q_1) + \sigma(\tau_{2m} - \tau_{1m}) e^{\sigma\tau_{2m}} \lambda_{\max}(Q_2) + \sigma(\tau_{1M} - \tau_{1m}) e^{\sigma\tau_{1M}} \\
&\quad \times \lambda_{\max}(Q_3) + \sigma\tau_{1m}^3 e^{\sigma\tau_{1m}} \lambda_{\max}(R_1) \|\bar{A}\|^2, \\
\rho_2 &= \sigma(\tau_{2M} - \tau_{2m}) e^{\sigma\tau_{2M}} \lambda_{\max}(Q_4) + \sigma\tau_{2m}(\tau_{2m} - \tau_{1m})^2 e^{\sigma\tau_{2m}} \lambda_{\max}(R_2) \|\bar{A}\|^2, \\
\kappa &= \sigma\tau_{1m}^3 e^{\sigma\tau_{1m}} \lambda_{\max}(R_1) + \sigma\tau_{2m}(\tau_{2m} - \tau_{1m})^2 e^{\sigma\tau_{2m}} \lambda_{\max}(R_2) + \sigma\tau_{1M}(\tau_{1M} - \tau_{1m})^2 \\
&\quad \times e^{\sigma\tau_{1M}} \lambda_{\max}(R_3) + \tau_{2M}\sigma(\tau_{2M} - \tau_{2m})^2 e^{\sigma\tau_{2M}} \lambda_{\max}(R_4).
\end{aligned}$$

Similar to an analysis method in [149, 57], if there exists a sufficiently small constant  $\sigma > 0$  such that  $\zeta_i < 0$  ( $i = 1, 2, 3$ ), we have

$$\|\xi(t)\|^2 \leq \frac{s_1 + s_2}{\lambda_{\min}(P)} \sup_{t_0 - \tau_M \leq s \leq t_0} \|\phi(s)\|^2 e^{-\sigma(t-t_0)} \quad (3.34)$$

where

$$\begin{aligned}
s_1 &= \sigma\tau_{1m} e^{\sigma\tau_{1m}} \lambda_{\max}(Q_1) + \sigma(2\tau_{2m} - \tau_{1m}) \lambda_{\max}(Q_2) e^{\sigma\tau_{2m}} + \sigma(2\tau_{1M} - \tau_{1m}) \lambda_{\max}(Q_3) e^{\sigma\tau_{1M}} \\
&\quad + \sigma(2\tau_{2M} - \tau_{2m}) \lambda_{\max}(Q_4) e^{\sigma\tau_{2M}} + \kappa \|\bar{A}\|^2, \\
s_2 &= \lambda_{\max}(P) + \tau_{1m} \lambda_{\max}(Q_1) + (\tau_{2m} - \tau_{1m}) \lambda_{\max}(Q_2) + (\tau_{1M} - \tau_{1m}) \lambda_{\max}(Q_3) \\
&\quad + (\tau_{2M} - \tau_{2m}) \lambda_{\max}(Q_4) + 0.5\tau_{1m}^3 \lambda_{\max}(R_1) \|\bar{A}\|^2 + 0.5(\tau_{2m} - \tau_{1m})^2 \\
&\quad \times (\tau_{2m} + \tau_{1m}) \lambda_{\max}(R_2) \|\bar{A}\|^2 + 0.5(\tau_{1M} - \tau_{1m})^2 (\tau_{1M} + \tau_{1m}) \lambda_{\max}(R_3) \|\bar{A}\|^2 \\
&\quad + 0.5(\tau_{2M} - \tau_{2m})^2 (\tau_{2M} + \tau_{2m}) \lambda_{\max}(R_4) \|\bar{A}\|^2.
\end{aligned}$$

Then it can be concluded from (3.34) that the closed-loop system described by (3.9)-(3.10) with  $\bar{\omega}(t) = 0$  is exponentially stable on  $[t_0, \infty)$  if the LMIs (3.15)-(3.18) hold.

Next, we consider the  $H_\infty$  tracking performance (3.12) of the system (3.9)-(3.10)

for all nonzero  $\bar{\omega}(t) \in \mathcal{L}_2[t_0, \infty)$ . It is clear to see that

$$\begin{aligned}
& \int_{l_k h + \tau_{l_k}}^t e^T(s) U e(s) ds \\
&= V(\xi_{l_k h + \tau_{l_k}}) - V(\xi_t) + \int_{l_k h + \tau_{l_k}}^t [e^T(s) U e(s) + \dot{V}(\xi_s) - \gamma^2 \bar{\omega}^T(s) \bar{\omega}(s) + \gamma^2 \bar{\omega}^T(s) \bar{\omega}(s)] ds \\
&+ \sum_{i=1}^{v_k-1} \int_{b_k^{i+1} h + \tau_{l_k}^{sc}}^{b_k^{i+2} h + \tau_{l_k}^{sc}} [e^T(s) U e(s) + \dot{V}(\xi_s) - \gamma^2 \bar{\omega}^T(s) \bar{\omega}(s) + \gamma^2 \bar{\omega}^T(s) \bar{\omega}(s)] ds \\
&+ \int_{b_k^{v_k+1} h + \tau_{l_k}^{sc}}^t [e^T(s) U e(s) + \dot{V}(\xi_s) - \gamma^2 \bar{\omega}^T(s) \bar{\omega}(s) + \gamma^2 \bar{\omega}^T(s) \bar{\omega}(s)] ds \quad (3.35)
\end{aligned}$$

for  $t \in [l_k h + \tau_{l_k}, l_{k+1} h + \tau_{l_{k+1}})$ ,  $\forall k \in \mathbb{N}$ . If the LMIs (3.15)-(3.18) hold, it can be seen from (3.31) that

$$\dot{V}(\xi_t) \leq \gamma^2 \bar{\omega}^T(t) \bar{\omega}(t) - e^T(t) U e(t), \quad t \in \Omega_i, \quad i = 1, 2, \dots, d_k + 1, \quad \forall k \in \mathbb{N}.$$

It follows that

$$\begin{aligned}
\int_{l_k h + \tau_{l_k}}^t e^T(s) U e(s) ds &\leq V(\xi_{l_k h + \tau_{l_k}}) - V(\xi_t) + \gamma^2 \int_{l_k h + \tau_{l_k}}^t \bar{\omega}^T(s) \bar{\omega}(s) ds \\
t &\in [l_k h + \tau_{l_k}, l_{k+1} h + \tau_{l_{k+1}}), \quad \forall k \in \mathbb{N}. \quad (3.36)
\end{aligned}$$

When  $d_k = 0$ , define  $\tau_1(t) = t - b_k^1 h$  on  $[l_k h + \tau_{l_k}, l_{k+1} h + \tau_{l_{k+1}})$ . The proof of the conditions (3.34) and (3.36) is a routine case and omitted. Notice that  $V(\xi_t)$  is continuous on  $[t_0, \infty)$  and  $\bigcup_{k=1}^{k=T} [l_k h + \tau_{l_k}, l_{k+1} h + \tau_{l_{k+1}}) = [t_0, t_f)$ , where  $T$  is the serial number of the last updating instant at the actuator,  $l_T$  is the nonnegative integer that indicates the last updating signal  $\tilde{u}(l_T h)$  at the actuator and  $l_T h$  is the time stamp of  $\tilde{u}(l_T h)$ . Using the LMIs (3.15)-(3.18), it follows from (3.36) that

$$\int_{t_0}^{t_f} e^T(t) M e(t) dt \leq V(\xi_{t_0}) + \gamma^2 \int_{t_0}^{t_f} \bar{\omega}^T(t) \bar{\omega}(t) dt$$

where  $t_0 = l_1 h + \tau_{l_1}$ , and  $t_f = l_{T+1} h + \tau_{l_{T+1}}$ . Then one can see the  $H_\infty$  tracking performance (3.12) is ensured for the system (3.9)-(3.10), which completes the proof.

In the literature, there are some results on network-based  $H_\infty$  output tracking control. For example, in [28], [54] and [137], the output tracking control is implemented by a state feedback controller which requires the controlled plant to be

completely measurable. In this chapter, an observer-based controller is utilized to estimate the states of the plant, which are not measurable, and to perform the output tracking control. The use of the observer-based controller makes a major difference in modeling and stability analysis of an NCS. More specifically, the two-channel NCS in [28], [54] and [137] is equivalent to the sensor-to-actuator channel NCS without affecting the system stability, and therefore the NCS is described by a system with one input delay. However, the two-channel NCS in this paper is modeled as the system (3.11) with two interval time-varying input delays since the updated inputs of the plant (3.1) and the observer-based controller (3.4) are subjected to network-induced delays and packet dropouts in the sensor-to-actuator channel and the sensor-to-controller channel, respectively.

It should be mentioned that there are some results available on systems with two additive delays arising from an NCS [29], [70], [114]. However, the two additive delays are essentially different from  $\tau_i(t)$  ( $i=1, 2$ ). In [29], [70] and [114],  $d_1(t)$  and  $d_2(t)$  are two differentiable time-varying delays and lumped together as one input delay in the closed-loop system, while  $\tau_1(t)$  and  $\tau_2(t)$  are two interval time-varying sawtooth delays and have different effects on the stability of the system (3.11). Due to network-induced delays and packet dropouts in the controller-to-actuator channel, it is not rational to assume  $\tau_1(t) = \tau_2(t)$  for the system (3.11) on  $[l_k h + \tau_{l_k}, l_{k+1} h + \tau_{l_{k+1}})$  ( $\forall k \in \mathbb{N}$ ). So the existing results on stability analysis of systems with one input delay in [29], [70] and [114] are not suitable for the system (3.11). Instead, we derive a new criterion for  $H_\infty$  tracking performance analysis. In the derivation of the criterion, we first decompose the interval  $[l_k h + \tau_{l_k}, l_{k+1} h + \tau_{l_{k+1}})$  into  $\Omega_i$  ( $i=1, 2, \dots, d_k+1, \forall k \in \mathbb{N}$ ) according to different values of  $\tau_1(t)$ ; second, we consider the stability of the system (3.9)-(3.10) on each  $\Omega_i$ ; third, we prove that the system (3.9)-(3.10) is exponentially stable on  $[t_0, t_\infty)$ , where  $[t_0, t_\infty) = \bigcup_{k=1}^{\infty} \bigcup_{i=1}^{d_k+1} \Omega_i$ ; finally, we show that the  $H_\infty$  tracking performance (3.12) is ensured if the LMIs (3.15)-(3.18) are satisfied.

Notice that the observer-based tracking control objective in this chapter is to search for the gains  $F$  and  $L$  such that the  $H_\infty$  tracking performance  $\gamma$  is minimized for the closed-loop system (3.9)-(3.10). For given  $\tau_{1m}, \tau_{2m}, \tau_{1M}, \tau_{2M}, U, L$  and  $F$ , one can employ Proposition 3.1 to determine the minimum  $\gamma$ , which can be obtained by solving the following optimization problem:

$$\begin{aligned} & \text{minimize} && \gamma \\ & \text{subject to} && P > 0, Q_i > 0, R_i > 0 \ (i=1, 2, 3, 4), \text{ and LMIs (3.15) -- (3.18)}. \end{aligned}$$

### 3.4 Observer-based tracking control design

In this section, we focus on the design problem of the observer-based controller (3.4) for the system (3.1)-(3.2). It is a common way to adopt a separation principle to design an observer-based tracking controller for traditional point-to-point systems ([72], [75], [81], [131]). However, a separation principle *can not be employed* to design an observer-based tracking controller for the system (3.1)-(3.2) because of the asynchronous inputs between the controlled plant (3.1) and the tracking controller (3.4). Notice that  $\bar{B}_1$  (coupling  $F$  and  $L$ ) and  $\bar{B}_2$  (including  $F$ ) in the system (3.9)-(3.10) are associated with two interval time-varying delays which are different due to network-induced delays and packet dropouts in the controller-to-actuator channel. So it is impossible to decouple  $F$  and  $L$  by using a separation principle. Instead, we present a new control design algorithm, which utilizes a particle swarm optimization (PSO) technique with the feasibility of Proposition 3.1, to obtain the minimum  $H_\infty$  tracking performance  $\gamma$  and the observer gain  $L$  and the control gain  $F$ . In the PSO technique, a particle status is characterized by two factors: its velocity and position, which are updated by the following equations ([18], [62])

$$\nu_{ij}(k+1) = \omega\nu_{ij}(k) + c_1r_1(\text{pbest}_{ij}(k) - f_{ij}(k)) + c_2r_2(\text{gbest}_j(k) - f_{ij}(k)) \quad (3.38)$$

$$f_{ij}(k+1) = f_{ij}(k) + \nu_{ij}(k+1) \quad (3.39)$$

$$\omega = (\omega_M - \omega_m)(m_\eta - c_\eta)/m_\eta + \omega_m \quad (3.40)$$

for  $i = 1, 2, \dots, n_p$  and  $j = 1, 2, \dots, d$ , where  $n_p$  and  $d$  are the number of particles in a group and the number of members in a particle, respectively;  $\nu_{ij}(k)$  is the  $j$ th dimensional velocity of the  $i$ th particle at iteration  $k$ , and  $\nu_j^{min} \leq \nu_{ij}(k) \leq \nu_j^{max}$ ;  $f_{ij}(k)$  is the  $j$ th dimensional position of the  $i$ th particle at iteration  $k$ ;  $\text{pbest}_i = (\text{pbest}_{i1}, \text{pbest}_{i2}, \dots, \text{pbest}_{ij})$  is the previous best position of the  $i$ th particle;  $\text{gbest}$  is the global best position of the group;  $r_1$  and  $r_2$  are two random numbers uniformly distributed in  $[0, 1]$ ;  $c_1$  and  $c_2$  are two acceleration coefficients;  $\omega$  is the inertia weight;  $\omega_M$  and  $\omega_m$  represent the maximum and minimum inertia weight, respectively;  $m_\eta$  is the maximum number of iterations and  $c_\eta$  is the current number of iterations. The design algorithm of the observer-based tracking controller (3.4) is described as follows

**Algorithm 3.1.**

Step 1. Initialization

- 1.1 Randomly initialize a group with  $n_p$  particles. Each particle consists of members  $f_{ij}(0)$  in  $F_i^0$  and  $L_i^0$ ; and  $f_{ij}(0)$  lies in the range  $[\alpha_j, \beta_j]$ , where  $i = 1, 2, \dots, n_p$ , and  $j = 1, 2, \dots, n(m + v)$ ;
- 1.2 Initialize some tuning parameters  $c_1, c_2, \omega_M, \omega_m, m_\eta, \nu_j^{min}$  and  $\nu_j^{max}$ , where  $j = 1, 2, \dots, n(m + v)$ ;
- 1.3 Initialize the velocity of  $n_p$  particles and  $\nu_j^{min} \leq \nu_{ij}(0) \leq \nu_j^{max}$ , where  $i = 1, 2, \dots, n_p$  and  $j = 1, 2, \dots, n(m + v)$ , and set  $k = 0$ ;
- 1.4 Initialize the fitness value  $\gamma_i^0 = l_p$ , where  $l_p$  is a positive constant and  $i = 1, 2, \dots, n_p$ . Solve the minimization problem mentioned at the end of Section 3 to obtain the minimum  $\gamma_i^0$  for given  $F_i^0$  and  $L_i^0, i \in \{1, 2, \dots, n_p\}$ ;

- 1.4.1 Assign the minimum  $\gamma_i^0$  to  $\gamma_{ip}$  and  $f_{ij}(0)$  to  $\text{pbest}_{ij}$ , respectively, where  $\gamma_{ip}$  is the fitness value of the particle  $\text{pbest}$ ,  $i = 1, 2, \dots, n_p$ , and  $j = 1, 2, \dots, n(m+v)$ , and set  $k = 0$ ;
- 1.4.2 Assign  $\min_i \{\gamma_i^0 \mid i \in \{1, 2, \dots, n_p\}\}$  to  $\gamma_g^0$  and  $f_{gj}(0)$  to  $\text{gbest}_j(0)$ , respectively, where  $\gamma_g^0$  is the fitness value of the particle  $\text{gbest}$ , and  $j = 1, 2, \dots, n(m+v)$  and set  $k = 0$ ;

Step 2. Fitness evaluation of particles

- 2.1 Obtain  $F_i^k$  and  $L_i^k$  from  $f_{ij}(k)$  in  $n_p$  particles, where  $i = 1, 2, \dots, n_p$ ,  $j = 1, 2, \dots, n(m+v)$ , and  $k \geq 1$ ,  $k \in \mathbb{N}$ ;
- 2.2 Solve the minimization problem mentioned at the end of Section 4.3 to obtain the minimum  $\gamma_i^k$  for given  $F_i^k$  and  $L_i^k$ , where  $i \in \{1, 2, \dots, n_p\}$ , and  $k \geq 1$ ,  $k \in \mathbb{N}$ ;
- 2.3 Record the previous best particles and their fitness values. If  $\gamma_i^k < \gamma_{ip}$ , then assign  $\gamma_i^k$  to  $\gamma_{ip}$  and  $f_{ij}(k)$  to  $\text{pbest}_{ij}$ , respectively, where  $i = 1, 2, \dots, n_p$ ,  $j = 1, 2, \dots, n(m+v)$ , and  $k \geq 1$ ,  $k \in \mathbb{N}$ ;
- 2.4 Record the global best particle and its fitness value. If  $\gamma_g^{k-1} > \min\{\gamma_i^k\}_{i=1}^{n_p}$ , then assign  $\min_i \{\gamma_i^k \mid i \in \{1, 2, \dots, n_p\}\}$  to  $\gamma_g^k$  and  $f_{gj}(k)$  to  $\text{gbest}_j(k)$ , respectively; otherwise, assign  $\gamma_g^{k-1}$  to  $\gamma_g^k$  and store the corresponding particle, where  $j = 1, 2, \dots, n(m+v)$ , and  $k \geq 1$ ,  $k \in \mathbb{N}$ . If  $|\gamma_g^k - \gamma_g^{k-1}| \leq \varepsilon$  is satisfied within  $m_\eta$  iterations, where  $\varepsilon > 0$  is a sufficiently small constant, then exit, and  $k \geq 1$ ,  $k \in \mathbb{N}$ ; otherwise, go to Step 2.5;
- 2.5 Update the velocity of  $n_p$  particles by (3.38) and (3.40). If  $\nu_{ij}(k) < \nu_j^{\min}$  or  $\nu_{ij}(k) > \nu_j^{\max}$ , then randomly generate  $\nu_{ij}(k)$  satisfying  $\nu_j^{\min} \leq \nu_{ij}(k) \leq \nu_j^{\max}$ , where  $i = 1, 2, \dots, n_p$ ,  $j = 1, 2, \dots, n(m+v)$ , and  $k \geq 1$ ,  $k \in \mathbb{N}$ ;

2.6 Update the position of  $n_p$  particles by (3.39). If  $f_{ij}(k) < \alpha_j$  or  $f_{ij}(k) > \beta_j$ , then randomly generate  $f_{ij}(k)$  satisfying  $\alpha_j \leq f_{ij}(k) \leq \beta_j$ , where  $i = 1, 2, \dots, n_p$ ,  $j = 1, 2, \dots, n(m + v)$ , and  $k \geq 1$ ,  $k \in \mathbb{N}$ ;

2.7 If  $k > m_\eta$ , where  $m_\eta$  is the maximum number of iterations, then exit; otherwise, set  $k = k + 1$  and go to Step 2.1;

Step 3. Obtain the minimum  $\gamma_g > 0$  and the corresponding  $F$  and  $L$  from the global best particle.

**Remark 3.3.** Applying a PSO technique with the feasibility of the LMIs of Proposition 3.1, Algorithm 3.1 is provided to search for the minimum  $H_\infty$  tracking performance  $\gamma_{min}$  and the observer gain  $L$  and the control gain  $F$ . The total row size of the LMIs of Proposition 3.1 is  $\mathcal{S} = 35p + 4q$  and the total number of scalar decision variables is  $\mathcal{N} = \frac{1}{2}[9p(p+1) + 8p(7p+q)]$ . Therefore, the computational complexity of Algorithm 3.1 is proportional to  $\mathcal{A} \mathcal{S} \mathcal{N}^3$ , where  $\mathcal{A} = (nm + nv)m_\eta n_p$ .

**Remark 3.4.** The convergence of Algorithm 3.1 is influenced by some tuning parameters  $n_p$ ,  $c_1$ ,  $c_2$ ,  $\omega_M$ ,  $\omega_m$ ,  $m_\eta$ ,  $\nu_j^{min}$  and  $\nu_j^{max}$  ( $j = 1, 2, \dots, n(m + v)$ ). Notice that a complete theoretical analysis of selecting parameters is given ([18], [130]). In this study, the tuning parameters are selected properly to ensure the convergence of Algorithm 3.1 according to [18] and [130]. The convergence speed of Algorithm 1 mainly depends on the search spaces  $[\alpha_j, \beta_j]$  ( $j = 1, 2, \dots, n(m + v)$ ). Usually, we can roughly determine  $\alpha_j$ ,  $\beta_j$  ( $j = 1, 2, \dots, n(m + v)$ ) in virtue of some traditional control strategies [72], [75], [81], [131].

**Remark 3.5.** The evolutionary process of Algorithm 3.1 will end when the search process converges in a given  $H_\infty$  tracking performance or will repeat for  $n_p$  particles until the maximum number of iterations  $m_\eta$  is reached. If the search process is ended by  $l_p$  and the conditions of Proposition 3.1 are not satisfied for given  $l_p$ ,  $F_g^k$

and  $L_g^k$  ( $k \in \mathbb{N}$ ), which means that no optimal solution is found, then one can adjust the search spaces or initialize the tuning parameters for another new search.

### 3.5 An example

Choose a mobile robot moving in one direction in [109] as a controlled plant, which is given by

$$\begin{cases} \dot{x}(t) = \begin{bmatrix} 0 & 1 \\ 0 & -11.32 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 11.32 \end{bmatrix} u(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \omega(t) \\ y(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} x(t) \end{cases} \quad (3.41)$$

where  $\omega(t)$  is an external disturbance.

The following reference model is described by

$$\begin{cases} \dot{x}_r(t) = \begin{bmatrix} 0 & 1 \\ -6 & -5 \end{bmatrix} x_r(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} r(t) \\ y_r(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} x_r(t) \end{cases} \quad (3.42)$$

where  $r(t) = 2.46\sin(t)$ .

We first design a traditional observer-based tracking controller of the form (3.3) by using the method in [81]. Using Theorem 2 with a parameter  $\alpha = 1$  and fuzzy rule  $r = 1$  in [81], we can easily obtain the minimum  $H_\infty$  tracking performance  $\gamma_{min} = 0.73$  and the corresponding gain matrices  $F = [-2.6456 \quad -0.2076]$  and  $L = [49.9480 \quad 345.8116]^T$ , denoted by  $GM_1$ .

Second, we design the network-based tracking controller (3.4) by using Algorithm 3.1. Following the modeling process in Section 4.2, the resulting system is represented by (3.9)-(3.10) with the following matrices

$$\begin{aligned} A &= \begin{bmatrix} 0 & 1 \\ 0 & -11.32 \end{bmatrix}, \quad A_r = \begin{bmatrix} 0 & 1 \\ -6 & -5 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 11.32 \end{bmatrix}, \\ D &= \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad B_r = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad C = C_r = [1 \quad 0]. \end{aligned}$$

In Algorithm 3.1, the search spaces are roughly set to be  $[\alpha_1, \beta_1] = [\alpha_2, \beta_2] = [-20, 0]$  and  $[\alpha_3, \beta_3] = [\alpha_4, \beta_4] = [0, 20]$ ; the tuning parameters are initialized to be  $n_p = 20$ ,  $c_1 = 2$ ,  $c_2 = 2$ ,  $\omega_M = 0.7$ ,  $\omega_m = 0.4$ ,  $m_\eta = 20$  and  $v_i^{max} = 2$  ( $i = 1, 2, 3, 4$ ), which

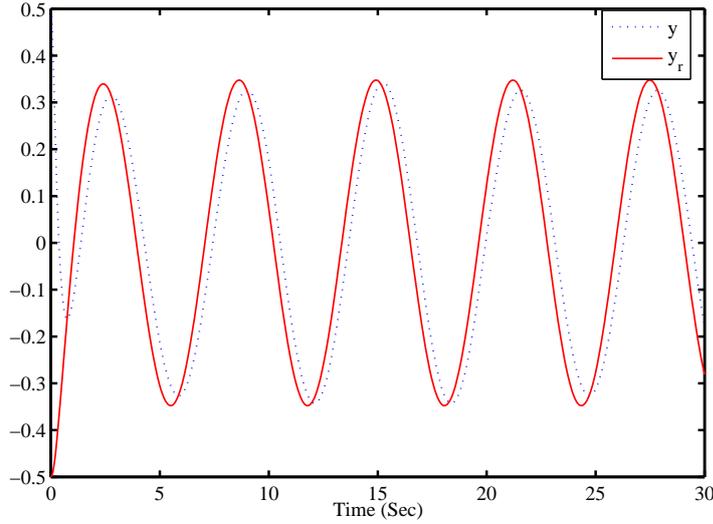


Figure 3.3: The outputs of the system (3.41) under a traditional controller [81] and the system (3.42) with  $r(t)$

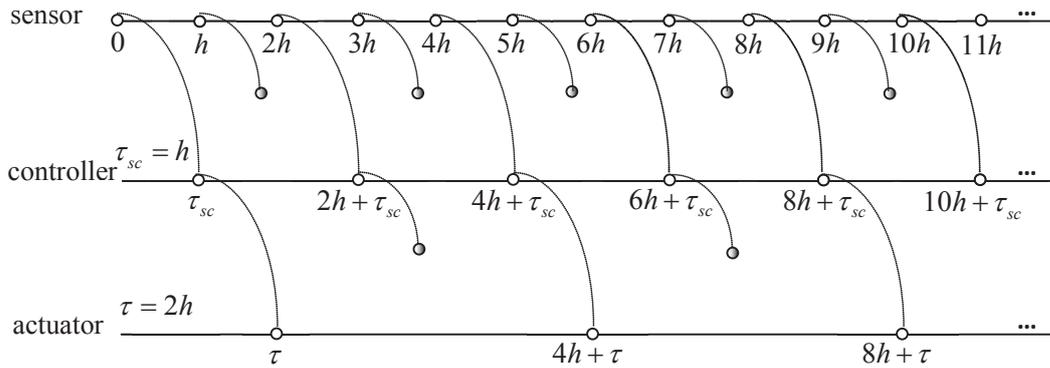


Figure 3.4: Time diagram of effective packets in simulation

satisfy the convergence of a PSO algorithm [18]; network index and a weighting matrix are given by  $\tau_{1m} = 20(ms)$ ,  $\tau_{2m} = 40(ms)$ ,  $\tau_{1M} = 60(ms)$ ,  $\tau_{2M} = 120(ms)$  and  $U = 1$ , respectively. Using Algorithm 3.1, we obtain the minimum  $H_\infty$  tracking performance  $\gamma_{min} = 0.1072$ , the gain matrices  $F = [-3.2465 \quad -0.2596]$  and  $L = [7.7941 \quad 0.8648]^T$ , denoted by  $GM_2$ .

Third, we compare the tracking effect via a traditional controller with  $GM_1$  and a network-based controller with  $GM_2$  by simulation of output responses. In simulation, we choose initial states  $x(0) = [0.2 \quad 0]$ ,  $\hat{x}(0) = [0 \quad 0]$ ,  $x_r(0) = [-0.5 \quad 0]$  and the external disturbance  $\omega(t) = 0.5\sin(t)$ . When there does not exist a network

between the plant (3.41) and the controller (3.3) with  $GM_1$ , the outputs  $y(t)$  and  $y_r(t)$  are shown in Figure 3.3, which demonstrates a stable tracking effect. Suppose that the plant (3.41) and the controller (3.3) are interconnected via a communication network. We choose a sampling period  $h = 20(ms)$  and assume that the sequences of packets successfully received by the controller and the actuator are  $\{b_i\}_{i=1}^{\infty}$  ( $b_i = 2i - 2$ ) and  $\{l_k\}_{k=1}^{\infty}$  ( $l_k = 4k - 4$ ), respectively, and network-induced delays of these packets are  $\tau_{b_i}^{sc} = 20(ms)$  and  $\tau_{l_k} = 40(ms)$ . Time diagram of effective packets with these assumptions is shown in Figure 3.4. It is easy to see that  $20(ms) \leq t - b_i \leq 60(ms)$  for  $t \in [2(i - 1)h + h, 2ih + h) \forall i \in \mathbb{N}$  and  $40(ms) \leq t - l_k \leq 120(ms)$  for  $t \in [4(k - 1)h + 2h, 4kh + 2h), \forall k \in \mathbb{N}$ . In this network setting, we depict the outputs  $y(t)$  and  $y_r(t)$  under the traditional controller with  $GM_1$  and the network-based controller with  $GM_2$  in Figure 3.5 and Figure 3.6, respectively.

Clearly, the traditional controller with  $GM_1$  can not ensure a stable tracking control in the network environment, however, the proposed network-based controller with  $GM_2$  achieves a stable and satisfactory tracking control. Moreover, in Figure 3.6, it is calculated that  $\|e(t)\|_2 = 0.6512$  and  $\bar{\omega}(t) = \|r(t)\|_2 + \|\omega(t)\|_2 = 9.5537$  for  $t \in [0, 30s]$ , which yields

$$\frac{\|e(t)\|_2}{\|\bar{\omega}(t)\|_2} = 0.0682 < \gamma_{min} = 0.1072. \quad (3.43)$$

Now, we introduce another reference input  $\tilde{r}(t)$  and another disturbance input  $\tilde{\omega}(t)$  as follows

$$\tilde{r}(t) = \begin{cases} 2, & 5s \leq t \leq 15s \\ -2, & 15s < t \leq 25s \\ 0, & \text{otherwise,} \end{cases} \quad \tilde{\omega}(t) = \begin{cases} 0.05 \sin(t), & 10s \leq t \leq 20s \\ 0, & \text{otherwise.} \end{cases}$$

Using the proposed controller, we show the outputs  $y(t)$  and  $y_r(t)$  in Figure 3.7. Similarly, we can calculate  $\|e(t)\|_2 = 0.5548$  and  $\bar{\omega}(t) = \|\tilde{r}(t)\|_2 + \|\tilde{\omega}(t)\|_2 = 8.9446$  for  $t \in [0, 30s]$ . It follows that

$$\frac{\|e(t)\|_2}{\|\bar{\omega}(t)\|_2} = 0.0620 < \gamma_{min} = 0.1072. \quad (3.44)$$

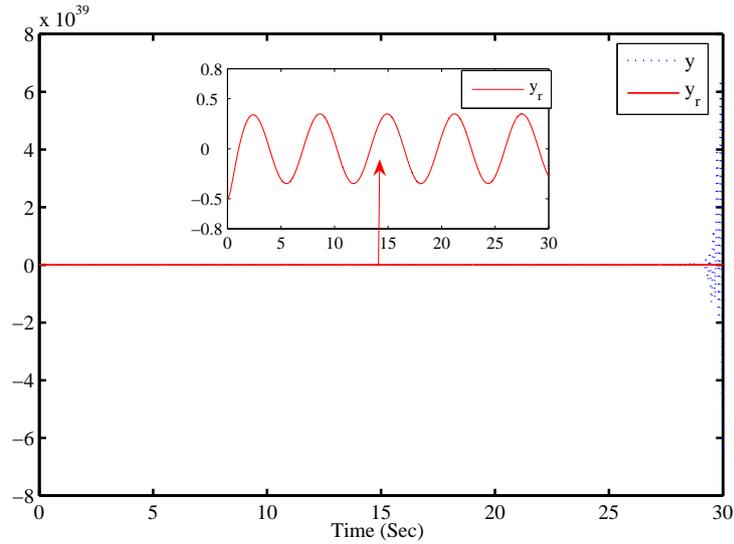


Figure 3.5: The outputs of the system (3.41) under a traditional controller [81] in a network environment and the system (3.42) with  $r(t)$

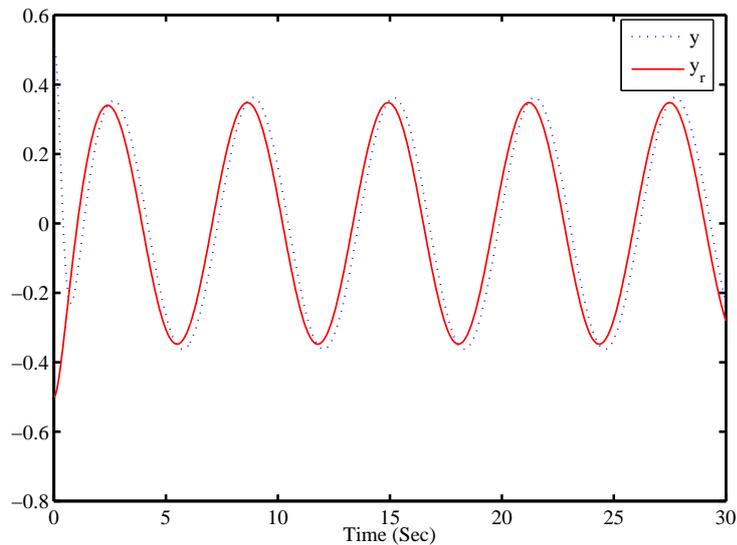


Figure 3.6: The outputs of the system (3.41) under the proposed controller and the system (3.42) with  $r(t)$

In Figure 3.8,  $e_1$  and  $e_2$  depict output tracking errors of the plant (3.41) under the obtained controller and the reference model (3.42) with two cases of reference inputs  $r(t)$  and  $\tilde{r}(t)$ , respectively, which show a satisfactory tracking control.

Lastly, we can conclude from Figure 3.3 and Figure 3.5 that a traditional controller (3.3) developed by the method in [81] can not be used to achieve a stable tracking control for the system (3.41)-(3.42) in a network environment. However, a

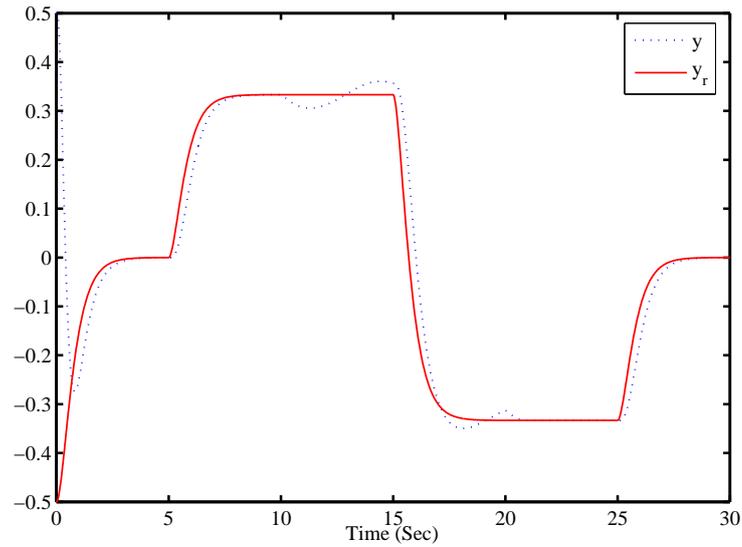


Figure 3.7: The outputs of the system (3.41) under the proposed controller and the system (3.42) with  $\tilde{r}(t)$

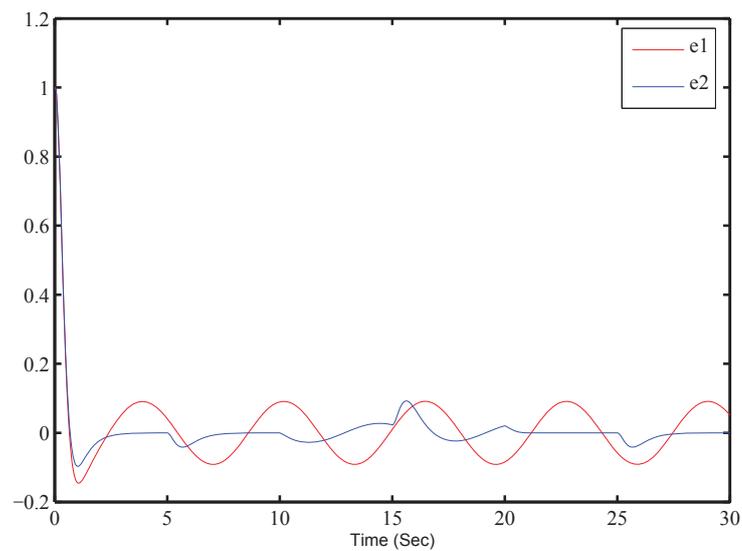


Figure 3.8: The output tracking errors between the system (3.41) under the proposed controller and the system (3.42) with different reference inputs

stable and satisfactory tracking control can be produced by a network-based controller (3.4) designed by Algorithm 3.1, which can be clearly seen from (3.43)-(3.44) and Figure 3.6, Figure 3.7 and Figure 3.8.

## 3.6 Summary

In this chapter, we have considered network-based output tracking control for a linear system via an observer-based controller. The inputs of the linear system and the observer-based tracking controller are asynchronously updated by available data that suffer network-induced delays and packet dropouts in the sensor-to-actuator channel and the sensor-to-controller channel, respectively. The resulting closed-loop system has been modeled as a system with two different interval time-varying delays. To ensure the closed-loop system with a prescribed  $H_\infty$  tracking performance, a new delay-dependent criterion has been derived in terms of linear matrix inequalities. Notice that a separation principle can not be employed to design the observer gain and the control gain due to the asynchronous inputs of the linear system and the controller. Instead, a new design method has been proposed by applying the particle swarm optimization technique with feasibility of the  $H_\infty$  performance criterion. An illustrative example has been given to show the effectiveness of the proposed method.

## Chapter 4

# Network-based fuzzy state feedback tracking control using asynchronous constraints

### 4.1 Introduction

Output tracking control for nonlinear systems has been applied in many fields of science and engineering, including industrial and military systems, biology, economy and other areas. Since many nonlinear systems can be represented by Takagi-Sugeno (T-S) fuzzy models [100], [124], much work has been done to deal with the output tracking control for a traditional T-S fuzzy system in the past decade, see [75], [81], [82] and [131]. In these references, the nonlinear system via a T-S fuzzy model and the fuzzy tracking controller are located together and connected in a point-to-point manner, which means that the T-S fuzzy model and the fuzzy controller share same IF-THEN rules and same continuous measurement of premise variables.

As a result of rapid advances in the network technology, modern feedback control systems become spatially distributed and dependent on a shared communication network ([36], [153]). The use of the network in the feedback control systems increases system flexibility, reduces cost of installation and maintenance, and enables a remote execution of control tasks. *Motivated by* the above mentioned advantages and the universal approximation property of a T-S fuzzy model, some researchers de-

vote themselves to investigating modeling, stability and control of a network-based T-S fuzzy system. The main feature of a network-based T-S fuzzy system is that the T-S fuzzy model and the fuzzy controller exchange feedback and control signals in the form of information packets through a digital network. The presence of a control network inevitably leads to network-induced delays and packet dropouts in the feedback control loop. Taking the effect of network-induced delays and packet dropouts into account, much attention has recently been paid to solving several control problems of a network-based T-S fuzzy system, for example, stabilization [57], guaranteed cost control [156], [98],  $H_\infty$  control [128], [157] and tracking control [54]. Notice that the network-based fuzzy controllers in [57], [98] and [156] depend on available sampled-data measurement of feedback states and *continuous measurement of premise variables*. However, these fuzzy controllers are not practical since their premise variables are *impossible* to be continuously measured in a network environment due to sampling behaviors and data transmission. *So the control design methods in [57], [98] and [156] must be reevaluated before they are applied in practice.* By using the network-based fuzzy controllers associated with available sampled-data measurement of both feedback states and premise variables, the network-based T-S fuzzy systems in [54], [128] and [157] are represented by asynchronous T-S fuzzy systems with an interval time-varying delay. It should be mentioned that in [128] and [157], some routine relaxation methods for a traditional T-S fuzzy system in [66], [80] and [122] are used to analyze  $H_\infty$  performance and design a fuzzy controller. But it is shown in [65] that *these relaxation methods do not work since the common product terms of membership functions in an asynchronous T-S fuzzy system can not be grouped.* Without using the relaxation methods in [66], [80] and [122], a fuzzy tracking control design method is proposed in [54]. However, As pointed out in [93], *the design method in [54] does not show any advantage over a linear tracking controller for the asynchronous fuzzy system.* In addition, the

knowledge of membership functions is not considered in the above control problems of a network-based T-S fuzzy system.

In this chapter, we will investigate network-based state feedback tracking control for T-S fuzzy systems by taking the knowledge of membership functions into consideration. Using a network-based fuzzy controller that depends on available sampled-data measurement of feedback states and premise variables, the network-based T-S fuzzy system will be formulated by an asynchronous T-S fuzzy system with an interval time-varying delay. Due to the asynchronous characteristic, a routine relaxation method for stability analysis and controller design of a traditional T-S fuzzy system can not be employed to analyze the  $H_\infty$  tracking performance and design the network-based fuzzy tracking controller of the asynchronous system. Instead, a new relaxation method will be proposed by applying asynchronous constraints on membership functions to introduce some free-weighting matrices. By using the proposed relaxation method and a discontinuous Lyapunov-Krasovskii functional, some new criteria for  $H_\infty$  tracking performance analysis and controller design will be derived in terms of linear matrix inequalities. It is worth pointing out that the fuzzy controller designed by using asynchronous constraints is essentially nonlinear and can ensure a better  $H_\infty$  tracking performance over a network-based linear controller provided by [54]. The effectiveness of the design method will be illustrated by performing network-based tracking control for the Duffing forced-oscillation system.

## 4.2 Modeling of network-based T-S fuzzy systems

Consider a smooth nonlinear control system on a compact region  $\mathbb{D}$ , which can be represented by the following T-S fuzzy model

*Plant Rule  $\mathcal{R}^i$* : IF  $\theta_1(t)$  is  $M_{i1}$  and  $\theta_2(t)$  is  $M_{i2}$  and  $\dots$  and  $\theta_g(t)$  is  $M_{ig}$ , THEN

$$\begin{cases} \dot{x}(t) = A_i x(t) + B_i u(t) + E_i \omega(t) \\ y(t) = C_i x(t) \end{cases} \quad (4.1)$$

where  $i = 1, 2, \dots, r$  and  $r$  denotes the number of IF-THEN rules;  $x(t) \in \mathbb{R}^n$  is

the state vector,  $u(t) \in \mathbb{R}^m$  is the control input vector,  $y(t) \in \mathbb{R}^l$  is the output vector, and  $\omega(t) \in \mathbb{R}^v$  is the external disturbance acting on the T-S fuzzy model and  $\omega(t) \in L_2[0, \infty)$ ;  $\theta(t) = [\theta_1(t), \theta_2(t), \dots, \theta_g(t)]$  is the premise variable;  $M_{ij}$  ( $i = 1, 2, \dots, r$ ;  $j = 1, 2, \dots, g$ ) are the fuzzy sets corresponding to premise variables  $\theta_i(t)$  ( $i = 1, 2, \dots, g$ ) and plant rules;  $A_i$ ,  $B_i$ ,  $C_i$  and  $E_i$  ( $i = 1, 2, \dots, r$ ) are constant matrices of appropriate dimensions.

By using a center average defuzzifier, a product fuzzy inference, and a singleton fuzzifier, the global dynamic of the fuzzy system (4.1) is inferred

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^r \mu_i(\theta(t)) [A_i x(t) + B_i u(t) + E_i \omega(t)] \\ y(t) = \sum_{i=1}^r \mu_i(\theta(t)) C_i x(t) \end{cases} \quad (4.2)$$

where

$$0 \leq \mu_i(\theta(t)) \leq 1, \quad i = 1, 2, \dots, r, \quad \sum_{i=1}^r \mu_i(\theta(t)) = 1.$$

In this chapter, we consider the network-based state feedback tracking control for the system (4.2) with membership functions under the following assumptions

**Assumption 4.1.** [53] *The time derivative of premise variable  $\theta(t)$  in the fuzzy membership functions is upper bounded, i.e.,  $|\dot{\theta}(t)| \leq \rho$ , where  $\rho > 0$  is a known positive constant.*

**Assumption 4.2.** *The fuzzy membership functions  $\mu_i(\theta(t))$  are Lipschitz continuous functions of premise variable  $\theta(t)$  and with known Lipschitz constants  $\epsilon_i$  on the compact region  $\mathbb{D}$ , where  $i = 1, 2, \dots, r$ .*

**Remark 4.1.** Notice that the common types of fuzzy membership functions are Gaussian, Triangular, S-shaped, Trapezoidal and Bell curves. So it is reasonable to assume that the membership functions ( $0 \leq \mu_i(\theta(t)) \leq 1$ ) are Lipschitz continuous functions of  $\theta(t)$  on the compact region  $\mathbb{D}$  and the Lipschitz constants  $\epsilon_i$  ( $i = 1, 2, \dots, r$ ) can be easily computed.

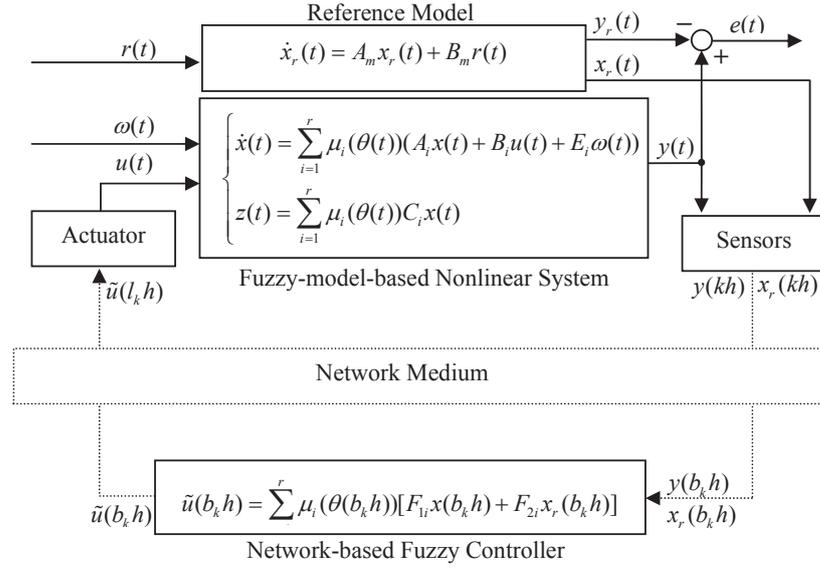


Figure 4.1: A setup of a network-based fuzzy tracking control system

Consider the following reference model

$$\begin{cases} \dot{x}_r(t) = A_m x_r(t) + B_m r(t) \\ y_r(t) = C_m x_r(t) \end{cases} \quad (4.3)$$

where  $x_r(t) \in \mathbb{R}^{\bar{n}}$ ,  $r(t) \in \mathbb{R}^{\bar{v}}$  and  $y_r(t) \in \mathbb{R}^{\bar{l}}$  are the state, the energy bounded input and the output, respectively;  $A_m$ ,  $B_m$  and  $C_m$  are given constant matrices of appropriate dimensions, and  $A_m$  is a Hurwitz matrix.

The objective of this chapter is to design a network-based fuzzy controller which can drive the output  $y(t)$  of the system (4.2) to follow the output  $y_r(t)$  of the reference model (4.3) as close as possible, see Figure 4.1. We assume that the fuzzy controller shares same fuzzy sets with the fuzzy model (4.1) in the premise parts. The sampled-data measurement of feedback states and premise variables ( $x(kh)$ ,  $x_r(kh)$  and  $\theta(kh)$ ,  $\forall k \in \mathbb{Z}$ , where  $h$  is the sampling period) is augmented as a single packet with a time stamp and transmitted to the controller in the sensor-to-controller channel. Due to the effects of network-induced delays and packet dropouts,  $x(b_k h)$ ,  $x_r(b_k h)$  and  $\theta(b_k h)$  ( $\forall k \in \mathbb{N}$ ) are available to update the fuzzy controller at the time instants  $\{b_k h + \tau_{b_k}^{sc}\}_{k=1}^{\infty}$ , where  $b_k$  ( $\forall k \in \mathbb{N}$ ) are some nonnegative integers which indicate the packets that successfully update the controller and  $\tau_{b_k}^{sc}$  ( $\forall k \in \mathbb{N}$ )

denote the corresponding sensor-to-controller delays. Then the network-based fuzzy controller can be described by

*Control Rule  $\mathcal{R}^i$* : IF  $\theta_1(b_k h)$  is  $M_{i1}$  and  $\theta_2(b_k h)$  is  $M_{i2}$  and  $\dots$  and  $\theta_g(b_k h)$  is  $M_{ig}$ ,  
THEN

$$u(t^+) = F_{1i}x(b_k h) + F_{2i}x_r(b_k h), \quad t \in \{b_k h + \tau_{b_k}^{sc}\}_{k=1}^{\infty} \quad (4.4)$$

where  $u(t^+) = \lim_{\delta \rightarrow t+0} u(\delta)$ ,  $\lim_{\delta \rightarrow t+0}$  is a limit taken from the left, and  $F_{1i}$  and  $F_{2i}$  ( $i = 1, 2, \dots, r$ ) are feedback gains to be determined. Analogous to (4.2), the network-based fuzzy controller is given by

$$\begin{cases} u(t^+) = \sum_{i=1}^r \mu_i(\theta(b_k h)) [F_{1i}x(b_k h) + F_{2i}x_r(b_k h)] \\ t \in \{b_k h + \tau_{b_k}^{sc}\}_{k=1}^{\infty}, \quad \forall k \in \mathbb{N}. \end{cases} \quad (4.5)$$

The control signal  $u(t^+)$  is transmitted to the actuator over a controller-to-actuator channel. The actuator has a computational hardware which can actively drop outdated packets by using time stamped information. In consequence,  $u(t^+)$  tagged with  $l_k$  ( $\forall k \in \mathbb{N}$ ) is available for the actuator at the time instant  $l_k h + \tau_{l_k}$ , where  $l_k$  ( $\forall k \in \mathbb{N}$ ) are some nonnegative integers which indicate the control signals that successfully update the actuator,  $\{l_1, l_2, l_3, \dots\} \subseteq \{b_1, b_2, b_3, \dots\}$ ,  $\{l_k\}$  is strictly increasing,  $\tau_{l_k} = \tau_{l_k}^{sc} + \tau_{l_k}^{ca}$  and  $\tau_{l_k}^{ca}$  denotes the controller-to-actuator delay. The actuator holds the available data until next update. Let  $\tau(t) = t - l_k h$  for  $t \in [l_k h + \tau_{l_k}, l_{k+1} h + \tau_{l_{k+1}})$  ( $\forall k \in \mathbb{N}$ ). One obtains the input of the system (4.1)

$$\begin{cases} u(t) = \sum_{i=1}^r \mu_i(\theta(l_k h)) [F_{1i}x(t - \tau(t)) + F_{2i}x_r(t - \tau(t))] \\ t \in [l_k h + \tau_{l_k}, l_{k+1} h + \tau_{l_{k+1}}), \quad \forall k \in \mathbb{N}. \end{cases} \quad (4.6)$$

Defining  $\tau_m = \min_{k \in \mathbb{N}} \{\tau_{l_k}\}$  and  $\tau_M = \max_{k \in \mathbb{N}} \{(l_{k+1} - l_k)h + \tau_{l_{k+1}}\}$ , we have

$$0 < \tau_m \leq \tau(t) \leq \tau_M, \quad t \in [l_k h + \tau_{l_k}, l_{k+1} h + \tau_{l_{k+1}}) \quad (4.7)$$

where  $\tau(t)$  is discontinuous at the time instants  $\{l_k h + \tau_{l_k}\}$  ( $\forall k \in \mathbb{N}$ ) and piecewise-linear with  $\dot{\tau}(t) = 1$  for  $t \neq \{l_k h + \tau_{l_k}\}_{k=1}^{\infty}$ .

Defining the output tracking error  $e(t) = y(t) - y_r(t)$  and using (4.2), (4.3) and (4.6), one can describe the augmented system by

$$\begin{cases} \dot{\xi}(t) = \sum_{i=1}^r \sum_{j=1}^r \mu_i \mu_j^k [\bar{A}_i \xi(t) + \bar{B}_i \bar{F}_j \xi(t - \tau(t)) + \bar{E}_i \bar{\omega}(t)] \\ e(t) = \sum_{i=1}^r \mu_i \bar{C}_i \xi(t), \quad t \in [l_k h + \tau_{l_k}, l_{k+1} h + \tau_{l_{k+1}}) \end{cases} \quad (4.8)$$

where

$$\begin{aligned} \xi(t) &= [x^T(t) \quad x_r^T(t)]^T, \quad \bar{\omega}(t) = [\omega^T(t) \quad r^T(t)]^T, \\ \bar{A}_i &= \begin{bmatrix} A_i & 0 \\ 0 & A_m \end{bmatrix}, \quad \bar{B}_i = \begin{bmatrix} B_i \\ 0 \end{bmatrix}, \quad \bar{E}_i = \begin{bmatrix} E_i & 0 \\ 0 & B_m \end{bmatrix}, \\ \bar{C}_i &= [C_i \quad -C_m], \quad \bar{F}_i = [F_{1i} \quad F_{2i}], \\ \mu_i &= \mu_i(\theta(t)) \geq 0, \quad \mu_i^k = \mu_i(\theta(l_k h)) \geq 0, \quad i = 1, 2, \dots, r, \\ \sum_{i=1}^r \mu_i &= \sum_{i=1}^r \mu_i^k = 1, \quad \forall k \in \mathbb{N}. \end{aligned}$$

We supplement the initial condition of the system (4.8) as  $\xi(t) = \phi(t)$ ,  $t \in [t_0 - \tau_M, t_0]$ , where  $\phi(t)$  is a continuous function on  $[t_0 - \tau_M, t_0]$ ,  $\phi(t_0) = [x_0^T \quad x_{r0}^T]^T$ , and  $x_0 = x(t_0)$  and  $x_{r0} = x_r(t_0)$  are initial states of the system (4.2) and the reference model (4.3), respectively. For the system (4.8), we choose the following  $H_\infty$  tracking performance

$$\int_{t_0}^{t_f} e^T(t) U e(t) dt \leq V(\xi_{t_0}) + \gamma^2 \int_{t_0}^{t_f} \bar{\omega}^T(t) \bar{\omega}(t) dt \quad (4.9)$$

where  $t_0$  is the initial time that the actuator starts to work,  $t_f$  is the terminal time,  $\gamma > 0$  is the prescribed  $H_\infty$  tracking performance,  $U > 0$  is the weighting matrix, and  $V(\xi_{t_0})$  is the energy function of initial states.

To achieve the output tracking control objective, the network-based fuzzy controller (4.5) is designed such that the augmented system (4.8) is asymptotically stable with a prescribed  $H_\infty$  tracking performance, which means that

- 1) the system (4.8) with  $\bar{\omega}(t) = 0$  is asymptotically stable;
- 2) the output tracking error  $e(t)$  satisfies the  $H_\infty$  tracking performance (4.9), for all nonzero  $\bar{\omega}(t) \in \mathcal{L}_2[t_0, \infty)$ .

**Remark 4.2.** The network-based fuzzy controller (4.5) depends on available measurement of premise variables and feedback states which are involved in the network transmission and subjected to network-induced delays and packet dropouts. Using such a fuzzy controller, the network-based T-S fuzzy system is represented by the asynchronous T-S fuzzy system (4.8). As pointed out in [65], the asynchronous membership functions of the system (4.8) make it difficult to analyze the  $H_\infty$  performance and design the fuzzy controller. Notice that there are some results available on fuzzy  $H_\infty$  control for a network-based nonlinear system which is modeled as an asynchronous T-S fuzzy system [54], [128] and [157]; however, the knowledge of asynchronous membership functions is not considered in the  $H_\infty$  performance analysis and controller design, which may lead to conservative results, see [93]. To utilize the asynchronous characteristic, we introduce the following asynchronous constraints on membership functions

$$|\mu_i - \mu_i^k| \leq \delta_i, \quad i = 1, 2, \dots, r \quad (4.10)$$

where  $\delta_i > 0$  ( $i = 1, 2, \dots, r$ ) are some positive constants. It follows from Assumption 4.1 and 4.2 that

$$\begin{aligned} |\theta(t) - \theta(l_k h)| &\leq \int_{t-\tau(t)}^t |\dot{\theta}(s)| ds \leq \rho \tau_M, \\ |\mu_i(\theta(t)) - \mu_i(\theta(l_k h))| &\leq \epsilon_i |\theta(t) - \theta(l_k h)|, \quad i = 1, 2, \dots, r. \end{aligned}$$

Then we have  $\delta_i = \min\{1, \epsilon_i \rho \tau_M\}$  ( $i = 1, 2, \dots, r$ ). It should be mentioned that if the T-S fuzzy system has available upper bounds of the time derivatives of membership functions [123], i.e., there exist  $\varphi_i > 0$  ( $i = 1, 2, \dots, r$ ) such that

$$|\dot{\mu}_i| = \left| \frac{\partial \mu_i(\theta(t))}{\partial \theta(t)} \dot{\theta}(t) \right| \leq \varphi_i, \quad i = 1, 2, \dots, r,$$

one can easily calculate that

$$|\mu_i - \mu_i^k| = \left| \int_{s-\tau(s)}^s \dot{\mu}_i dt \right| \leq \tau_M \varphi_i, \quad i = 1, 2, \dots, r.$$

Then we have  $\delta_i = \min\{1, \tau_M \varphi_i\}$  ( $i = 1, 2, \dots, r$ ).

### 4.3 Performance analysis of state feedback tracking control

In this section, we will analyze  $H_\infty$  tracking performance for the fuzzy system (4.8) by using the asynchronous constraint (4.10). Lemma 2.1 and Lemma 2.2 are needed to derive a delay-dependent criterion for  $H_\infty$  tracking performance analysis.

In the following, by finding a Lyapunov-Krasovskii functional (LKF)  $V(t)$  satisfying the condition (2.7)-(2.10), we will derive a new delay-dependent criterion. For simplicity of presentation, let

$$\begin{aligned}\eta^T(t) &= [\eta_1^T(t) \ \eta_2^T(t)], \\ \eta_1^T(t) &= [\xi^T(t) \ \dot{\xi}^T(t) \ \xi^T(t - \tau(t))], \\ \eta_2^T(t) &= [\xi^T(t - \bar{\tau}(t)) \ \xi^T(t - \tau_m) \ \xi^T(t - \tau_M)], \\ e_1 &= [I \quad 0 \quad 0 \quad 0 \quad 0 \quad 0]_{p \times 6p}, \\ e_2 &= [0 \quad I \quad 0 \quad 0 \quad 0 \quad 0]_{p \times 6p}, \\ e_3 &= [0 \quad 0 \quad I \quad 0 \quad 0 \quad 0]_{p \times 6p}, \\ e_4 &= [0 \quad 0 \quad 0 \quad I \quad 0 \quad 0]_{p \times 6p}, \\ e_5 &= [0 \quad 0 \quad 0 \quad 0 \quad I \quad 0]_{p \times 6p}, \\ e_6 &= [0 \quad 0 \quad 0 \quad 0 \quad 0 \quad I]_{p \times 6p}\end{aligned}$$

where  $e_i$  ( $i = 1, 2, \dots, 6$ ) are  $p \times 6p$  matrices;  $I$  denotes a  $p \times p$  identity matrix, the others in  $e_i$  ( $i = 1, 2, \dots, 6$ ) are  $p \times p$  zero matrices;  $p$  is the dimension of  $\xi(t)$  and  $p = n + \bar{n}$ . The delay-dependent criterion is given by

**Proposition 4.1.** *Given positive scalars  $\gamma$ ,  $\tau_m$ ,  $\tau_M$  and  $\delta_i$  ( $i = 1, 2, \dots, r$ ), gain matrices  $\bar{F}_i$  ( $i = 1, 2, \dots, r$ ) and a weighting matrix  $U > 0$ , the system (4.8) is asymptotically stable with an  $H_\infty$  tracking performance if there exist symmetric matrices  $P > 0$ ,  $Q_i > 0$  ( $i = 1, 2$ ),  $R_i > 0$  ( $i = 1, 2, 3, 4$ ) and matrices  $S_i$  ( $i = 1, 2, 3$ ),  $X_i$  ( $i = 1, 2$ ),  $M_i$ ,  $N_i$  ( $i = 1, 2, \dots, r$ ) and  $T_{ij}$  ( $i, j = 1, 2, \dots, 2r$ ) such that the following linear matrix*

inequalities (LMIs) hold for  $i, j = 1, 2, \dots, r$ :

$$\begin{bmatrix} \Xi_{ij}^1 & * & * & * \\ \Gamma_i & -\gamma^2 I & * & * \\ \delta S_1 & 0 & -\delta R_2 & * \\ \delta S_3 & 0 & 0 & -\delta R_3 \end{bmatrix} < 0 \quad (4.11)$$

$$\begin{bmatrix} \Xi_{ij}^2 & * & * \\ \Gamma_i & -\gamma^2 I & * \\ \delta S_2 & 0 & -\delta R_2 \end{bmatrix} < 0 \quad (4.12)$$

$$\begin{bmatrix} T_{11} & * & \cdots & * \\ T_{21} & T_{22} & \cdots & * \\ \vdots & \vdots & \ddots & \vdots \\ T_{2r,1} & T_{2r,2} & \cdots & T_{2r,2r} \end{bmatrix} < 0 \quad (4.13)$$

$$T_{ij} + T_{ji} - 2M_i \leq 0 \quad (4.14)$$

$$-2N_j - T_{(j+r)(i+r)} - T_{(i+r)(j+r)} \leq 0 \quad (4.15)$$

where

$$\Xi_{ij}^1 = \Omega_{ij}^0 + e_1^T \bar{C}_i^T U \bar{C}_i e_1 - \mathcal{E}_0 \Upsilon_{ij} \mathcal{E}_0^T,$$

$$\Xi_{ij}^2 = \Omega_{ij}^0 + \delta \Omega_{ij}^1 + e_1^T \bar{C}_i^T U \bar{C}_i e_1 - \mathcal{E}_0 \Upsilon_{ij} \mathcal{E}_0^T,$$

$$\begin{aligned} \Omega_{ij}^0 = & e_1^T (X_1^T \bar{A}_i + \bar{A}_i^T X_1 + Q_1 + Q_2 - R_1 - R_4) e_1 + e_1^T P e_2 \\ & + e_1^T (\bar{A}_i^T X_2 - X_1^T) e_2 + e_1^T X_1^T \bar{B}_i \bar{F}_j e_3 + e_1^T R_4 e_4 \\ & + e_1^T R_1 e_5 + e_2^T (P - X_1 + X_2^T \bar{A}_i) e_1 + e_3^T \bar{F}_j^T \bar{B}_i^T X_1 e_1 \\ & + e_4^T R_4 e_1 + e_5^T R_1 e_1 + e_2^T (\tau_m^2 R_1 + \delta R_2 - X_2 - X_2^T) e_2 \\ & + e_2^T X_2^T \bar{B}_i \bar{F}_j e_3 + e_3^T \bar{F}_j^T \bar{B}_i^T X_2 e_2 - e_4^T R_4 e_4 - e_5^T Q_1 e_5 \\ & - e_5^T R_1 e_5 + (e_5 - e_3)^T S_1 + S_1^T (e_5 - e_3) + S_2^T (e_3 - e_6) \\ & - e_6^T Q_2 e_6 + (e_3 - e_6)^T S_2 + S_3^T (e_1 - e_4) + (e_1 - e_4)^T S_3, \end{aligned}$$

$$\Omega_{ij}^1 = e_1^T R_4 e_2 + e_2^T R_4 e_1 + e_2^T R_3 e_2 - e_2^T R_4 e_4 - e_4^T R_4 e_2,$$

$$\begin{aligned} \Upsilon_{ij} = & \frac{T_{ij} + T_{ji} + T_{(i+r)(j+r)} + T_{(j+r)(i+r)}}{2} + T_{i(j+r)} + T_{(j+r)i} \\ & - \sum_{k=1}^r \delta_k \left[ M_i - \frac{T_{ik} + T_{ki}}{2} + N_j + \frac{T_{(j+r)(k+r)} + T_{(k+r)(j+r)}}{2} \right], \end{aligned}$$

$$\Gamma_i = \bar{E}_i^T X_1 e_1 + \bar{E}_i^T X_2 e_2, \quad \delta = \tau_M - \tau_m, \quad \mathcal{E}_0 = \begin{bmatrix} e_1^T & e_2^T & e_3^T \end{bmatrix}.$$

*Proof:* Choose the following LKF for the system (4.8)

$$\begin{aligned}
V(t) &= \int_{-\tau_m}^0 \xi^T(t+s)Q_1\xi(t+s)ds + \int_{-\tau_M}^0 \xi^T(t+s)Q_2\xi(t+s)ds \\
&+ \xi^T(t)P\xi(t) + \tau_m \int_{-\tau_m}^0 \int_s^0 \dot{\xi}^T(t+\theta)R_1\dot{\xi}(t+\theta)d\theta ds \\
&+ \int_{-\tau_M}^{-\tau_m} \int_s^0 \dot{\xi}^T(t+\theta)R_2\dot{\xi}(t+\theta)d\theta ds \\
&+ (\tau_M - \tau(t)) \int_{-\bar{\tau}(t)}^0 \dot{\xi}^T(t+\theta)R_3\dot{\xi}(t+\theta)d\theta \\
&+ (\tau_M - \tau(t))\chi^T(t)R_4\chi(t)
\end{aligned} \tag{4.16}$$

where  $P > 0$ ,  $Q_i > 0$  ( $i = 1, 2$ ) and  $R_i > 0$  ( $i = 1, 2, 3, 4$ ),  $\chi(t) = x(t) - x(t - \bar{\tau}(t))$ ,  $\bar{\tau}(t) = \tau(t) - \tau_{l_k}$  and  $\tau(t) = t - l_k h$  for  $t \in [l_k h + \tau_{l_k}, l_{k+1} h + \tau_{l_{k+1}})$ ,  $\forall k \in \mathbb{N}$ .

We first show that the LKF (4.16) satisfies the condition (2.7) for  $P > 0$ ,  $Q_i > 0$  ( $i = 1, 2$ ) and  $R_i > 0$  ( $i = 1, 2, 3, 4$ ). It is clear to see that  $V(t) \geq \lambda_{l_k} \|\xi(t)\|^2$  for  $t \in [l_k h + \tau_{l_k}, l_{k+1} h + \tau_{l_{k+1}})$ , where  $\lambda_{l_k} > 0$  ( $\forall k \in \mathbb{N}$ ). Notice that  $\bigcup_{k=0}^{\infty} [l_k h + \tau_{l_k}, l_{k+1} h + \tau_{l_{k+1}}) = [t_0, \infty)$ , we have  $V(t) \geq \varepsilon_1 \|\xi(t)\|^2$  for  $t \in [l_T h + \tau_{l_T}, \infty)$ , where  $\varepsilon_1 = \sum_{k=1}^{k=T} \{\lambda_{l_k}\} > 0$ . For  $P > 0$ ,  $Q_i > 0$  ( $i = 1, 2$ ) and  $R_i > 0$  ( $i = 1, 2, 3, 4$ ), we have

$$V(t) \leq \kappa_1 \max_{\theta \in [-\tau_M, 0]} \|\xi_t(\theta)\|^2 + \kappa_2 \max_{\theta \in [-\tau_M, 0]} \|\dot{\xi}_t(\theta)\|^2 \tag{4.17}$$

where

$$\begin{aligned}
\kappa_1 &= \lambda_{\max}(P) + \tau_m \lambda_{\max}(Q_1) + \tau_M \lambda_{\max}(Q_2), \\
\kappa_2 &= \tau_m^3 \lambda_{\max}(R_1) + \tau_M (\tau_M - \tau_m) \lambda_{\max}(R_2) \\
&+ (\tau_M - \tau_m)^3 [\lambda_{\max}(R_3) + \lambda_{\max}(R_4)].
\end{aligned}$$

Second, we show that the LKF (4.16) satisfies the condition (2.8) for some matrices  $P > 0$ ,  $Q_i > 0$  ( $i = 1, 2$ ) and  $R_i > 0$  ( $i = 1, 2, 3, 4$ ). In the LKF (4.16),  $R_3$ ,  $R_4$ -dependent terms are discontinuous at updating instants  $\{l_k h + \tau_{l_k}\}_{k=1}^{\infty}$  and do not increase along  $\{l_k h + \tau_{l_k}\}_{k=1}^{\infty}$  since they are non-negative before  $l_k h + \tau_{l_k}$  and become zero just after  $l_k h + \tau_{l_k}$ ,  $\forall k \in \mathbb{N}$ ; the other terms are continuous on  $[t_0, \infty)$ . Thus, we obtain the condition (2.8).

Third, we consider the asymptotic stability for the system (4.8) with  $\bar{\omega}(t) = 0$ . Taking the derivative of the LKF (4.16) along the trajectory of the system (4.8) with  $\bar{\omega}(t) = 0$ , we have

$$\begin{aligned}
 \dot{V}(t) = & 2 \sum_{i=1}^r \sum_{j=1}^r \mu_i \mu_j^k \eta^T(t) (e_1^T X_1^T + e_2^T X_2^T) (\bar{A}_i e_1 + \bar{B}_i \bar{F}_j e_3 - e_2) \eta(t) \\
 & + \eta^T(t) [e_1^T (Q_1 + Q_2 - R_1 - R_4) e_1 + e_1^T P e_2 + e_1^T R_4 e_4] \eta(t) \\
 & + \eta^T(t) [(\tau_M - \tau(t)) (e_1^T R_4 e_2 + e_2^T R_4 e_1) + e_1^T R_1 e_5 + e_2^T P e_1] \eta(t) \\
 & + \eta^T(t) [e_4^T R_4 e_1 + e_5^T R_1 e_1 + e_2^T (\tau_m^2 R_1 + \delta R_2) e_2] \eta(t) \\
 & - \eta^T(t) [e_4^T R_4 e_4 + e_5^T (R_1 + Q_1) e_5 + e_6^T Q_2 e_6 \eta(t)] \eta(t) \\
 & - \eta^T(t) [(\tau_M - \tau(t)) (e_2^T R_4 e_4 + e_4^T R_4 e_2 - e_2^T R_3 e_2)] \eta(t) \\
 & - \tau_m \int_{t-\tau_m}^t \dot{\xi}^T(s) R_1 \dot{\xi}(s) ds - \int_{t-\tau(t)}^{t-\tau_m} \dot{\xi}^T(s) R_2 \dot{\xi}(s) ds \\
 & - \int_{t-\tau_M}^{t-\tau(t)} \dot{\xi}^T(s) R_2 \dot{\xi}(s) ds - \int_{t-\bar{\tau}(t)}^t \dot{\xi}^T(s) R_3 \dot{\xi}(s) ds
 \end{aligned} \tag{4.18}$$

for  $t \in [l_k h + \tau_{l_k}, l_{k+1} h + \tau_{l_{k+1}})$ ,  $\forall k \in \mathbb{N}$ .

Using Jensen integral inequality, we obtain

$$-\tau_m \int_{t-\tau_m}^t \dot{\xi}^T(s) R_1 \dot{\xi}(s) ds \leq -\eta^T(t) (e_1 - e_5)^T R_1 (e_1 - e_5) \eta(t). \tag{4.19}$$

Applying Lemma 2.2 with  $\mathcal{E} = e_5 - e_3$ ,  $\psi = \eta(t)$  and  $Z = S_1$  in Chapter 2 to the term  $-\int_{t-\tau(t)}^{t-\tau_m} \dot{\xi}^T(s) R_2 \dot{\xi}(s) ds$  yields

$$\begin{aligned}
 -\int_{t-\tau(t)}^{t-\tau_m} \dot{\xi}^T(s) R_2 \dot{\xi}(s) ds & \leq \eta^T(t) [(e_5 - e_3)^T S_1 + S_1^T (e_5 - e_3)] \eta(t) \\
 & + (\tau(t) - \tau_m) \eta^T(t) S_1^T R_2^{-1} S_1 \eta(t).
 \end{aligned} \tag{4.20}$$

Similarly, the following inequalities hold

$$\begin{aligned}
 -\int_{t-\tau_M}^{t-\tau(t)} \dot{\xi}^T(s) R_2 \dot{\xi}(s) ds & \leq \eta^T(t) [(e_3 - e_6)^T S_2 + S_2^T (e_3 - e_6)] \eta(t) \\
 & + (\tau_M - \tau(t)) \eta^T(t) S_2^T R_2^{-1} S_2 \eta(t)
 \end{aligned} \tag{4.21}$$

$$\begin{aligned}
 -\int_{t-\bar{\tau}(t)}^t \dot{\xi}^T(s) R_3 \dot{\xi}(s) ds & \leq \eta^T(t) [(e_1 - e_4)^T S_3 + S_3^T (e_1 - e_4)] \eta(t) \\
 & + (\tau(t) - \tau_m) \eta^T(t) S_3^T R_3^{-1} S_3 \eta(t).
 \end{aligned} \tag{4.22}$$

Then it follows from (4.18)-(4.22) that

$$\dot{V}(t) \leq \sum_{i=1}^r \sum_{j=1}^r \mu_i \mu_j^k \eta^T(t) \Omega_{ij}^0(t) \eta(t) \quad (4.23)$$

for  $t \in [l_k h + \tau_{l_k}, l_{k+1} h + \tau_{l_{k+1}})$ ,  $\forall k \in \mathbb{N}$ , where

$$\begin{aligned} \Omega_{ij}^0(t) &= \Omega_{ij}^1(t) + \Omega_{ij}^2(t) + \Omega_{ij}^0, \\ \Omega_{ij}^1(t) &= (\tau_M - \tau(t))(S_2^T R_2^{-1} S_2 + \Omega_{ij}^1), \\ \Omega_{ij}^2(t) &= (\tau(t) - \tau_m)(S_1^T R_1^{-1} S_1 + S_3^T R_3^{-1} S_3). \end{aligned}$$

We are now in a position to prove that

$$\sum_{i=1}^r \sum_{j=1}^r \mu_i \mu_j^k \Omega_{ij}^0(t) < 0 \quad (4.24)$$

for  $t \in [l_k h + \tau_{l_k}, l_{k+1} h + \tau_{l_{k+1}})$ ,  $\forall k \in \mathbb{N}$ . Since  $\Omega_{ij}^0(t)$  is a convex combination of  $\Omega_{ij}^1(t)$  and  $\Omega_{ij}^2(t)$  on  $\tau(t) \in [\tau_m, \tau_M]$ , one can see that (4.24) holds if (4.13) and the following inequalities are satisfied

$$\begin{aligned} \sum_{i=1}^r \sum_{j=1}^r \mu_i \mu_j^k \Phi_{ij}^l &< - \sum_{i=1}^r \sum_{j=1}^r \mu_i^k \mu_j^k \mathcal{E}_0^T T_{(i+r)(j+r)} \mathcal{E}_0^T - \sum_{i=1}^r \sum_{j=1}^r \mu_i \mu_j \mathcal{E}_0^T T_{ij} \mathcal{E}_0^T \\ &+ \sum_{i=1}^r \sum_{j=1}^r \mu_i \mu_j^k [\Phi_{ij}^l - \mathcal{E}_0^T (T_{i(j+r)} + T_{(j+r)i}) \mathcal{E}_0^T] < 0 \end{aligned} \quad (4.25)$$

for  $l = 1, 2$  and  $t \in [l_k h + \tau_{l_k}, l_{k+1} h + \tau_{l_{k+1}})$ ,  $\forall k \in \mathbb{N}$ , where

$$\begin{aligned} \Phi_{ij}^1 &= \Omega_{ij}^0 + \Omega_{ij}^1(t)|_{\tau(t)=\tau_m}, \\ \Phi_{ij}^2 &= \Omega_{ij}^0 + \Omega_{ij}^2(t)|_{\tau(t)=\tau_M}. \end{aligned}$$

Notice that

$$- \sum_{i=1}^r \sum_{j=1}^r \mu_i \mu_j T_{ij} = - \sum_{i=1}^r \sum_{j=1}^r [\mu_i \mu_j^k + \mu_i (\mu_j - \mu_j^k)] \frac{T_{ij} + T_{ji}}{2} \quad (4.26)$$

$$- \sum_{i=1}^r \sum_{j=1}^r \mu_i^k \mu_j^k T_{(i+r)(j+r)} = - \sum_{i=1}^r \sum_{j=1}^r [\mu_i \mu_j^k - (\mu_i - \mu_i^k) \mu_j^k] \frac{T_{(i+r)(j+r)} + T_{(j+r)(i+r)}}{2}. \quad (4.27)$$

And some free-weighting matrices  $M_i$  and  $N_i$  ( $i=1, 2, \dots, r$ ) with appropriate dimensions are introduced by

$$\sum_{i=1}^r \sum_{j=1}^r \mu_i(\mu_j - \mu_j^k) M_i = 0 \quad (4.28)$$

$$\sum_{i=1}^r \sum_{j=1}^r (\mu_i - \mu_i^k) \mu_j^k N_j = 0. \quad (4.29)$$

Under Assumption 4.1 and Assumption 4.2, we obtain  $|\mu_i - \mu_i^k| \leq \delta_i$ . If the LMIs (4.11)-(4.13) are satisfied, it follows from (4.25)-(4.29) that

$$\sum_{i=1}^r \sum_{j=1}^r \mu_i \mu_j^k (\Phi_{ij}^l - \mathcal{E}_0 \Upsilon_{ij} \mathcal{E}_0^T) < 0 \quad (4.30)$$

for  $t \in [l_k h + \tau_{l_k}, l_{k+1} h + \tau_{l_{k+1}})$ ,  $\forall k \in \mathbb{N}$ . Using Schur complement to the LMIs (4.13)-(4.14), one can obtain (4.30), which implies that the inequality (4.24) holds. Then the system (4.8) is asymptotically stable if the LMIs (4.11)-(4.14) are satisfied.

Lastly, we consider the  $H_\infty$  tracking performance (4.9) for the system (4.8). Taking the derivative of the LKF (4.16) along (4.8), we have

$$\begin{aligned} \dot{V}(t) \leq & \sum_{i=1}^r \sum_{j=1}^r \mu_i \mu_j^k \begin{bmatrix} \eta(t) \\ \omega(t) \end{bmatrix}^T \begin{bmatrix} \tilde{\Phi}_{ij}^0(t) & * \\ \Gamma_i & -\gamma^2 I \end{bmatrix} \begin{bmatrix} \eta(t) \\ \omega(t) \end{bmatrix} \\ & - e^T(t) U e(t) + \gamma^2 \omega^T(t) \omega(t) \end{aligned} \quad (4.31)$$

for  $t \in [l_k h + \tau_{l_k}, l_{k+1} h + \tau_{l_{k+1}})$ ,  $\forall k \in \mathbb{N}$ , where

$$\tilde{\Phi}_{ij}^0(t) = \Phi_{ij}^0(t) + e_1^T \bar{C}_i^T U \bar{C}_i e_1.$$

Using the convex combination technique and Schur complement to the LMIs (4.11)-(4.14), we have

$$\begin{bmatrix} \tilde{\Phi}_{ij}^0(t) - \mathcal{E}_0 \Upsilon_{ij} \mathcal{E}_0^T & * \\ \Gamma_i & -\gamma^2 I \end{bmatrix} < 0. \quad (4.32)$$

From (4.31) and (4.32), one can see that  $\dot{V}(t) + e^T(t) U e(t) - \gamma^2 \bar{\omega}^T(t) \bar{\omega}(t) < 0$  holds for  $t \in [l_k h + \tau_{l_k}, l_{k+1} h + \tau_{l_{k+1}})$  ( $\forall k \in \mathbb{N}$ ). Then the  $H_\infty$  tracking performance (4.9) is ensured for the system (4.8), which completes the proof.

**Remark 4.3.** Notice that in the proof of Proposition 4.1, (1) the inherent piecewise-linear time-varying delay information  $\dot{\tau}(t) = 1$  on  $[l_k h + \tau_{l_k}, l_{k+1} h + \tau_{l_{k+1}})$  ( $\forall k \in \mathbb{N}$ ) is fully used; (2) the knowledge about premise variables and fuzzy membership functions ( $\rho$  and  $\epsilon_i$ ,  $i = 1, 2, \dots, r$ ) is considered; and (3) some free-weighting matrices  $M_i$ ,  $N_i$  ( $i = 1, 2, \dots, r$ ) and  $T_{ij}$  ( $i, j = 1, 2, \dots, 2r$ ) are introduced by using the asynchronous constraint (4.10) on membership functions. Thus, it is expected that Proposition 4.1 is of less conservatism.

**Remark 4.4.** For network-based T-S fuzzy systems in [128] and [156], some routine relaxation methods in [66], [80] and [122] are used to analyze the stability and design the fuzzy controller. However, this is not the case for the system (4.8) because all the relaxation methods in [66], [80] and [122] are based on the following partition

$$\xi^T(t) \left( \sum_{i=1}^r \mu_i^2 \Phi_{ii} + \sum_{i=1}^r \sum_{i < j \leq r} \mu_i \mu_j (\Phi_{ij} + \Phi_{ji}) \right) \xi(t), \quad (4.33)$$

and this partition does not hold for the system (4.8) with  $\mu_i \neq \mu_j^k$  for  $i = j$ . Instead, we use the asynchronous constraint (4.10) to introduce some free-weighting matrices in the  $H_\infty$  tracking performance analysis of the system (4.8). The introduction of these matrices in Proposition 4.1 can allow the existence of a network-based fuzzy controller with different control gains for different fuzzy control rules at the cost of computational complexity, see [93].

If the lower bound of network-induced delays is assumed to zero, i.e.,  $\tau_m = 0$ , one can obtain the following result.

**Proposition 4.2.** *Given positive scalars  $\gamma$ ,  $\tau_M$ , and  $\delta_i$  ( $i = 1, 2, \dots, r$ ), gain matrices  $\bar{F}_i$  ( $i = 1, 2, \dots, r$ ) and a weighting matrix  $U > 0$ , the system (4.8) is asymptotically stable with an  $H_\infty$  tracking performance if there exist symmetric matrices  $P > 0$ ,  $Q_2 > 0$ ,  $R_i > 0$  ( $i = 2, 3, 4$ ) and matrices  $S_i$  ( $i = 1, 2, 3$ ),  $X_i$  ( $i = 1, 2$ ),  $M_i$ ,  $N_i$  ( $i = 1, 2, \dots, r$ ) and  $T_{ij}$  ( $i, j = 1, 2, \dots, 2r$ ) such that (4.11)-(4.15) hold for  $i, j = 1, 2, \dots, r$ ,*

where  $\delta = \tau_M$  and

$$\begin{aligned}\Omega_{ij}^0 = & e_1^T (X_1^T \bar{A}_i + \bar{A}_i^T X_1 + Q_2 - R_4) e_1 + e_1^T (P - X_1^T + e_1^T \bar{A}_i^T X_2) e_2 \\ & + e_1^T X_1^T \bar{B}_i \bar{F}_j e_3 + e_3^T \bar{F}_j^T \bar{B}_i^T X_1 e_1 + e_1^T R_4 e_4 + e_4^T R_4 e_1 - e_4^T R_4 e_4 \\ & + e_2^T (\tau_M R_2 - X_2 - X_2^T) e_2 + e_2^T X_2^T \bar{B}_i \bar{F}_j e_3 + e_3^T \bar{F}_j^T \bar{B}_i^T X_2 e_2 \\ & - e_5^T Q_2 e_5 + (e_1 - e_3)^T S_1 + S_1^T (e_1 - e_3) + S_2^T (e_3 - e_5) \\ & + (e_3 - e_5)^T S_2 + S_3^T (e_1 - e_4) + (e_1 - e_4)^T S_3,\end{aligned}$$

$$e_1 = [I \quad 0 \quad 0 \quad 0 \quad 0]_{p \times 5p},$$

$$e_2 = [0 \quad I \quad 0 \quad 0 \quad 0]_{p \times 5p},$$

$$e_3 = [0 \quad 0 \quad I \quad 0 \quad 0]_{p \times 5p},$$

$$e_4 = [0 \quad 0 \quad 0 \quad I \quad 0]_{p \times 5p},$$

$$e_5 = [0 \quad 0 \quad 0 \quad 0 \quad I]_{p \times 5p}.$$

*Proof:* The proof is similar to that of Proposition 4.1 and omitted.

## 4.4 Fuzzy state feedback tracking control design

We now establish a delay-dependent criterion on the existence of a network-based  $H_\infty$  fuzzy tracking controller for the systems (4.2)-(4.3). The criterion is given by

**Proposition 4.3.** *Given some positive scalars  $\gamma$ ,  $\tau_m$ ,  $\tau_M$  and  $\delta_i$  ( $i = 1, 2, \dots, r$ ), a tuning parameter  $\sigma$ , and a weighting matrix  $U > 0$ , the system (4.8) is asymptotically stable with the  $H_\infty$  tracking performance  $\gamma$  if there exist symmetric matrices  $\bar{P} > 0$ ,  $\bar{Q}_i > 0$  ( $i = 1, 2$ ),  $\bar{R}_i > 0$  ( $i = 1, 2, 3, 4$ ), and some matrices  $\bar{X}$ ,  $\bar{S}_i$  ( $i = 1, 2, 3$ ),  $Y_i$ ,  $\bar{M}_i$ ,  $\bar{N}_i$  ( $i = 1, 2, \dots, r$ ),  $\bar{T}_{ij}$  ( $i, j = 1, 2, \dots, 2r$ ) such that the following inequalities hold for  $i, j = 1, 2, \dots, r$ :*

$$\begin{bmatrix} \bar{\Xi}_{ij}^1 & * & * & * & * \\ \bar{\Gamma}_i & -\gamma^2 I & * & * & * \\ \delta \bar{S}_1 & 0 & -\delta \bar{R}_2 & * & * \\ \delta \bar{S}_3 & 0 & 0 & -\delta \bar{R}_3 & * \\ \bar{C}_i \bar{X} e_1 & 0 & 0 & 0 & -U^{-1} \end{bmatrix} < 0 \quad (4.34)$$

$$\begin{bmatrix} \bar{\Xi}_{ij}^2 & * & * & * \\ \bar{\Gamma}_i & -\gamma^2 I & * & * \\ \delta \bar{S}_2 & 0 & -\delta \bar{R}_2 & * \\ \bar{C}_i \bar{X} e_1 & 0 & 0 & -U^{-1} \end{bmatrix} < 0 \quad (4.35)$$

$$\begin{bmatrix} \bar{T}_{11} & * & \cdots & * \\ \bar{T}_{21} & \bar{T}_{22} & \cdots & * \\ \vdots & \vdots & \ddots & \vdots \\ \bar{T}_{2r,1} & \bar{T}_{2r,2} & \cdots & \bar{T}_{2r,2r} \end{bmatrix} < 0 \quad (4.36)$$

$$\bar{T}_{ij} + \bar{T}_{ji} - 2\bar{M}_i \leq 0 \quad (4.37)$$

$$-2\bar{N}_j - \bar{T}_{(j+r)(i+r)} - \bar{T}_{(i+r)(j+r)} \leq 0 \quad (4.38)$$

where  $e_i$  ( $i = 1, 2, \dots, 6$ ) are given in Proposition 4.1 and

$$\bar{\Xi}_{ij}^1 = \bar{\Omega}_{ij}^0 - \mathcal{E}_0 \bar{\Upsilon}_{ij} \mathcal{E}_0^T,$$

$$\bar{\Xi}_{ij}^2 = \bar{\Omega}_{ij}^0 + \delta \bar{\Omega}_{ij}^1 - \mathcal{E}_0 \bar{\Upsilon}_{ij} \mathcal{E}_0^T,$$

$$\begin{aligned} \bar{\Omega}_{ij}^0 = & e_1^T (\bar{A}_i \bar{X} + \bar{X}^T \bar{A}_i^T + \bar{Q}_1 + \bar{Q}_2 - \bar{R}_1 - \bar{R}_4) e_1 + e_1^T (\bar{P} - \bar{X} + \sigma \bar{X}^T \bar{A}_i^T) e_2 \\ & + e_2^T (\bar{P} - \bar{X}^T + \sigma \bar{A}_i \bar{X}) e_1 + e_1^T \bar{B}_i Y_j e_3 + e_3^T Y_j^T \bar{B}_i^T e_1 + e_1^T \bar{R}_4 e_4 + e_4^T \bar{R}_4 e_1 \\ & + e_1^T \bar{R}_1 e_5 + e_5^T \bar{R}_1 e_1 + e_2^T (\tau_m^2 \bar{R}_1 - \sigma \bar{X} - \sigma \bar{X}^T) e_2 + e_2^T \delta \bar{R}_2 e_2 + e_2^T \sigma \bar{B}_i Y_j e_3 \\ & + e_3^T \sigma Y_j^T \bar{B}_i^T e_2 - e_4^T \bar{R}_4 e_4 - e_5^T (\bar{Q}_1 + \bar{R}_1) e_5 - e_6^T \bar{Q}_2 e_6 + (e_5 - e_3)^T \bar{S}_1 \\ & + \bar{S}_1^T (e_5 - e_3) + \bar{S}_2^T (e_3 - e_6) + (e_3 - e_6)^T \bar{S}_2 + \bar{S}_3^T (e_1 - e_4) + (e_1 - e_4)^T \bar{S}_3, \end{aligned}$$

$$\bar{\Omega}_{ij}^1 = e_1^T \bar{R}_4 e_2 + e_2^T \bar{R}_4 e_1 + e_2^T \bar{R}_3 e_2 - e_2^T \bar{R}_4 e_4 - e_4^T \bar{R}_4 e_2, \quad \bar{\Gamma}_i = \bar{E}_i^T e_1 + \sigma \bar{E}_i^T e_2,$$

$$\begin{aligned} \bar{\Upsilon}_{ij} = & (\bar{T}_{ij} + \bar{T}_{ji} + \bar{T}_{(i+r)(j+r)} + \bar{T}_{(j+r)(i+r)})/2 + \bar{T}_{i(j+r)} + \bar{T}_{(j+r)i} \\ & - \sum_{k=1}^r \delta_k [\bar{M}_i - (\bar{T}_{ik} + \bar{T}_{ki})/2 + \bar{N}_j + (\bar{T}_{(j+r)(k+r)} + \bar{T}_{(k+r)(j+r)})/2]. \end{aligned}$$

Moreover, the control gains  $\bar{F}_i$  are given by  $\bar{F}_i = Y_i \bar{X}^{-1}$  ( $i = 1, 2, \dots, r$ ).

*Proof:* From Proposition 4.1, we can see that the system (4.8) is asymptotically stable with an  $H_\infty$  tracking performance if the LMIs (4.11)-(4.15) are satisfied. Pre- and post-multiplying both sides of the inequality (4.11) with  $\text{diag}\{\Delta, \Delta, I, \bar{X}, \bar{X}\}^T$  and its transpose, the inequality (4.12) with  $\text{diag}\{\Delta, \Delta, I, \bar{X}\}^T$  and its transpose, the inequality (4.13) with  $\text{diag}\{\Delta, \Delta, \dots, \Delta\}_{6pr \times 6pr}^T$  and its transpose, the inequalities (4.14) and (4.15) with  $\Delta^T$  and its transpose, respectively, and introducing

$\Delta = \text{diag}\{\bar{X}, \bar{X}, \bar{X}\}$ ,  $\bar{X} = X_1^{-1} = \sigma X_2^{-1}$ ,  $\bar{P} = \bar{X}^T P \bar{X}$ ,  $\bar{Q}_i = \bar{X}^T Q_i \bar{X}$  ( $i = 1, 2$ ),  $\bar{R}_i = \bar{X}^T R_i \bar{X}$  ( $i = 1, 2, 3, 4$ ),  $\bar{S}_i^T = \text{diag}\{\Delta, \Delta\}^T S_i^T \bar{X}$  ( $i = 1, 2, 3$ ),  $\bar{T}_{ij} = \Delta^T T_{ij} \Delta$  ( $i, j = 1, 2, \dots, 2r$ ),  $Y_i = F_i \bar{X}$ ,  $\bar{M}_i = \Delta^T M_i \Delta$  and  $\bar{N}_i = \Delta^T N_i \Delta$  ( $i = 1, 2, \dots, r$ ), we can obtain the LMIs (4.34)-(4.38) by using Schur complement.

If information about asynchronous membership functions is not considered in the tracking control design, one has the following corollary.

**Corollary 4.1.** *Given some positive scalars  $\gamma$ ,  $\tau_m$  and  $\tau_M$ , a tuning parameter  $\sigma$ , and a weighting matrix  $U > 0$ , the system (4.8) is asymptotically stable with the  $H_\infty$  tracking performance  $\gamma$  if there exist symmetric matrices  $\bar{P} > 0$ ,  $\bar{Q}_i > 0$  ( $i = 1, 2$ ),  $\bar{R}_i > 0$  ( $i = 1, 2, 3, 4$ ), and some matrices  $\bar{X}$ ,  $\bar{S}_i$  ( $i = 1, 2, 3$ ),  $Y_i$ ,  $\bar{M}_i$ ,  $\bar{N}_i$  ( $i = 1, 2, \dots, r$ ),  $\bar{T}_{ij}$  ( $i, j = 1, 2, \dots, 2r$ ) such that*

$$\Pi_{ij}^0 < 0, \quad i, j = 1, 2, \dots, r \quad (4.39)$$

$$\Pi_{ij}^1 < 0, \quad i, j = 1, 2, \dots, r \quad (4.40)$$

where

$$\Pi_{ij}^0 = \begin{bmatrix} \bar{\Omega}_{ij}^0 & * & * & * & * \\ \bar{\Gamma}_i & -\gamma^2 I & * & * & * \\ \delta \bar{S}_1 & 0 & -\delta \bar{R}_2 & * & * \\ \delta \bar{S}_3 & 0 & 0 & -\delta \bar{R}_3 & * \\ \bar{C}_i \bar{X} e_1 & 0 & 0 & 0 & -U^{-1} \end{bmatrix},$$

$$\Pi_{ij}^1 = \begin{bmatrix} \bar{\Omega}_{ij}^0 + \delta \bar{\Omega}_{ij}^1 & * & * & * \\ \bar{\Gamma}_i & -\gamma^2 I & * & * \\ \delta \bar{S}_2 & 0 & -\delta \bar{R}_2 & * \\ \bar{C}_i \bar{X} e_1 & 0 & 0 & -U^{-1} \end{bmatrix},$$

and  $\bar{\Omega}_{ij}^0$ ,  $\bar{\Omega}_{ij}^1$  and  $\bar{\Gamma}_i$  ( $i, j = 1, 2, \dots, r$ ) are defined in Proposition 4.3.

The following proposition establishes the relationship between Proposition 4.3 and Corollary 4.1.

**Proposition 4.4.** *If the conditions of Corollary 4.1 are satisfied, then the conditions of Proposition 4.3 are satisfied.*

*Proof:* From (4.39) and (4.40), it can be seen that there exist some small scalars  $\varepsilon_i > 0$  ( $i = 1, 2, \dots, r$ ) such that  $\Pi_{ij}^0 < -(\varepsilon_i + \varepsilon_j)I_{q \times q}$  and  $\Pi_{ij}^1 < -(\varepsilon_i + \varepsilon_j)I_{q \times q}$  for  $i, j = 1, 2, \dots, r$ , where  $q = 8p + v + \bar{v} + l$ . On the other hand, choosing  $M_i = -\varepsilon_i I_{3p \times 3p}$ ,  $N_j = -\varepsilon_j I_{3p \times 3p}$ ,  $\bar{T}_{ij} = 0$  for  $i \neq j$ ,  $\bar{T}_{ii} = -\varepsilon_i I_{3p \times 3p}$ ,  $\bar{T}_{(j+r)(i+r)} = 0$  for  $i \neq j$ ,  $\bar{T}_{(j+r)(j+r)} = -\varepsilon_j I_{3p \times 3p}$ ,  $\bar{T}_{i(j+r)} = \bar{T}_{(j+r)i} = 0$ , where  $i, j = 1, 2, \dots, r$ , one can see that the LMIs (4.34)-(4.38) are satisfied. This completes the proof.

**Remark 4.5.** Proposition 4.3 can be effectively used to design the network-based fuzzy tracking controller such that the system (4.8) with available  $\delta_i$  ( $i = 1, 2, \dots, r$ ) is asymptotically stable with a prescribed  $H_\infty$  tracking performance. If  $\delta_i$  are unknown or  $\delta_i > 1$ , one can choose Corollary 4.1 for tracking controller design. However, in this case, only a network-based linear controller can be developed and the control design method is conservative, see [93].

In the case  $\tau_m = 0$ , we have the following proposition

**Proposition 4.5.** *Given positive scalars  $\gamma$ ,  $\tau_M$  and  $\delta_i$  ( $i = 1, 2, \dots, r$ ), a tuning parameter  $\sigma$ , and a weighting matrix  $U > 0$ , the system (4.8) is asymptotically stable with the  $H_\infty$  tracking performance  $\gamma$  if there exist symmetric matrices  $\bar{P} > 0$ ,  $\bar{Q}_2 > 0$ ,  $\bar{R}_i > 0$  ( $i = 2, 3, 4$ ), and matrices  $\bar{X}$ ,  $\bar{S}_i$  ( $i = 1, 2, 3$ ),  $Y_i$ ,  $\bar{M}_i$ ,  $\bar{N}_i$  ( $i = 1, 2, \dots, r$ ),  $\bar{T}_{ij}$  ( $i, j = 1, 2, \dots, 2r$ ) such that (4.34)-(4.38) hold for  $i, j = 1, 2, \dots, r$ , where  $\delta = \tau_M$ ,  $e_i$  ( $i = 1, 2, \dots, 5$ ) are given in Proposition 4.2 and*

$$\begin{aligned} \bar{\Omega}_{ij}^0 = & e_1^T (\bar{A}_i \bar{X} + \bar{X}^T \bar{A}_i^T + \bar{Q}_2 - \bar{R}_4) e_1 + e_1^T (\sigma \bar{X}^T \bar{A}_i^T - \bar{X}) e_2 \\ & + e_1^T \bar{P} e_2 + e_1^T \bar{B}_i Y_j e_3 + e_1^T \bar{R}_4 e_4 + e_2^T (\sigma \bar{A}_i \bar{X} - \bar{X}^T) e_1 \\ & + e_2^T \bar{P} e_1 + e_3^T Y_j^T \bar{B}_i^T e_1 + e_4^T \bar{R}_4 e_1 - e_2^T (\sigma \bar{X} + \sigma \bar{X}^T) e_2 \\ & + e_2^T \tau_M \bar{R}_2 e_2 + e_2^T \sigma \bar{B}_i Y_j e_3 + e_3^T \sigma Y_j^T \bar{B}_i^T e_2 - e_4^T \bar{R}_4 e_4 \\ & - e_5^T \bar{Q}_2 e_5 + (e_1 - e_3)^T \bar{S}_1 + \bar{S}_1^T (e_1 - e_3) + \bar{S}_2^T (e_3 - e_5) \\ & + (e_3 - e_5)^T \bar{S}_2 + \bar{S}_3^T (e_1 - e_4) + (e_1 - e_4)^T \bar{S}_3. \end{aligned}$$

Moreover, the control gains  $\bar{F}_i$  are given by  $\bar{F}_i = Y_i \bar{X}^{-1}$  ( $i = 1, 2, \dots, r$ ).

**Remark 4.6.** In [54],  $x(t)$  and  $x_r(t)$  are assumed to be with same dimensions. Defining  $e(t) = x(t) - x_r(t)$ , the network-based control system is modeled as

$$\begin{cases} \dot{e}(t) = \sum_{i=1}^r \sum_{j=1}^r \mu_i \mu_j^k [A_i e(t) + B_i F_j e(l_k h) + \omega_e(t)] \\ t \in [l_k h + \tau_{l_k}, l_{k+1} h + \tau_{l_{k+1}}), \forall k \in \mathbb{N} \end{cases} \quad (4.41)$$

where

$$\omega_e(t) = \sum_{i=1}^r \mu_i [(A_i - A_m)x_r(t) + E_i \omega(t) - B_m r(t)].$$

It should be mentioned that Proposition 4.3 and 4.5 with alterations  $\bar{A}_i = A_i$ ,  $\bar{B}_i = B_i$ ,  $\bar{F}_i = F_i$ ,  $\bar{C}_i = I$  and  $\bar{E}_i = I$  ( $i = 1, 2, \dots, r$ ) can be used to design a network-based fuzzy controller such that the tracking error system (4.41) with available  $\delta_i$  ( $i = 1, 2, \dots, r$ ) is asymptotically state with a prescribed  $H_\infty$  tracking performance. An example will be given to show the advantage of the proposed method over the result in [54].

## 4.5 An example

In this section, we will illustrate the effectiveness of the proposed design method by performing the network-based tracking control for the Duffing forced-oscillation system, which is described by

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^2 \mu_i(x_1(t)) [A_i x(t) + B_i u(t) + E_i \omega(t)] \\ y(t) = \sum_{i=1}^2 \mu_i(x_1(t)) C_i x(t) \end{cases} \quad (4.42)$$

where

$$A_1 = \begin{bmatrix} 0 & 1 \\ 0 & -0.1 \end{bmatrix}, A_2 = \begin{bmatrix} 0 & 1 \\ -25 & -0.1 \end{bmatrix}, B_i = E_i = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, C_i = [1 \ 0], i = 1, 2,$$

$$x(t) = [x_1^T(t) \ x_2^T(t)]^T, \mu_1(x_1(t)) = 1 - x_1^2(t)/25, \mu_2(x_1(t)) = x_1^2(t)/25.$$

In the following, we consider the tracking control for two cases of network-based T-S fuzzy systems, i.e., the system described by (4.8) and the system described by

(4.41). In the first case, we perform the output tracking control through a network-based fuzzy controller designed by Proposition 4.3. In the second case, we show the advantage of Proposition 4.5 over Theorem 1 in [54].

Case I: choose the following reference model

$$\begin{cases} \dot{x}_r = -x_r(t) + r(t) \\ y_r(t) = x_r(t). \end{cases} \quad (4.43)$$

Suppose that a communication network is inserted between the system (4.42) and the following fuzzy tracking controller

$$u(t) = \sum_{i=1}^2 \mu_i(x_1(t)) [F_{1i}x(t) + F_{2i}x_r(t)]. \quad (4.44)$$

Using the modeling process in Section II, we have the augmented system (4.8) with the following matrices

$$\begin{aligned} \bar{A}_1 &= \begin{bmatrix} 0 & 1 & 0 \\ 0 & -0.1 & 0 \\ 0 & 0 & -1 \end{bmatrix}, \quad \bar{B}_1 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \quad \bar{E}_1 = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}, \\ \bar{A}_2 &= \begin{bmatrix} 0 & 1 & 0 \\ -25 & -0.1 & 0 \\ 0 & 0 & -1 \end{bmatrix}, \quad \bar{B}_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \quad \bar{E}_2 = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}, \\ \bar{C}_1 &= [1 \ 0 \ -1], \quad \bar{C}_2 = [1 \ 0 \ -1]. \end{aligned}$$

It is assumed that the compact region  $\mathbb{D} = [-5, 5] \times [-4, 4]$ . For  $t \in [l_k h + \tau_{l_k}, l_{k+1} h + \tau_{l_{k+1}})$  ( $\forall k \in \mathbb{N}$ ), we have

$$\begin{aligned} |x_1(t) - x_1(l_k h)| &\leq \int_{t-\tau(t)}^t |\dot{x}_1(s)| ds \leq 4\tau_M, \\ |\mu_i - \mu_i^k| &\leq |x_1(t) + x_1(l_k h)| |x_1(t) - x_1(l_k h)| / 25 \leq 1.6\tau_M, \quad i = 1, 2. \end{aligned}$$

Then  $\delta_i = \min\{1, 1.6\tau_M\}$  ( $i = 1, 2$ ). Given  $\tau_m = 30(ms)$ ,  $\tau_M = 90(ms)$  and  $U = I_{2 \times 2}$ . Applying Proposition 4.3 with  $\sigma = 0.2$  and  $\delta_i = 0.144$  ( $i = 1, 2$ ), one can obtain the minimum  $H_\infty$  tracking performance  $\gamma_{min} = 0.38$  and the corresponding fuzzy gains  $F_{11} = [-33.7161 \ -8.7109]$ ,  $F_{21} = 27.5881$ ,  $F_{12} = [-9.8056 \ -8.6784]$ ,  $F_{22} = 27.6524$ . When  $\delta_i$  ( $i = 1, 2$ ) are unavailable, one can use Corollary 4.1 with  $\sigma = 0.2$  to obtain

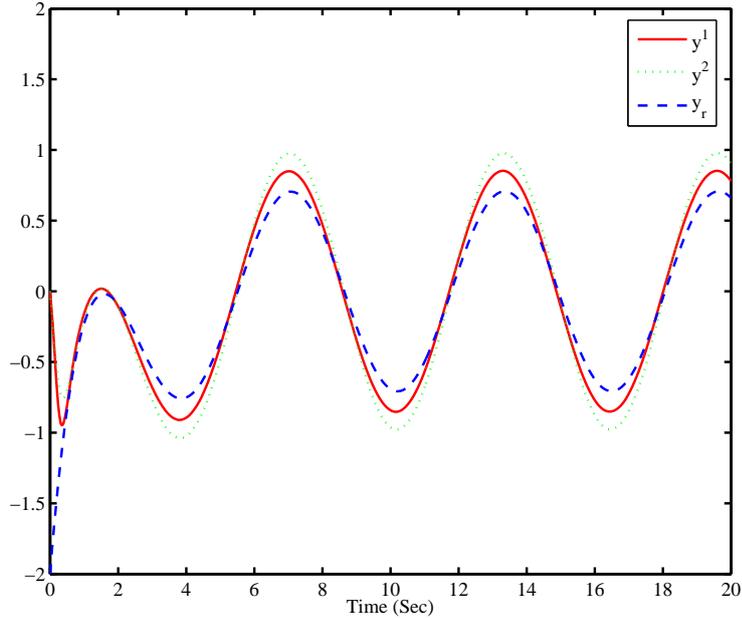


Figure 4.2: The outputs of the system (4.42)-(4.43)

$\gamma_{min} = 0.64$  and  $F_{11} = F_{12} = [-23.8429 \quad -8.6894]$ ,  $F_{21} = F_{22} = 21.2722$ . Clearly, the obtained fuzzy controller with  $F_{j1} \neq F_{j2}$  ( $j = 1, 2$ ) is nonlinear and the fuzzy controller with  $F_{j1} = F_{j2}$  ( $j = 1, 2$ ) is essentially linear.

We now compare the tracking effect between the obtained nonlinear controller and the linear controller in simulation. Choose the sampling period  $h = 30(ms)$ , the initial states  $x(0) = [0 \quad -3]^T$ ,  $x_r(0) = -2$ , and the reference input  $r(t) = \cos(t)$ . We assume that network-induced delay  $\tau_k$  ( $\forall k \in \mathbb{N}$ ) vary in  $[30(ms), 60(ms)]$  and no packet dropout occurs in the network transmission. Figure 4.2 and Figure 4.3 shows the output responses of the systems (4.42)-(4.43) and the corresponding output tracking error, respectively. In Figure 4.2,  $y^1$  and  $y^2$  are the output responses of the system (4.42) controlled by the controller with  $F_{j1} \neq F_{j2}$  and the controller with  $F_{j1} = F_{j2}$ , respectively, and  $y_r$  denotes the output of the reference model (4.43). Figure 4.3 compares the corresponding output tracking errors, where  $e^1 = y^1 - y_r$  and  $e^2 = y^2 - y_r$ . It is easy to see that the network-based fuzzy controller designed by Proposition 4.3 with available  $\delta_i$  can achieve a better  $H_\infty$  tracking performance than

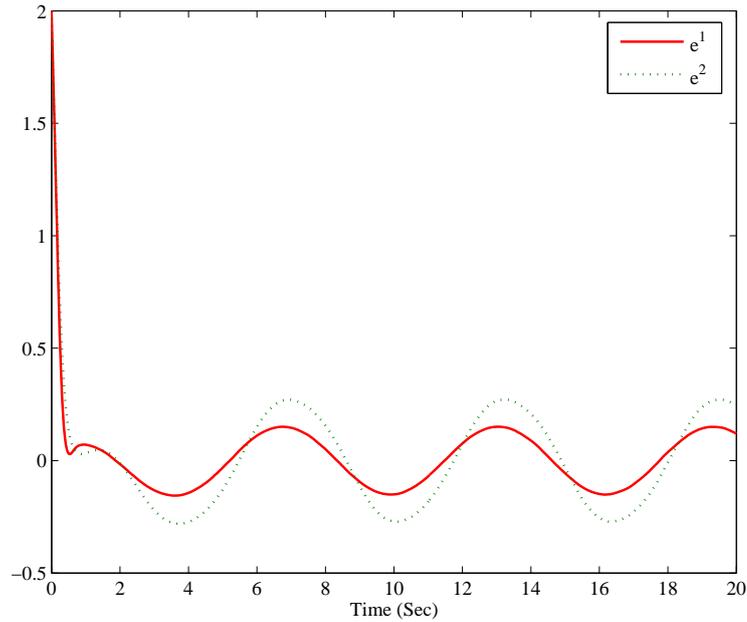


Figure 4.3: The comparison of the output tracking errors

the controller designed by Proposition 4.3 with unavailable  $\delta_i$ , where  $i=1, 2$ .

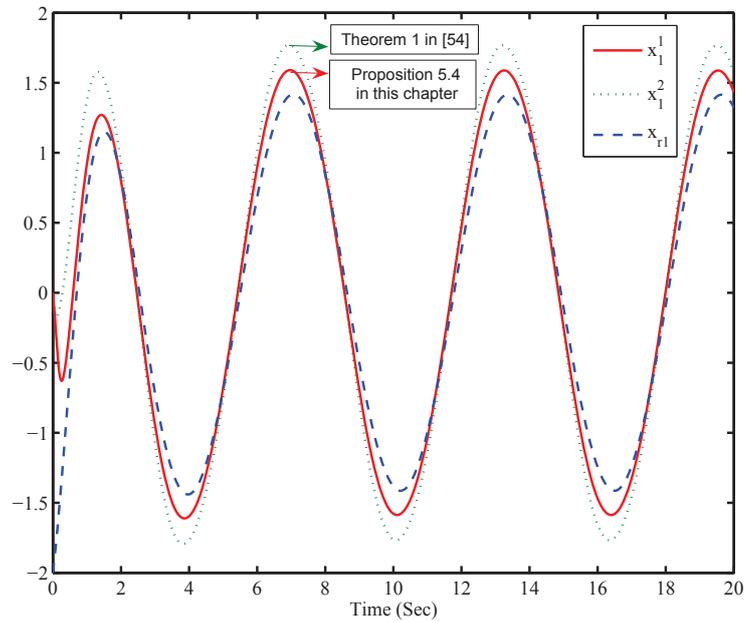


Figure 4.4: The state trajectories  $x_1(t)$  and  $x_{r1}(t)$  of the system (4.42) and (4.45)

Case II: choose the reference model in [54]

$$\dot{x}_r = \begin{bmatrix} 0 & 1 \\ -3 & -2 \end{bmatrix} x_r + \begin{bmatrix} 0 \\ 1 \end{bmatrix} r(t). \quad (4.45)$$

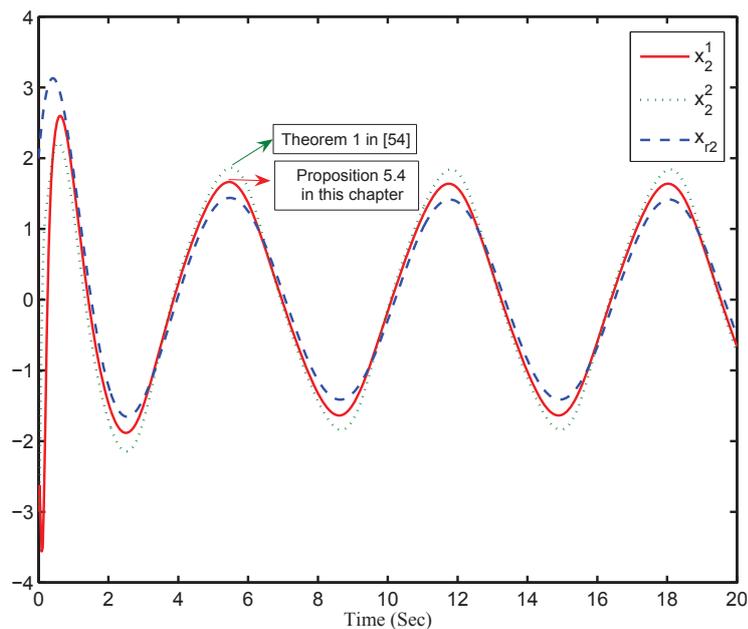


Figure 4.5: The state trajectories  $x_2(t)$  and  $x_{r2}(t)$  of the system (4.42) and (4.45)

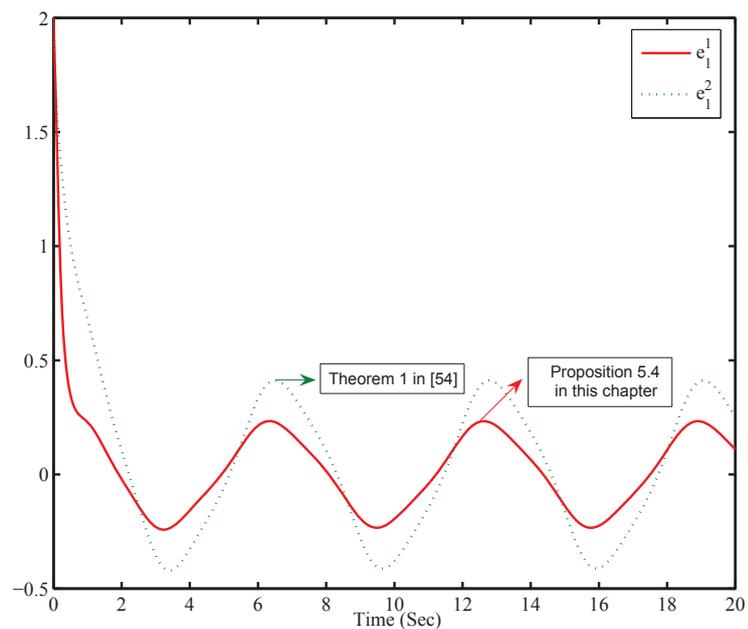


Figure 4.6: The comparison of the tracking errors  $e_1^i(t)$  ( $i = 1, 2$ ) of the system (4.42) and the reference model (4.45)

Similar to [54], the input of the system (4.42) on  $[l_k h + \tau_{l_k}, l_{k+1} h + \tau_{l_{k+1}})$  ( $k \in \mathbb{N}$ ) is

$$u(t) = \sum_{i=1}^2 \mu_i(x_1(l_k h)) F_i(x(l_k h) - x_r(l_k h)). \quad (4.46)$$

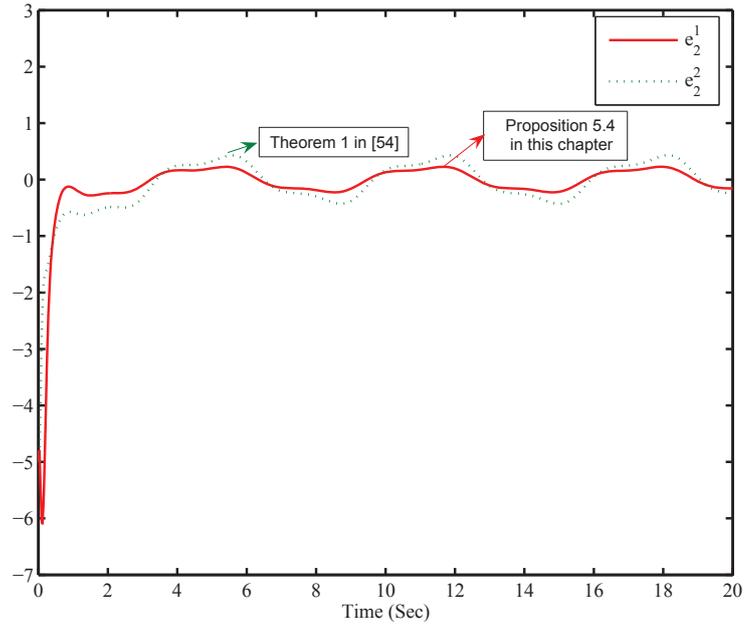


Figure 4.7: The comparison of the tracking errors  $e_2^i(t)$  ( $i = 1, 2$ ) of the system (4.42) and the reference model (4.45)

And the resulting closed-loop system can be described by (4.41) with system matrices  $A_1$ ,  $A_2$ ,  $B_1$ ,  $B_2$ ,  $E_1$  and  $E_2$ , which can be found below (4.42).

For the system (4.41), using Theorem 1 with  $U = I_{2 \times 2}$ ,  $\tau_m = 0$  and  $\tau_M = 60(ms)$  in [54] to obtain the minimal  $H_\infty$  tracking performance  $\gamma_{min} = 2$  and corresponding control gains  $F_1 = F_2 = [-21.7667 \quad -13.7008]$ . Applying Proposition 4.5 with  $\sigma = 0.1$  and some alterations (see, Remark 4.6), one obtains  $\gamma_{min} = 1.06$  and  $F_1 = [-48.5220 \quad -13.1135]$ ,  $F_2 = [-23.9771 \quad -13.1700]$ . Clearly, the fuzzy tracking controller designed by Proposition 4.5 is essentially nonlinear and can ensure a better  $H_\infty$  tracking performance  $\gamma_{min} = 1.06$  for the system (4.41) than the linear network-based controller provided by [54].

Then we compare the tracking effect via the proposed nonlinear controller and the linear controller [54]. In simulation, the initial states of the systems (4.42) and (4.45) are  $x(0) = [0 \quad -3]^T$  and  $x_r(0) = [-2 \quad 2]^T$ ; the sampling period is  $h = 10(ms)$ ; and the reference input is  $r(t) = 4 \cos(t)$ . Choose the same network constraints as those in [54], i.e., the maximum allowable delay bound is  $30(ms)$  and the maximum allowable

number of consecutive packet dropouts is 2. We depict the state trajectories of the systems (4.42)-(4.45) via a controller designed by Proposition 4.5 and Theorem 1 in [54] by Figure 4.4 and Figure 4.5. Figure 4.6 and Figure 4.7 show the corresponding tracking errors. From Figure 4.4-4.7, one can clearly see that the proposed method provides a better tracking control than the one in [54].

## 4.6 Summary

In this chapter, we have considered network-based state feedback tracking control for T-S fuzzy systems by using the asynchronous constraints on fuzzy membership functions. Since premise variables are involved in network transmission, the membership functions of a T-S fuzzy model and a fuzzy tracking controller are asynchronously fired. Taking the asynchronous characteristic into account, the network-based T-S fuzzy system has been represented by an asynchronous T-S fuzzy system with an interval time-varying sawtooth delay. Notice that a routine relaxation method for  $H_\infty$  performance analysis and controller design of a traditional T-S fuzzy system can not be used for the asynchronous fuzzy system since the common product of asynchronous membership functions can not be grouped. We have proposed a new relaxation method by using the asynchronous constraints on fuzzy membership functions to introduce some free-weighting matrices. By using the proposed relaxation method and a discontinuous Lyapunov-Krasovskii functional method, some delay-dependent criteria for  $H_\infty$  performance tracking analysis and tracking controller design have been established in terms of linear matrix inequalities. An illustrative example has been given to show that a better  $H_\infty$  tracking performance can be achieved by the proposed method.

## Chapter 5

# Network-based static output feedback tracking control for linear systems

### 5.1 Introduction

Consider the following linear system

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) + E\omega(t) \\ y(t) = Cx(t) \\ x(t_0) = x_0 \end{cases} \quad (5.1)$$

where  $x(t) \in \mathbb{R}^n$  is the state,  $u(t) \in \mathbb{R}^m$  is the control input,  $y(t) \in \mathbb{R}^l$  is the output,  $\omega(t) \in \mathbb{R}^v$  is the external disturbance acting on the system (5.1) and  $\omega(t) \in \mathcal{L}_2[t_0, \infty)$ ;  $x(t_0) = x_0$  is the initial state;  $A$ ,  $B$ ,  $C$  and  $E$  are constant system matrices of appropriate dimensions; the pairs  $(A, B)$  and  $(A, C)$  are assumed to be controllable and observable, respectively. It is assumed that the system (5.1) with  $\omega(t) = 0$  can not be stabilized by a non-delayed static output feedback controller, but can be stabilized by a delayed static output feedback controller. Such a typical system is a driven damped harmonic oscillator [84], which is described by

$$\ddot{z}(t) + 2\zeta\omega_0\dot{z}(t) + \omega_0^2z(t) = u(t), \quad y(t) = z(t) \quad (5.2)$$

where  $\zeta$  ( $\zeta < 0$ ) is the damping ratio,  $\omega_0$  ( $\omega_0 > 0$ ) is the undamped angular frequency, and  $z(t)$  is the position of the oscillator. Let  $x(t) = [z^T(t) \dot{z}^T(t)]^T$ . The system (5.2)

can be expressed in the following state-space representation

$$\begin{cases} \dot{x}(t) = \begin{bmatrix} 0 & 1 \\ -\omega_0^2 & -2\zeta\omega_0 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t) \\ y(t) = [1 \ 0]x(t). \end{cases} \quad (5.3)$$

Clearly, the oscillator (5.2) can not be stabilized by a static output feedback controller  $u(t) = fy(t)$  because there does not exist a feedback gain  $f$  such that the matrix  $\begin{bmatrix} 0 & 1 \\ f-\omega_0^2 & -2\zeta\omega_0 \end{bmatrix}$  is Hurwitz. But the oscillator (5.2) can be stabilized by introducing a time-delay  $\tau > 0$  in the controller  $u(t) = fy(t-\tau)$ .

In this chapter, the objective of static output feedback tracking control is to drive the output  $y(t)$  of the system (5.1), via a static output feedback controller, to follow a reference signal  $y_r(t)$  as close as possible. The signal  $y_r(t)$  is generated by the following reference model

$$\begin{cases} \dot{x}_r(t) = A_r x_r(t) + B_r r(t) \\ y_r(t) = C_r x_r(t) \\ x_r(t_0) = x_{r0} \end{cases} \quad (5.4)$$

where  $x_r(t) \in \mathbb{R}^r$ ,  $r(t) \in \mathbb{R}^{\bar{v}}$  and  $y_r(t) \in \mathbb{R}^l$  are the state, the energy bounded input and the output, respectively;  $x_r(t_0) = x_{r0}$  is the initial state;  $A_r$ ,  $B_r$  and  $C_r$  are constant matrices, and  $A_r$  is a Hurwitz matrix. Since the system (5.1) with  $\omega(t) = 0$  under consideration can not be stabilized by a non-delayed static output feedback controller, it is *impossible* to fulfil the objective of output feedback tracking control by the following non-delayed static output feedback controller

$$u(t) = F_1 y(t) + F_2 y_r(t) \quad (5.5)$$

where  $F_1$  and  $F_2$  are output feedback gains. But the system (5.1) with  $\omega(t) = 0$  can be stabilized by a delayed static output feedback controller. So a feasible way to achieve the objective of static output feedback tracking control is intentionally introducing a time-delay in the controller (5.5). To develop the delayed feedback control, we insert a communication network between the system (5.1) and the controller (5.5). The *main motivation* of introducing the network is to investigate

whether network-induced delays have positive effects on system stability and tracking performance. Usually, network-induced delay is regarded as the source of poor performance and system instability (deterioration effects: negative effects), see [28], [36], [46], [54], [133], [137] and the references therein. In this chapter, we will take a *different and novel view*. For the system (5.1) under consideration, we will *purposefully* produce network-induced delays in the feedback control loop by inserting the network to provide a stable and satisfactory tracking control. The introduction of the network in a feedback control system has several advantages such as wiring reduction, ease of maintenance and installation, and remote execution of tracking control [36]. When exchanging data among system components through a communication network, there exist four types of essential delays: queueing time  $\tau_{qt}$ , processing time  $\tau_{pt}$ , propagation delay  $\tau_{pd}$  and transmission delay  $\tau_{td}$ . For the purpose of clarity, we classify the network-induced delays as the sensor-to-controller delay  $\tau^{sc}$  and the controller-to-actuator delay  $\tau^{ca}$ , where  $\tau^{sc} = \tau_{qt}^s + \tau_{pt}^s + \tau_{pd}^{sc} + \tau_{td}^{sc}$ ,  $\tau^{ca} = \tau_{qt}^c + \tau_{pt}^c + \tau_{pd}^a + \tau_{td}^{ca}$ , and the superscripts  $s, c, a, sc$  and  $ca$  denote the sensor, the controller, the actuator, between the sensor and the controller, and between the controller and the actuator, respectively. It is assumed that the sampled-data  $y(kh)$  and  $y_r(kh)$  ( $\forall k \in \mathbb{N}$ ), where  $h$  is the sampling period and  $\mathbb{N}$  is the set of non-negative integers, are transmitted in a single packet and no packet dropout occurs in transmission. Taking into consideration the network-induced delays  $\tau_k^{sc}$  and  $\tau_k^{ca}$ , the update input of the actuator is

$$u(t) = F_1 y(kh) + F_2 y_r(kh), \quad t \in \{kh + \tau_k\}_{k=0}^{\infty} \quad (5.6)$$

where  $\tau_k = \tau_k^{sc} + \tau_k^{ca}$  ( $\forall k \in \mathbb{Z}$ ). Defining  $\tau(t) = t - kh$  for  $t \in [kh + \tau_k, (k+1)h + \tau_{k+1})$ ,  $\tau_m \triangleq \min_{k \in \mathbb{Z}} \{\tau_k\}$  and  $\tau_M \triangleq h + \max_{k \in \mathbb{Z}} \{\tau_k\}$  [56], we have

$$0 < \tau_m \leq \tau(t) \leq \tau_M. \quad (5.7)$$

The actuator holds the control signal to input the system (5.1) until next update.

The input of the system (5.1) is described by

$$\begin{cases} u(t) = F_1 y(t - \tau(t)) + F_2 y_r(t - \tau(t)) \\ t \in [kh + \tau_k, (k+1)h + \tau_{k+1}), \forall k \in \mathbb{Z}. \end{cases} \quad (5.8)$$

Obviously,  $\tau(t)$  is discontinuous at  $\{kh + \tau_k\}_{k=0}^{\infty}$  and piecewise-linear with derivative  $\dot{\tau}(t) = 1$  for  $t \neq kh + \tau_k$  ( $\forall k \in \mathbb{Z}$ ). From the definitions of  $\tau_m$  and  $\tau_M$ , one can see that  $\tau(t)$  is an interval time-varying sawtooth delay induced by sample-and-hold behaviors and the network-induced delay  $\tau_k$  satisfying  $0 < \tau_m \leq \tau_k \leq \tau_M - h$  ( $\forall k \in \mathbb{Z}$ ).

Defining the output tracking error  $e(t) = y(t) - y_r(t)$  and using (5.1), (5.4) and (5.6), one has the following augmented system

$$\begin{cases} \dot{\xi}(t) = \bar{A}\xi(t) + \bar{B}\bar{F}\xi(t - \tau(t)) + \bar{E}\bar{\omega}(t) \\ e(t) = \bar{C}\xi(t) \\ \tau(t) = t - kh, t \in [kh + \tau_k, (k+1)h + \tau_{k+1}), \forall k \in \mathbb{Z} \end{cases} \quad (5.9)$$

where

$$\begin{aligned} \xi(t) &= \begin{bmatrix} x(t) \\ x_r(t) \end{bmatrix}, \bar{\omega}(t) = \begin{bmatrix} \omega(t) \\ r(t) \end{bmatrix}, \bar{A} = \begin{bmatrix} A & 0 \\ 0 & A_r \end{bmatrix}, \bar{B} = \begin{bmatrix} B \\ 0 \end{bmatrix}, \\ \bar{E} &= \begin{bmatrix} E & 0 \\ 0 & B_r \end{bmatrix}, \bar{F} = [F_1 C \quad F_2 C_r], \bar{C} = [C \quad -C_r]. \end{aligned}$$

The initial condition of the system (5.9) is supplemented as  $\xi(t) = \phi(t)$  with  $\phi(t_0) = [x_0^T \ x_{r0}^T]^T$ ,  $t \in [t_0 - \tau_M, t_0]$ . We choose the following  $H_\infty$  tracking performance

$$\int_{t_0}^{t_f} e^T(t) U e(t) dt \leq V(t_0) + \gamma^2 \int_{t_0}^{t_f} \bar{\omega}^T(t) \bar{\omega}(t) dt \quad (5.10)$$

where  $t_f$  is the terminal time,  $\gamma > 0$  is the desired  $H_\infty$  tracking performance level,  $U > 0$  is the weighting matrix, and  $V(t_0)$  is the energy function of initial states.

The purpose of this chapter is to design a network-based static output feedback controller such that the augmented system (5.9) is asymptotically stable with a prescribed  $H_\infty$  tracking performance, which means that

- 1) the system (5.9) with  $\bar{\omega}(t) = 0$  is asymptotically stable;
- 2) the output tracking error  $e(t)$  satisfies the  $H_\infty$  tracking performance (5.10), for all nonzero  $\bar{\omega}(t) \in \mathcal{L}_2[t_0, \infty)$ .

Notice that the system (5.1) can not be stabilized by a non-delayed static output feedback controller, which implies that  $A+BF_1C$  is not Hurwitz. Since  $\bar{A}+\bar{B}\bar{F} = \begin{bmatrix} A+BF_1C & BF_2C_r \\ 0 & A_r \end{bmatrix}$ , the system (5.9) with  $\bar{\omega}(t)=0$  is not stable when  $\tau(t)\equiv 0$ . As a result, a stable tracking performance can not be ensured for the system (5.1) by using the static output feedback controller (5.5) without a time-delay. By inserting a communication network that between the system (5.1) and the controller (5.5), a network-induced delay is purposefully introduced in the feedback control loop, which results in the delayed control input (5.6). Since the system (5.1) can be stabilized by a delayed static output feedback controller, it is possible that the system (5.9) with an interval time-varying delay  $\tau(t)$  satisfying (5.7) is asymptotically stable with a prescribed  $H_\infty$  tracking performance. It should be mentioned that  $\tau_m$  is both the lower bound of the network-induced delay and the lower bound of the sawtooth delay, and therefore  $\tau_m > 0$  is essential for the network-based output tracking control. In addition, *network-based static output feedback tracking control for the system (5.1) has not been investigated in the existing literature*, which motivates the present study.

## 5.2 A complete LKF method for tracking performance analysis

In this section, we will analyze  $H_\infty$  tracking performance for the system (5.9). We need the following lemma

**Lemma 5.1.** *There exist some positive scalars  $\varepsilon_i > 0$  ( $i = 1, 2$ ) and a functional  $V(t, \xi_t(\theta), \dot{\xi}_t(\theta)) : \mathbb{R} \times \mathbb{R}^n \times \mathbb{R}^n \rightarrow \mathbb{R}$  such that*

$$\varepsilon_1 \|\xi(t)\|^2 \leq V(t, \xi_t(\theta), \dot{\xi}_t(\theta)) \leq \varepsilon_2 \|\xi\|_{\mathbb{W}}^2 \quad (5.11)$$

where  $\xi_t(\theta) = \xi(t+\theta)$  and  $\dot{\xi}_t(\theta) = \dot{\xi}(t+\theta)$ ,  $\forall \theta \in [-\tau_M, 0]$ , and the space of functions  $\xi_t(\theta)$  and  $\dot{\xi}_t(\theta)$  is denoted by  $\mathbb{W}$  with the norm

$$\|\xi\|_{\mathbb{W}} = \sup_{\theta \in [-\tau_M, 0]} \{\|\xi_t(\theta)\|, \|\dot{\xi}_t(\theta)\|\}.$$

Let the functional  $V(t) = V(t, \xi_t(\theta), \dot{\xi}_t(\theta))$  be absolutely continuous for  $t \neq kh + \tau_k$  and satisfy

$$V(kh + \tau_k) \leq \lim_{t \rightarrow (kh + \tau_k)^-} V(t), \quad \forall k \in \mathbb{Z} \quad (5.12)$$

where  $\lim_{t \rightarrow (kh + \tau_k)^-} V(t)$  is a limit taken from the left,  $t \in [(k-1)h + \tau_{k-1}, kh + \tau_k)$ .

Define  $\dot{V}(t) = \limsup_{\epsilon \rightarrow 0} \frac{1}{\epsilon} [V(t + \epsilon) - V(t)]$ , where  $V(t + \epsilon) = V(t + \epsilon, \xi_{t+\epsilon}(\theta), \dot{\xi}_{t+\epsilon}(\theta))$ .

Then one can see that

- (i) the system (5.9) with  $\bar{\omega}(t) = 0$  is asymptotically stable if there exists an  $\varepsilon_3 > 0$  such that the time-derivative of  $V(t)$  along (5.9) with  $\bar{\omega}(t) = 0$  satisfies

$$\dot{V}(t) \leq -\varepsilon_3 \|\xi(t)\|^2 \quad (5.13)$$

for  $t \in [kh + \tau_k, (k+1)h + \tau_{k+1})$ ,  $\forall k \in \mathbb{Z}$ .

- (ii) the  $H_\infty$  tracking performance (5.10) can be ensured for all nonzero  $\bar{\omega}(t) \in \mathcal{L}_2[t_0, \infty)$  if along (5.9), the following inequality holds

$$\dot{V}(t) + e^T(t)W e(t) - \gamma^2 \bar{\omega}^T(t)\bar{\omega}(t) < 0 \quad (5.14)$$

for  $t \in [kh + \tau_k, (k+1)h + \tau_{k+1})$ ,  $\forall k \in \mathbb{Z}$ .

*Proof:* It can be directly obtained from Lemma 2.1 with  $i_k = k$  ( $\forall k \in \mathbb{Z}$ ).

Notice that the existing simple Lyapunov-Krasovskii functionals (LKFs) can not be used to derive a delay-dependent criterion for  $H_\infty$  tracking performance analysis since they require the system (5.9) with  $\tau(t) = 0$  to be stable, where the definition of a simple LKF can be found in [40]. Instead, a new discontinuous complete LKF, which makes use of the lower bound of the network-induced delay  $\tau_m$ , the sawtooth delay  $\tau(t)$  and its upper bound  $\tau_M$ , is constructed as follows

$$V(t) = V_1(t) + V_2(t) + V_3(t) \quad (5.15)$$

where

$$\begin{aligned}
V_1(t) &= \frac{1}{2} \xi^T(t) P \xi(t) + \xi^T(t) \int_{-\tau_M}^0 Q(\theta) \xi(t+\theta) d\theta \\
&\quad + \frac{1}{2} \int_{-\tau_M}^0 \int_{-\tau_M}^0 \xi^T(t+\theta) R(\theta, s) \xi(t+s) d\theta ds \\
&\quad + \frac{1}{2} \int_{-\tau_M}^0 \xi^T(t+\theta) S(\theta) \xi(t+\theta) d\theta, \\
V_2(t) &= \frac{1}{2} \int_{-\tau_m}^0 \xi^T(t+\theta) Z_1 \xi(t+\theta) d\theta \\
&\quad + \frac{\tau_m}{2} \int_{-\tau_m}^0 \int_s^0 \dot{\xi}^T(t+\theta) Z_2 \dot{\xi}(t+\theta) d\theta ds \\
&\quad + \frac{\tau_M - \tau_m}{2} \int_{-\tau_M}^{-\tau_m} \int_s^0 \dot{\xi}^T(t+\theta) Z_3 \dot{\xi}(t+\theta) d\theta ds, \\
V_3(t) &= \frac{\tau_M - \tau(t)}{2} \int_{-\bar{\tau}(t)}^0 \dot{\xi}^T(t+\theta) Z_4 \dot{\xi}(t+\theta) d\theta \\
&\quad + \frac{\tau_M - \tau(t)}{2} (x(t) - x(t - \bar{\tau}(t)))^T Z_5 (x(t) - x(t - \bar{\tau}(t))),
\end{aligned}$$

$$\bar{\tau}(t) = \tau(t) - \tau_k, \quad t \in [kh + \tau_k, (k+1)h + \tau_{k+1}), \quad \forall k \in \mathbb{Z},$$

$$Z_i \in \mathbb{R}^{p \times p}, \quad Z_i = Z_i^T (i=1, 2, 3, 4, 5), \quad P \in \mathbb{R}^{p \times p}, \quad P = P^T,$$

$$Q : [-\tau_M, 0] \rightarrow \mathbb{R}^{p \times p}, \quad Q(\theta) = Q^T(\theta),$$

$$S : [-\tau_M, 0] \rightarrow \mathbb{R}^{p \times p}, \quad S(\theta) = S^T(\theta),$$

$$R : [-\tau_M, 0] \times [-\tau_M, 0] \rightarrow \mathbb{R}^{p \times p}, \quad R(\theta, s) = R^T(s, \theta),$$

and  $Q$ ,  $S$  and  $R$  are continuously differential matrix functions.

Choose  $Q$ ,  $S$  and  $R$  to be continuous piecewise linear [33], i.e.,

$$Q^i(\alpha) = Q(\theta_i + \alpha v) = (1-\alpha)Q_{i-1} + \alpha Q_i, \quad i=1, 2, \dots, N \quad (5.16)$$

$$S^i(\alpha) = S(\theta_i + \alpha v) = (1-\alpha)S_{i-1} + \alpha S_i, \quad i=1, 2, \dots, N \quad (5.17)$$

$$\begin{aligned}
R^{ij}(\alpha, \beta) &= R(\theta_{i-1} + \alpha v, \theta_{j-1} + \beta v) \\
&= \begin{cases} (1-\alpha)R_{i-1, j-1} + \beta R_{ij} + (\alpha-\beta)R_{i, j-1}, & \alpha \geq \beta, i, j=1, 2, \dots, N \\ (1-\beta)R_{i-1, j-1} + \alpha R_{ij} + (\beta-\alpha)R_{i-1, j}, & \alpha < \beta, i, j=1, 2, \dots, N \end{cases} \quad (5.18)
\end{aligned}$$

where  $0 \leq \alpha \leq 1$ ,  $0 \leq \beta \leq 1$ ,  $[-\tau_M, 0]$  is divided into  $N$  segments  $[\theta_{i-1}, \theta_i]$  ( $i=1, 2, \dots, N$ ) of an equal length  $v = \tau_M/N$  and  $\theta_i = -\tau_M + iv$  ( $i=0, 1, \dots, N$ ).

The following lemma is introduced to ensure the LKF (5.15) satisfying (5.11).

**Lemma 5.2.** *For piecewise linear  $Q$ ,  $S$  and  $R$  described by (5.16)-(5.18), there exist  $\varepsilon_i > 0$  ( $i = 1, 2$ ) such that the Lyapunov-Krasovskii functional (5.15) satisfies  $\varepsilon_1 \|\xi(t)\|^2 \leq V(t) \leq \varepsilon_2 \|\xi\|_{\mathbb{W}}^2$  if*

$$P > 0 \tag{5.19}$$

$$Z_i > 0, \quad i = 1, 2, 3, 4, 5 \tag{5.20}$$

$$S_i > 0, \quad i = 0, 1, \dots, N \tag{5.21}$$

$$\begin{bmatrix} P & * \\ \tilde{Q}^T & \tilde{S} + \tilde{R} \end{bmatrix} > 0 \tag{5.22}$$

where

$$\begin{aligned} \tilde{Q} &= (Q_0, Q_1, \dots, Q_N), \\ \tilde{S} &= \text{diag}\{v^{-1}S_0, v^{-1}S_1, \dots, v^{-1}S_N\}, \\ \tilde{R} &= \begin{bmatrix} R_{00} & R_{01} & \cdots & R_{0N} \\ R_{10} & R_{11} & \cdots & R_{1N} \\ \vdots & \vdots & \ddots & \vdots \\ R_{N0} & R_{N1} & \cdots & R_{NN} \end{bmatrix}. \end{aligned}$$

*Proof:* We first prove that the LKF (5.15) satisfies  $V(t) \geq \varepsilon_1 \|\xi(t)\|^2$ , where  $\varepsilon_1 > 0$ .

Using (5.16)-(5.18) to  $V_1(t)$  yields

$$\begin{aligned} V_1(t) &= \frac{1}{2} \int_0^1 \left[ \xi^T(t) \quad v(\tilde{\psi}^T(1)I_r - \tilde{\psi}^T(\alpha)I_d) \right] \\ &\quad \times \begin{bmatrix} P & * \\ \tilde{Q}^T & \tilde{R} \end{bmatrix} \begin{bmatrix} \xi(t) \\ v(I_r^T \tilde{\psi}(1) - I_d^T \tilde{\psi}(\alpha)) \end{bmatrix} d\alpha \\ &\quad + \frac{v}{2} \sum_{i=1}^N \int_0^1 \varphi^{iT}(\alpha) S^i(\alpha) \varphi^i(\alpha) d\alpha \end{aligned} \tag{5.23}$$

where

$$\begin{aligned} I_r &= [0_{N \times 1} \quad I], \quad I_d = [I \quad 0_{N \times 1}] - [0_{N \times 1} \quad I], \\ \tilde{\psi}(\alpha) &= [\psi^{1T}(\alpha) \quad \psi^{2T}(\alpha) \quad \cdots \quad \psi^{NT}(\alpha)]^T, \\ \psi^i(\alpha) &= \int_0^\alpha \varphi^i(s) ds, \quad i = 1, 2, \dots, N, \\ \varphi^i(\alpha) &= \xi(t - i v + \alpha v), \quad i = 1, 2, \dots, N. \end{aligned}$$

Using a similar method in [39], we have

$$V_1(t) \geq \frac{1}{2} \int_0^1 \left[ \xi^T(t) \quad v(\tilde{\psi}^T(1)I_r - \tilde{\psi}^T(\alpha)I_d) \right] \\ \times \begin{bmatrix} P & * \\ \tilde{Q}^T & \tilde{S} + \tilde{R} \end{bmatrix} \begin{bmatrix} \xi(t) \\ v(I_r^T \tilde{\psi}(1) - I_d^T \tilde{\psi}(\alpha)) \end{bmatrix} d\alpha.$$

For the terms  $V_i(t)$  ( $i = 2, 3$ ), it follows from (5.20)-(5.21) that  $V_i(t) \geq 0$  ( $i = 2, 3$ ).

Then it is clear to see that there exists an  $\varepsilon_1 > 0$  such that  $V(t) = V_1(t) + V_2(t) + V_3(t) \geq \varepsilon_1 \|\xi(t)\|^2$  if the linear matrix inequalities (LMIs) (5.19)-(5.22) are satisfied.

Second, we show that there exists an upper bound for the LKF (5.15). It can be seen from (5.23) that

$$V_1(t) \leq \frac{1}{2} \left[ \|\xi(t)\|^2 + \int_0^1 f(\alpha) d\alpha \right] \lambda_{\max} \left( \begin{bmatrix} P & * \\ \tilde{Q}^T & \tilde{R} \end{bmatrix} \right) \\ + \frac{Nv}{2} \max_{i \in \{1, 2, \dots, N\}} \lambda_{\max}(S^i) \int_0^1 \varphi^{iT}(\alpha) \varphi^i(\alpha) d\alpha \\ \leq \kappa_1 \max_{\theta \in [-\tau_M, 0]} \|\dot{\xi}_t(\theta)\|^2 \quad (5.24)$$

where

$$f(\alpha) = v^2 \int_{\alpha}^1 \varphi^{1T}(s) ds \int_{\alpha}^1 \varphi^1(s) ds + v^2 \int_0^{\alpha} \varphi^{NT}(s) ds \int_0^{\alpha} \varphi^N(s) ds \\ + 2v^2 \sum_{i=2}^N \int_{\alpha}^1 \varphi^{iT}(s) ds \int_{\alpha}^1 \varphi^i(s) ds + 2v^2 \sum_{i=1}^{N-1} \int_0^{\alpha} \varphi^{iT}(s) ds \int_0^{\alpha} \varphi^i(s) ds, \\ \kappa_1 = \left( \frac{1}{2} + \frac{2N-1}{3} v^2 \right) \lambda_{\max} \left( \begin{bmatrix} P & * \\ \tilde{Q}^T & \tilde{R} \end{bmatrix} \right) + \frac{Nv}{2} \max_{i \in \{1, 2, \dots, N\}} \lambda_{\max}(S^i).$$

For  $Z_i > 0$  ( $i = 1, 2, 3, 4, 5$ ), we have

$$V_2(t) + V_3(t) \leq \frac{\tau_m}{2} \lambda_{\max}(Z_1) \max_{\theta \in [-\tau_M, 0]} \|\xi_t(\theta)\|^2 + \kappa_2 \max_{\theta \in [-\tau_M, 0]} \|\dot{\xi}_t(\theta)\|^2 \quad (5.25)$$

where

$$\kappa_2 = \frac{\tau_m^3}{2} \lambda_{\max}(Z_2) + \frac{\tau_M(\tau_M - \tau_m)}{2} \lambda_{\max}(Z_3) + \frac{(\tau_M - \tau_m)^3}{2} [\lambda_{\max}(Z_4) + \lambda_{\max}(Z_5)].$$

From (5.24)-(5.25), we can see that

$$V(t) \leq \varepsilon_2 \|\xi\|_{\mathbb{W}}^2 \quad (5.26)$$

where  $\varepsilon_2 = (\kappa_1 + \kappa_2 + \frac{\tau_m}{2} \lambda_{\max}(Z_1)) \|\xi\|_{\mathbb{W}}^2$ . Then it is concluded that the LKF (5.15) satisfies the condition (5.11) if the LMIs (5.19)-(5.22) are satisfied.

We now show that the condition (5.12) holds for the LKF (5.15) satisfying the inequality (5.11). In the LKF (5.15), the terms  $V_i(t)$  ( $i = 1, 2$ ) are continuous on  $[t_0, \infty)$ , which means that  $V_i(kh + \tau_k) = \lim_{t \rightarrow (kh + \tau_k)^-} V_i(t)$  ( $i = 1, 2, \forall k \in \mathbb{Z}$ ). The term  $V_3(t)$  involving explicitly the sawtooth delay  $\tau(t)$  is discontinuous at  $kh + \tau_k$ . But for  $Z_4 > 0$  and  $Z_5 > 0$ ,  $V_3(t)$  does not increase along  $kh + \tau_k$  since it is non-negative before  $kh + \tau_k$  and becomes zero just after  $kh + \tau_k, \forall k \in \mathbb{Z}$ . Thus, we can obtain the inequality (5.12).

In the following, by utilizing the complete LKF (5.15) with the discretization technique (5.16)-(5.18), we will derive a new delay-dependent criterion such that the system (5.9) is asymptotically stable with a prescribed  $H_\infty$  tracking performance. For simplicity of presentation, let

$$\begin{aligned} \eta^T(t) &= [\eta_1^T(t) \quad \eta_2^T(t) \quad \bar{\omega}^T(t)], \\ \eta_1^T(t) &= [\xi^T(t) \quad \xi^T(t - \tau_m) \quad \xi^T(t - \tau(t))], \\ \eta_2^T(t) &= [\xi^T(t - \bar{\tau}(t)) \quad \xi^T(t - \tau_M)], \\ e_1 &= [I_{p \times p} \quad 0 \quad 0 \quad 0 \quad 0 \quad 0]_{p \times (5p+q)}, \\ e_2 &= [0 \quad I_{p \times p} \quad 0 \quad 0 \quad 0 \quad 0]_{p \times (5p+q)}, \\ e_3 &= [0 \quad 0 \quad I_{p \times p} \quad 0 \quad 0 \quad 0]_{p \times (5p+q)}, \\ e_4 &= [0 \quad 0 \quad 0 \quad I_{p \times p} \quad 0 \quad 0]_{p \times (5p+q)}, \\ e_5 &= [0 \quad 0 \quad 0 \quad 0 \quad I_{p \times p} \quad 0]_{p \times (5p+q)}, \\ e_6 &= [0 \quad 0 \quad 0 \quad 0 \quad 0 \quad I_{q \times q}]_{q \times (5p+q)} \end{aligned}$$

where  $e_i$  ( $i = 1, 2, 3, 4, 5$ ) are  $p \times (5p + q)$  matrices and  $e_6$  is a  $q \times (5p + q)$  matrix;  $I_{p \times p}$  and  $I_{q \times q}$  denote  $p \times p$  and  $q \times q$  identity matrices, respectively; the others in  $e_j$  ( $j = 1, 2, \dots, 6$ ) are zero matrices with appropriate dimensions;  $p$  and  $q$  are dimensions of  $\xi(t)$  and  $\bar{\omega}(t)$ , respectively. Then  $\xi(t) = e_1 \eta(t)$ ,  $\xi(t - \tau(t)) = e_3 \eta(t)$ ,

and  $\bar{\omega}(t) = e_6 \eta(t)$ . For  $t \in [kh + \tau_k, (k+1)h + \tau_{k+1})$  ( $\forall k \in \mathbb{Z}$ ), rewrite the augmented system (5.9) as

$$\begin{cases} \dot{\xi}(t) = (\bar{A}e_1 + \bar{B}\bar{F}e_3 + \bar{E}e_6)\eta(t) \\ e(t) = \bar{C}e_1\eta(t). \end{cases} \quad (5.27)$$

**Proposition 5.1.** *For given positive scalars  $\gamma$ ,  $\tau_m$  and  $\tau_M$ , and a weighting matrix  $U > 0$ , the system (5.9) is asymptotically stable with the  $H_\infty$  tracking performance (5.10), if there exist symmetric matrices  $P > 0$ ,  $Z_i > 0$  ( $i = 1, 2, 3, 4, 5$ ),  $S_i > 0$  ( $i = 0, 1, \dots, N$ ), and  $X_1, X_2, X_3, Q_i, R_{ij}$  ( $i, j = 0, 1, \dots, N$ ) such that (5.22) and*

$$\begin{bmatrix} \bar{\Xi}_{11}^0 & * & * & * \\ \bar{\Xi}_{21} & \bar{\Xi}_{22} & * & * \\ -\delta X_2 & 0 & \delta Z_3 & * \\ -\delta X_1 & 0 & 0 & \delta Z_4 \end{bmatrix} > 0 \quad (5.28)$$

$$\begin{bmatrix} \bar{\Xi}_{11}^0 + \delta \bar{\Xi}_{11}^1 & * & * \\ \bar{\Xi}_{21} & \bar{\Xi}_{22} & * \\ -\delta X_3 & 0 & \delta Z_3 \end{bmatrix} > 0 \quad (5.29)$$

where

$$\begin{aligned} \bar{\Xi}_{11}^0 = & -e_1^T(P\bar{A} + \bar{A}^T P + Q_N + Q_N^T + S_N + Z_1 - Z_2)e_1 \\ & - e_1^T(\bar{A}^T \Omega \bar{A} + 2\bar{C}^T U \bar{C} - Z_5)e_1 - e_1^T Z_2 e_2 \\ & - e_1^T(P + \bar{A}^T \Omega)\bar{B}\bar{F}e_3 - e_1^T Z_5 e_4 + e_1^T Q_0 e_5 \\ & - e_1^T(P + \bar{A}^T \Omega)\bar{E}e_6 - e_2^T Z_2 e_1 - e_3^T \bar{F}^T \bar{B}^T P e_1 \\ & - e_3^T \bar{F}^T \bar{B}^T \Omega \bar{A} e_1 - e_4^T Z_5 e_1 + e_5^T Q_0^T e_1 \\ & - e_6^T \bar{E}^T (P + \Omega \bar{A})e_1 + e_2^T (Z_1 + Z_2)e_2 \\ & - e_3^T (\bar{B}\bar{F})^T \Omega \bar{B}\bar{F}e_3 - e_3^T (\bar{B}\bar{F})^T \Omega \bar{E}e_6 \\ & - e_6^T \bar{E}^T \Omega \bar{B}\bar{F}e_3 + e_4^T Z_5 e_4 + e_5^T S_0 e_5 \\ & - e_6^T (\bar{E}^T \Omega \bar{E} - 2\gamma^2 I)e_6 - (e_1 - e_4)^T X_1 \\ & - X_1^T (e_1 - e_4) - X_2^T (e_2 - e_3) - (e_2 - e_3)^T X_2 \\ & - X_3^T (e_3 - e_5) - (e_3 - e_5)^T X_3, \end{aligned}$$

$$\begin{aligned}
\bar{\Xi}_{11}^1 &= -e_1^T (\bar{A}^T Z_4 \bar{A} + Z_5 \bar{A} + \bar{A}^T Z_5) e_1 - e_1^T Z_5 \bar{B} \bar{F} e_3 \\
&\quad - e_1^T \bar{A}^T Z_4 \bar{B} \bar{F} e_3 - e_3^T (\bar{B} \bar{F})^T (Z_4 \bar{A} + Z_5) e_1 \\
&\quad - e_3^T (\bar{B} \bar{F})^T Z_4 \bar{B} \bar{F} e_3 + e_1^T \bar{A}^T Z_5 e_4 + e_4^T Z_5 \bar{A} e_1 \\
&\quad - e_1^T (\bar{A}^T Z_4 + Z_5) \bar{E} e_6 + e_3^T (\bar{B} \bar{F})^T Z_5 e_4 \\
&\quad - e_3^T (\bar{B} \bar{F})^T Z_4 \bar{E} e_6 - e_6^T \bar{E}^T Z_4 \bar{B} \bar{F} e_3 \\
&\quad - e_6^T \bar{E}^T (Z_4 \bar{A} + Z_5) e_1 + e_4^T Z_5 \bar{B} \bar{F} e_3 \\
&\quad - e_6^T \bar{E}^T Z_4 \bar{E} e_6 + e_4^T Z_5 \bar{E} e_6 + e_6^T \bar{E}^T Z_5 e_4, \\
\bar{\Xi}_{21} &= \begin{bmatrix} \Gamma_1^{sT} & 0 & \Gamma_2^{sT} & 0 & \Gamma_3^{sT} & \Gamma_4^{sT} \\ \Gamma_1^{aT} & 0 & \Gamma_2^{aT} & 0 & \Gamma_3^{aT} & \Gamma_4^{aT} \end{bmatrix}, \\
\bar{\Xi}_{22} &= \text{diag} \{ R_d + S_d, 3S_d \}, \\
\Omega &= \tau_m^2 Z_2 + \delta Z_3, \quad \delta = \tau_M - \tau_m, \quad v = \tau_M / N, \\
\Gamma_i^s &= [\Gamma_{i1}^s \quad \Gamma_{i2}^s \quad \cdots \quad \Gamma_{iN}^s] \quad (i = 1, 2, 3, 4), \\
\Gamma_i^a &= [\Gamma_{i1}^a \quad \Gamma_{i2}^a \quad \cdots \quad \Gamma_{iN}^a] \quad (i = 1, 2, 3, 4), \\
\Gamma_{1i}^s &= Q_i - Q_{i-1} - \frac{v}{2} [\bar{A}^T (Q_i + Q_{i-1}) + (R_{i,N}^T + R_{i-1,N}^T)], \\
\Gamma_{1i}^a &= \frac{v}{2} [\bar{A}^T (Q_i - Q_{i-1}) + (R_{i,N}^T - R_{i-1,N}^T)], \\
\Gamma_{2i}^s &= \frac{v}{2} (\bar{B} \bar{F})^T (-Q_i - Q_{i-1}), \\
\Gamma_{2i}^a &= \frac{v}{2} (\bar{B} \bar{F})^T (Q_i - Q_{i-1}), \\
\Gamma_{3i}^s &= \frac{v}{2} (R_{i,0}^T + R_{i-1,0}^T), \\
\Gamma_{3i}^a &= \frac{v}{2} (R_{i-1,0}^T - R_{i,0}^T), \\
\Gamma_{4i}^s &= \frac{v}{2} \bar{E}^T (-Q_i - Q_{i-1}), \\
\Gamma_{4i}^a &= \frac{v}{2} \bar{E}^T (Q_i - Q_{i-1}), \\
R_d &= \begin{bmatrix} R_{d11} & R_{d12} & \cdots & R_{d1N} \\ R_{d21} & R_{d22} & \cdots & R_{d2N} \\ \vdots & \vdots & \ddots & \vdots \\ R_{dN1} & R_{dN2} & \cdots & R_{dNN} \end{bmatrix}, \\
R_{dij} &= v(R_{ij} - R_{i-1,j-1}) \quad (i, j = 1, 2, \dots, N), \\
S_d &= \text{diag} \{ S_1 - S_0, S_2 - S_1, \dots, S_N - S_{N-1} \}.
\end{aligned}$$

*Proof:* Using Lemma 5.2, we can see that the existence of the LKF (5.15) satisfying the conditions (5.13)-(5.14) can be ensured by the LMIs (5.19)-(5.22). Taking the time derivative of the LKF (5.15) on  $[kh+\tau_k, (k+1)h+\tau_{k+1})$  ( $\forall k \in \mathbb{Z}$ ) along the trajectory of the system (5.9), we have

$$\begin{aligned}
\dot{V}(t) = & \eta^T(t) e_1^T \int_{-\tau_M}^0 Q(\theta) \dot{\xi}(t+\theta) d\theta + \int_{-\tau_M}^0 \dot{\xi}^T(t+\theta) S(\theta) \xi(t+\theta) d\theta \\
& + \eta^T(t) (\bar{A}e_1 + \bar{B}\bar{F}e_3 + \bar{E}e_6)^T \int_{-\tau_M}^0 Q(\theta) \xi(t+\theta) d\theta \\
& + \int_{-\tau_M}^0 \int_{-\tau_M}^0 \dot{\xi}^T(t+\theta) R(\theta, s) \xi(t+s) d\theta ds \\
& + \frac{1}{2} \eta^T(t) [e_1^T (P\bar{A} + \bar{A}^T P + Z_1) e_1 + e_1^T P \bar{B} \bar{F} e_3] \eta(t) \\
& + \frac{1}{2} \eta^T(t) [e_3^T \bar{F}^T \bar{B}^T P e_1 + e_1^T P \bar{E} e_6 + e_6^T \bar{E}^T P e_1] \eta(t) \\
& - \frac{1}{2} \eta^T(t) [e_2^T Z_1 e_2 + (e_1 - e_4)^T Z_5 (e_1 - e_4)] \eta(t) \\
& + \frac{\tau_M - \tau(t)}{2} \left[ \eta^T(t) (e_1 - e_4)^T Z_5 \dot{\xi}(t) + \dot{\xi}^T(t) Z_5 (e_1 - e_4) \eta(t) \right] \\
& + \frac{1}{2} \dot{\xi}^T(t) [\tau_m^2 Z_2 + \delta Z_3 + (\tau_M - \tau(t)) Z_4] \dot{\xi}(t) \\
& - \frac{\tau_m}{2} \int_{t-\tau_m}^t \dot{\xi}^T(s) Z_2 \dot{\xi}(s) ds - \frac{1}{2} \int_{t-\tau(t)}^{t-\tau_m} \dot{\xi}^T(s) Z_3 \dot{\xi}(s) ds \\
& - \frac{1}{2} \int_{t-\tau_M}^{t-\tau(t)} \dot{\xi}^T(s) Z_3 \dot{\xi}(s) ds - \frac{1}{2} \int_{t-\bar{\tau}(t)}^t \dot{\xi}^T(s) Z_4 \dot{\xi}(s) ds. \tag{5.30}
\end{aligned}$$

Using Jensen integral inequality, we obtain

$$-\tau_m \int_{t-\tau_m}^t \dot{\xi}^T(s) Z_2 \dot{\xi}(s) ds \leq -\eta^T(t) (e_1 - e_2)^T Z_2 (e_1 - e_2) \eta(t). \tag{5.31}$$

Applying Lemma 2.2 to the term  $-\int_{t-\bar{\tau}(t)}^t \dot{\xi}^T(s) Z_4 \dot{\xi}(s) ds$  yields

$$\begin{aligned}
-\int_{t-\bar{\tau}(t)}^t \dot{\xi}^T(s) Z_4 \dot{\xi}(s) ds \leq & \eta^T(t) [(e_1 - e_4)^T X_1 + X_1^T (e_1 - e_4)] \eta(t) \\
& + (\tau(t) - \tau_m) \eta^T(t) X_1^T Z_4^{-1} X_1 \eta(t). \tag{5.32}
\end{aligned}$$

Similarly, the following inequalities hold

$$\begin{aligned}
-\int_{t-\tau(t)}^{t-\tau_m} \dot{\xi}^T(s) Z_3 \dot{\xi}(s) ds \leq & \eta^T(t) [(e_2 - e_3)^T X_2 + X_2^T (e_2 - e_3)] \eta(t) \\
& + (\tau(t) - \tau_m) \eta^T(t) X_2^T Z_3^{-1} X_2 \eta(t) \tag{5.33}
\end{aligned}$$

$$\begin{aligned}
-\int_{t-\tau_M}^{t-\tau(t)} \dot{\xi}^T(s) Z_3 \dot{\xi}(s) ds &\leq \eta^T(t) [(e_3 - e_5)^T X_3 + X_3^T (e_3 - e_5)] \eta(t) \\
&\quad + (\tau_M - \tau(t)) \eta^T(t) X_3^T Z_3^{-1} X_3 \eta(t). \tag{5.34}
\end{aligned}$$

Integrating by parts in (5.30) and applying the discretization scheme (5.16)-(5.18), one can see from (5.31)-(5.34) that

$$\begin{aligned}
\dot{V}(t) &\leq -\frac{1}{2} \eta^T(t) (\Xi_{11}(t) - 2\gamma^2 e_6^T e_6 + 2e_1^T \bar{C}^T U \bar{C} e_1) \eta(t) \\
&\quad - \frac{1}{2} \int_0^1 \tilde{\xi}^T(\alpha) S_d \tilde{\xi}(\alpha) d\alpha \\
&\quad - \eta^T(t) \int_0^1 [\Gamma^s + (1 - 2\alpha)\Gamma^a] \tilde{\xi}(\alpha) d\alpha \\
&\quad - \frac{1}{2} \int_0^1 \tilde{\xi}^T(\alpha) d\alpha R_d \int_0^1 \tilde{\xi}(\alpha) d\alpha \tag{5.35}
\end{aligned}$$

for  $t \in [kh + \tau_k, (k+1)h + \tau_{k+1})$  ( $\forall k \in \mathbb{Z}$ )

where

$$\begin{aligned}
\tilde{\xi}^T(\alpha) &= [\tilde{\xi}^T(\alpha_1) \ \tilde{\xi}^T(\alpha_2) \ \cdots \ \tilde{\xi}^T(\alpha_N)]^T, \\
\tilde{\xi}^T(\alpha_i) &= \xi^T(t - i\tau + \alpha\tau), \quad i = 1, 2, \dots, N, \\
\Xi_{11}(t) &= \bar{\Xi}_{11}^0 - \bar{\Xi}_{11}^1(t) - \bar{\Xi}_{11}^2(t), \\
\bar{\Xi}_{11}^1(t) &= (\tau(t) - \tau_m)(X_1^T Z_4^{-1} X_1 + X_2^T Z_3^{-1} X_2), \\
\bar{\Xi}_{11}^2(t) &= (\tau_M - \tau(t))(\bar{\Xi}_{11}^1 + X_3^T Z_3^{-1} X_3).
\end{aligned}$$

Applying Proposition 5.21 in [34] to the inequality (5.35), we have

$$\begin{aligned}
\dot{V}(t) &\leq -\frac{1}{2} \begin{bmatrix} \eta(t) \\ \int_0^1 \tilde{\xi}(\alpha) d\alpha \end{bmatrix}^T \begin{bmatrix} \bar{\Xi}_{11}(t) & \bar{\Xi}_{12} \\ \bar{\Xi}_{21} & \bar{\Xi}_{22} \end{bmatrix} \begin{bmatrix} \eta(t) \\ \int_0^1 \tilde{\xi}(\alpha) d\alpha \end{bmatrix} \\
&\quad + \eta^T(t) (\gamma^2 e_6^T e_6 - e_1^T \bar{C}^T U \bar{C} e_1) \eta(t) \tag{5.36}
\end{aligned}$$

for  $t \in [kh + \tau_k, (k+1)h + \tau_{k+1})$  ( $\forall k \in \mathbb{Z}$ ).

Let

$$\bar{\Xi}(t) = \begin{bmatrix} \bar{\Xi}_{11}(t) & \bar{\Xi}_{12} \\ \bar{\Xi}_{21} & \bar{\Xi}_{22} \end{bmatrix}, \quad \bar{\Xi}^i(t) = \begin{bmatrix} \bar{\Xi}_{11}^i(t) & \bar{\Xi}_{12} \\ \bar{\Xi}_{21} & \bar{\Xi}_{22} \end{bmatrix} \quad (i = 1, 2).$$

It is easy to see that  $\bar{\Xi}(t)$  is a convex combination of  $\bar{\Xi}^1(t)$  and  $\bar{\Xi}^2(t)$  on  $\tau(t) \in [\tau_m, \tau_M]$ . Using Schur complement to the LMIs (5.28)-(5.29), we obtain  $\bar{\Xi}(t) < 0$ . Then it follows from (5.36) that the condition (5.14) is obtained, which means that the  $H_\infty$  tracking performance (5.10) is ensured for the system (5.9) on  $[t_0, t_f)$ .

Next, we consider the asymptotic stability of the system (5.9) with  $\bar{\omega}(t) = 0$ . Taking the derivative of the LKF (5.15) along (5.9) with  $\bar{\omega}(t) = 0$ , we have

$$\dot{V}(t) \leq -\frac{1}{2} \begin{bmatrix} \tilde{\eta}(t) \\ \int_0^1 \tilde{\xi}(\alpha) d\alpha \end{bmatrix}^T \begin{bmatrix} \tilde{\Xi}_{11}(t) & \tilde{\Xi}_{12} \\ \tilde{\Xi}_{21} & \tilde{\Xi}_{22} \end{bmatrix} \begin{bmatrix} \tilde{\eta}(t) \\ \int_0^1 \tilde{\xi}(\alpha) d\alpha \end{bmatrix} \quad (5.37)$$

for  $t \in [kh + \tau_k, (k+1)h + \tau_{k+1})$  ( $\forall k \in \mathbb{Z}$ ). Applying Schur complement and the convex combination technique to the LMIs (5.28)-(5.29), we have

$$\begin{bmatrix} \tilde{\Xi}_{11}(t) & \tilde{\Xi}_{12} \\ \tilde{\Xi}_{21} & \tilde{\Xi}_{22} \end{bmatrix} < 0 \quad (5.38)$$

where

$$\begin{aligned} \tilde{\Xi}_{11}(t) &= \begin{bmatrix} e_1^T & e_2^T & e_3^T & e_4^T & e_5^T \end{bmatrix}^T \Xi_{11}(t) \begin{bmatrix} e_1^T & e_2^T & e_3^T & e_4^T & e_5^T \end{bmatrix}, \\ \tilde{\Xi}_{21} &= \begin{bmatrix} \Gamma_1^{sT} & 0 & \Gamma_2^{sT} & 0 & \Gamma_3^{sT} \\ \Gamma_1^{aT} & 0 & \Gamma_2^{aT} & 0 & \Gamma_3^{aT} \end{bmatrix}. \end{aligned}$$

From (5.37)-(5.38), one can see that there exists an  $\varepsilon_3 > 0$  such that  $\dot{V}(\xi_t) \leq -\varepsilon_3 \|\xi(t)\|^2 < 0$  for  $t \in [kh + \tau_k, (k+1)h + \tau_{k+1})$  ( $\forall k \in \mathbb{Z}$ ). Using Lemma 5.1, one can conclude that the system (5.9) with  $\bar{\omega}(t) = 0$  is asymptotically stable, which completes the proof.

Notice that the network-based static output feedback tracking control objective is to search for the gains  $F_1$  and  $F_2$  such that the  $H_\infty$  tracking performance  $\gamma$  is minimized for the system (5.9). For given  $\tau_m, \tau_M, U, F_1$  and  $F_2$ , one can employ Proposition 5.1 to determine the minimum  $\gamma$ , which can be obtained by solving the following optimization problem:

$$\begin{aligned} &\text{minimize} && \gamma \\ &\text{subject to} && P > 0, Z_i > 0 \ (i=1, 2, 3, 4, 5), S_i > 0 \ (i=0, 1, \dots, N), \\ & && \text{and LMIs (5.22), (5.28) - (5.29).} \end{aligned}$$

**Remark 5.1.** A new discontinuous complete LKF candidate (5.15), which includes the lower bound of the network-induced delay  $\tau_m$ , the sawtooth delay  $\tau(t)$  and its upper bound  $\tau_M$ , is constructed to derive a delay-dependent criterion for  $H_\infty$  tracking performance analysis of the system (5.9). Notice that the derivation involves a coupling property between the present state  $\xi(t)$  and the past state  $\xi(t - \tau_M)$ , the division of delay interval  $[-\tau_M, 0]$ , and the inherent piecewise-linear time-varying delay information  $\dot{\tau}(t) = 1$  on  $[kh + \tau_k, (k + 1)h + \tau_{k+1})$  ( $\forall k \in \mathbb{Z}$ ). So it is expected that the derived criterion is of less conservatism.

**Remark 5.2.** In [28] and [139], network-based output tracking control for linear systems via a state feedback controller is studied. In many practical situations, it is physically difficult to measure all the process variables of a system. Considering that a static output feedback controller can be easily implemented with low cost, we investigate network-based static output feedback tracking control for the system (5.1). Notice that the existing simple LKFs in [28] and [139] *can not be employed* for  $H_\infty$  tracking performance analysis of the system (5.9) because they require the system (5.1) with  $\omega(t) = 0$  can be stabilized by a static output feedback controller without a time-delay, see [40]. Instead, the complete LKF (5.15) is constructed to analyze the  $H_\infty$  tracking performance of the system (5.9). By using the LKF (5.15) with the discretization scheme (5.16)-(5.18), Proposition 5.1 is established to judge whether the system (5.9) is asymptotically stable with a prescribed  $H_\infty$  tracking performance.

For given control gains  $F_1, F_2$  and delay bounds  $\tau_m, \tau_M$ , one can employ Proposition 5.1 to judge whether an  $H_\infty$  tracking performance is ensured for the augmented system (5.9), which includes the system (5.1) and the reference model (5.4). Notice that the reference model (5.4) is asymptotically stable and the system (5.1) under consideration can not be stabilized by a non-delayed static output feedback controller, but can be stabilized by a delayed static output feedback controller. As a

result, from the stability point of view, there exist a lower delay bound  $\tau_{min}$  and an upper delay bound  $\tau_{max}$  such that the system (5.9) with an interval time-varying delay  $\tau(t)$  satisfying (5.7) is asymptotically stable with a prescribed  $H_\infty$  tracking performance. By intentionally inserting a communication network between the system (5.1) and the controller (5.5), network-induced delays  $\tau_k$  satisfying  $0 < \tau_m \leq \tau_k \leq \tau_M - h$  ( $\forall k \in \mathbb{N}$ ) are produced to make the interval  $[\tau_m, \tau_M - h]$  fall into  $(\tau_{min}, \tau_{max} - h)$ , then a stable tracking control performance can be achieved. In this study, Proposition 5.1 is a sufficient delay-dependent criterion and it can be applied to search for some local optimal values  $\tau_{min}^p$  and  $\tau_{max}^p$ . The specific steps are given by

*Step 1:* Given the  $H_\infty$  tracking performance  $\gamma$  and the sampling period  $h$ . Choose  $\tau_m = \tau_{min}^{ini}$  and  $\tau_M = \tau_{max}^{ini}$  such that (5.19)-(5.22), (5.28) and (5.29);

*Step 2:* Let  $\tau_m = \tau_m - \delta_1$  and  $\tau_M = \tau_M + \delta_2$ , where  $\delta_i$  ( $i = 1, 2$ ) are two step lengths;

*Step 3:* For given  $\tau_m$  and  $\tau_M$ , if the LMIs (5.19)-(5.22), (5.28) and (5.29) are satisfied,  $\tau_{min}^p = \tau_m$  and  $\tau_{max}^p = \tau_M$ , then go to Step 2; otherwise go to Step 4;

*Step 4:* Output the local optimal values  $\tau_{min}^p$  and  $\tau_{max}^p$ .

We consider the case that the system (5.1) involves time-varying parameter uncertainties  $\Delta A(t)$  and  $\Delta B(t)$  satisfying

$$[\Delta A(t) \ \Delta B(t)] = GF(t)[H_a \ H_b] \quad (5.40)$$

where  $G$ ,  $H_a$  and  $H_b$  are some constant matrices of appropriate dimensions,  $F(t)$  is an unknown time-varying matrix function with Lebesgue measurable elements and  $F^T(t)F(t) \leq I$  ([41], [42], [56], [57], [58]). For derivation purpose, we have

$$[\Delta \bar{A}(t) \ \Delta \bar{B}(t)] = GF(t)[\bar{H}_a \ \bar{H}_b] \quad (5.41)$$

where

$$\bar{H}_a = \begin{bmatrix} H_a & 0 \\ 0 & 0 \end{bmatrix}, \quad \bar{H}_b = \begin{bmatrix} H_b \\ 0 \end{bmatrix}.$$

Similar to the process in [41] and [42], we obtain the following criterion

**Proposition 5.2.** For given positive scalars  $\gamma$ ,  $\tau_m$  and  $\tau_M$ , a weighting matrix  $U > 0$  and control gains  $F_1$  and  $F_2$ , the system (5.9) with the uncertainties (5.41) is asymptotically stable with the  $H_\infty$  tracking performance (5.10), if there exist a positive scalar  $\lambda$ , symmetric matrices  $P > 0$ ,  $Z_i > 0$  ( $i = 1, 2, 3, 4, 5$ ),  $S_i > 0$  ( $i = 0, 1, \dots, N$ ), and  $X_1, X_2, X_3, Q_i, R_{ij}$  ( $i, j = 0, 1, \dots, N$ ) such that (5.22) and

$$\begin{bmatrix} \bar{\Xi}_{11}^0 + \Delta^0 & * & * & * & * \\ \bar{\Xi}_{21} & \bar{\Xi}_{22} & * & * & * \\ \bar{\Xi}_{31} & \bar{\Xi}_{32} & \lambda I & * & * \\ -\delta X_2 & 0 & 0 & \delta Z_3 & * \\ -\delta X_1 & 0 & 0 & 0 & \delta Z_4 \end{bmatrix} > 0 \quad (5.42)$$

$$\begin{bmatrix} \bar{\Xi}_{11}^0 + \delta \bar{\Xi}_{11}^1 + \Delta^1 & * & * & * \\ \bar{\Xi}_{21} & \bar{\Xi}_{22} & * & * \\ \bar{\Xi}_{31} & \bar{\Xi}_{32} & \lambda I & * \\ -\delta X_3 & 0 & 0 & \delta Z_3 \end{bmatrix} > 0 \quad (5.43)$$

where  $\bar{\Xi}_{11}^0, \bar{\Xi}_{11}^1, \bar{\Xi}_{21}$  and  $\bar{\Xi}_{22}$  can be found in Proposition 5.1 and

$$\Delta^0 = -\lambda e_1^T \bar{H}_a^T \bar{H}_a e_1 - \lambda e_1^T \bar{H}_a^T \bar{H}_b e_3 - \lambda e_3^T \bar{H}_b^T \bar{H}_a e_1 - \lambda e_3^T \bar{H}_b^T \bar{H}_b e_3,$$

$$\Delta^1 = -\lambda e_1^T \bar{H}_a^T \bar{H}_a e_1 - \lambda e_1^T \bar{H}_a^T \bar{H}_b e_3 - \lambda e_3^T \bar{H}_b^T \bar{H}_a e_1 - \lambda e_3^T \bar{H}_b^T \bar{H}_b e_3,$$

$$\bar{\Xi}_{31} = [e_1^T G^T P \quad 0 \quad 0 \quad 0 \quad 0],$$

$$\bar{\Xi}_{32} = [e_1^T G^T Q^s \quad -e_1^T G^T Q^a \quad \tau_m e_1^T G^T Z_2 \quad \delta e_1^T G^T Z_3 \quad 0]^T,$$

$$Q^s = [\frac{\nu}{2}(Q_1 + Q_0) \quad \frac{\nu}{2}(Q_2 + Q_1), \dots, \frac{\nu}{2}(Q_N + Q_{N-1})],$$

$$Q^a = [\frac{\nu}{2}(Q_1 - Q_0) \quad \frac{\nu}{2}(Q_2 - Q_1), \dots, \frac{\nu}{2}(Q_N - Q_{N-1})].$$

*Proof:* The proof is a routine case and omitted, see [41] and [42].

**Remark 5.3.** Notice that the delay-dependent criteria, i.e., Proposition 5.1 and Proposition 5.2, derived by using the complete LKF (5.15) can be used to judge whether the prescribed  $H_\infty$  tracking performance  $\gamma$  is ensured not only for a system that can not be stabilized by a non-delayed static output feedback controller, but can be stabilized by a delayed static output feedback controller (see, a damped harmonic oscillator, a structural system and an internal combustion engine [1], [34],

[84] and [132]), but also for a system that can be stabilized by a non-delayed static output feedback controller (see for example, the aircraft in [27]).

### 5.3 Static output feedback tracking control design

In this section, we are interested in designing a network-based static output feedback tracking controller such that the system (5.9) is asymptotically stable with a desired  $H_\infty$  tracking performance. It is pointed out in [20] that delay-dependent criteria for the existence of a controller (in both output feedback and state feedback cases) are expressed in terms of nonlinear matrix inequalities. To establish an LMI-based design result, it is inevitable to introduce some linearization techniques such as equality or inequality constraints ([29], [28], [54], [137], [139], [149]) and iterative algorithms ([28], [29], [57]), which bring some conservatism. Notice that less conservative  $H_\infty$  tracking performance criteria and control design methods can provide an appropriate tradeoff between the maximum allowable delay and the minimum  $H_\infty$  tracking performance for a network-based control system (NCS) ([28], [54], [137], [139]). Accordingly, it is of importance to develop a new design method without using the linearization techniques, which can determine output feedback gains by solving an optimization problem of an  $H_\infty$  tracking performance.

On the other hand, a particle swarm optimization (PSO) technique plays a key role in solving complex design optimization problems due to its global search ability, easy implementation, stable convergence characteristic and computational efficiency ([18], [62], [130]). In the PSO technique, its population is called a swarm and each individual is called a particle. Each particle evolves to an optimal solution in the multidimensional search space, adjusting its position by its own experience and that of neighboring particles. A particle therefore makes use of best position encountered by itself and that of its neighbors to position itself toward an optimal solution. The

status of a particle is characterized by two factors: its velocity and position, which are updated by the following equations

$$\nu_{ij}(k+1) = \omega\nu_{ij}(k) + c_1r_1(\text{pbest}_{ij}(k) - f_{ij}(k)) + c_2r_2(\text{gbest}_j(k) - f_{ij}(k)) \quad (5.44)$$

$$f_{ij}(k+1) = f_{ij}(k) + \nu_{ij}(k+1) \quad (5.45)$$

$$\omega = (\omega_M - \omega_m)(m_\eta - c_\eta)/m_\eta + \omega_m \quad (5.46)$$

for  $i = 1, 2, \dots, n_p$  and  $j = 1, 2, \dots, d$ , where  $n_p$  and  $d$  are the number of particles in a group and the number of members in a particle, respectively;  $\nu_{ij}(k)$  is the  $j$ th dimensional velocity of the  $i$ th particle at iteration  $k$ , and  $\nu_j^{\min} \leq \nu_{ij}(k) \leq \nu_j^{\max}$ ;  $f_{ij}(k)$  is the  $j$ th dimensional position of the  $i$ th particle at iteration  $k$ ;  $\text{pbest}_i = (\text{pbest}_{i1}, \text{pbest}_{i2}, \dots, \text{pbest}_{id})$  is the previous best position of the  $i$ th particle;  $\text{gbest}$  is the global best position of the group;  $r_1$  and  $r_2$  are two random numbers uniformly distributed in  $[0, 1]$ ;  $c_1$  and  $c_2$  are two acceleration coefficients;  $\omega$  is the inertia weight;  $\omega_M$  and  $\omega_m$  represent the maximum and minimum inertia weight, respectively;  $m_\eta$  is the maximum number of iterations and  $c_\eta$  is the current number of iterations.

Recently, the PSO technique has been applied to design a proportional-integral-derivative controller that minimizes the  $H_\infty$  performance for traditional point-to-point systems in a frequency domain ([64], [151]). However, the potential of a PSO technique in finding the solutions of a network-based tracking controller has not been explored. Therefore, we will try to propose a design algorithm of a network-based static output feedback tracking controller by applying the PSO technique with the feasibility of Proposition 5.1. The algorithm is given by

**Algorithm 5.1.** Step 1. Initialization

- 1.1 Randomly initialize a group with  $n_p$  particles. Each particle consists of members  $f_{ij}(0)$  in  $F_1^0$  and  $F_2^0$ ; and  $f_{ij}(0)$  lies in the range  $[\alpha_j, \beta_j]$ , where  $i = 1, 2, \dots, n_p$ , and  $j = 1, 2, \dots, 2ml$ ;

- 1.2 Initialize parameters  $c_1, c_2, \omega_M, \omega_m, m_\eta, \nu_j^{min}$  and  $\nu_j^{max}$ , where  $j = 1, 2, \dots, 2ml$ ;
- 1.3 Initialize the velocity of  $n_p$  particles and  $\nu_j^{min} \leq \nu_{ij}(0) \leq \nu_j^{max}$ , where  $i = 1, 2, \dots, n_p$  and  $j = 1, 2, \dots, 2ml$ , and set  $k = 0$ ;
- 1.4 Initialize the fitness value  $\gamma_i^0 = l_p$ , where  $l_p$  is a positive constant and  $i = 1, 2, \dots, n_p$ . Solve the minimization problem mentioned above Remark 5.1 to obtain the minimum  $\gamma_i^0$  for given  $F_{1i}^0$  and  $F_{2i}^0$ ,  $i \in \{1, 2, \dots, n_p\}$ ;
- 1.4.1 Assign the minimum  $\gamma_i^0$  to  $\gamma_{ip}$  and  $f_{ij}(0)$  to  $pbest_{ij}$ , respectively, where  $\gamma_{ip}$  is the fitness value of the particle  $pbest$ ,  $i = 1, 2, \dots, n_p$ , and  $j = 1, 2, \dots, 2ml$ , and set  $k = 0$ ;
- 1.4.2 Assign  $\min_i \{\gamma_i^0 \mid i \in \{1, 2, \dots, n_p\}\}$  to  $\gamma_g^0$  and  $f_{gj}(0)$  to  $gbest_j(0)$ , respectively, where  $\gamma_g^0$  is the fitness value of the particle  $gbest$ , and  $j = 1, 2, \dots, 2ml$  and set  $k = 0$ ;

Step 2. Fitness evaluation of particles

- 2.1 Obtain  $F_{1i}^k$  and  $F_{2i}^k$  from  $f_{ij}(k)$  in  $n_p$  particles, where  $i = 1, 2, \dots, n_p$ ,  $j = 1, 2, \dots, 2ml$ , and  $k \geq 1, k \in \mathbb{N}$ ;
- 2.2 Solve the minimization problem mentioned above Remark 5.1 to obtain the minimum  $\gamma_i^k$  for given  $F_{1i}^k$  and  $F_{2i}^k$ , where  $i \in \{1, 2, \dots, n_p\}$ , and  $k \geq 1, k \in \mathbb{N}$ ;
- 2.3 Record the previous best particles and their fitness values. If  $\gamma_i^k < \gamma_{ip}$ , then assign  $\gamma_i^k$  to  $\gamma_{ip}$  and  $f_{ij}(k)$  to  $pbest_{ij}$ , respectively, where  $i = 1, 2, \dots, n_p$ ,  $j = 1, 2, \dots, 2ml$ , and  $k \geq 1, k \in \mathbb{N}$ ;
- 2.4 Record the global best particle and its fitness value. If  $\gamma_g^{k-1} > \min\{\gamma_i^k\}_{i=1}^{n_p}$ , then assign  $\min_i \{\gamma_i^k \mid i \in \{1, 2, \dots, n_p\}\}$  to  $\gamma_g^k$  and  $f_{gj}(k)$  to  $gbest_j(k)$ , respectively; otherwise, assign  $\gamma_g^{k-1}$  to  $\gamma_g^k$  and store the corresponding particle, where  $j = 1, 2, \dots, 2ml$ , and  $k \geq 1, k \in \mathbb{N}$ . If  $|\gamma_g^k - \gamma_g^{k-1}| \leq \varepsilon$  is satisfied within  $m_\eta$

iterations, where  $\varepsilon > 0$  is a sufficiently small constant, then exit, and  $k \geq 1$ ,  $k \in \mathbb{N}$ ; otherwise, go to Step 2.5;

2.5 Update the velocity of  $n_p$  particles by (5.44) and (5.46). If  $\nu_{ij}(k) < \nu_j^{min}$  or  $\nu_{ij}(k) > \nu_j^{max}$ , then randomly generate  $\nu_{ij}(k)$  satisfying  $\nu_j^{min} \leq \nu_{ij}(k) \leq \nu_j^{max}$ , where  $i = 1, 2, \dots, n_p$ ,  $j = 1, 2, \dots, 2ml$ , and  $k \geq 1$ ,  $k \in \mathbb{N}$ ;

2.6 Update the position of  $n_p$  particles by (5.45). If  $f_{ij}(k) < \alpha_j$  or  $f_{ij}(k) > \beta_j$ , then randomly generate  $f_{ij}(k)$  satisfying  $\alpha_j \leq f_{ij}(k) \leq \beta_j$ , where  $i = 1, 2, \dots, n_p$ ,  $j = 1, 2, \dots, 2ml$ , and  $k \geq 1$ ,  $k \in \mathbb{N}$ ;

2.7 If  $k > m_\eta$ , where  $m_\eta$  is the maximum number of iterations, then exit; otherwise, set  $k = k + 1$  and go to Step 2.1;

Step 3. Obtain the minimum  $\gamma_g > 0$  and the corresponding  $F_1$  and  $F_2$  from the global best particle.

**Remark 5.4.** Applying a PSO technique with the feasibility of Proposition 5.1, Algorithm 5.1 is provided to search for the minimum  $H_\infty$  tracking performance  $\gamma_{min}$  and corresponding gains  $F_1$  and  $F_2$ . The total row size of the LMIs of Proposition 5.1 is  $\mathcal{S} = (4N + 12)p + q$  and the total number of scalar decision variables is  $\mathcal{N} = \frac{1}{2}[(N^2 + 5N + 40)p^2 + (2N + 8 + 6q)p]$ . Thus, the computational complexity of Algorithm 5.1 is proportional to  $\mathcal{A}\mathcal{S}\mathcal{N}^3$ , where  $\mathcal{A} = (nm + nv)m_\eta n_p$ .

**Remark 5.5.** The convergence of Algorithm 5.1 is influenced by some tuning parameters  $n_p$ ,  $c_1$ ,  $c_2$ ,  $\omega_M$ ,  $\omega_m$ ,  $m_\eta$ ,  $\nu_j^{min}$  and  $\nu_j^{max}$  ( $j = 1, 2, \dots, 2ml$ ). A complete theoretical analysis of selecting these parameters to cause convergence of a PSO algorithm is given in [18] and [130]. In this study, the tuning parameters are selected properly to ensure the convergence of Algorithm 5.1 according to [18] and [130]. Moreover, the search spaces  $[\alpha_i, \beta_i]$  ( $i = 1, 2, \dots, 2ml$ ) have an influence on the convergence speed of Algorithm 5.1. Some basic ideas of how to determine  $\alpha_i$  and  $\beta_i$  in

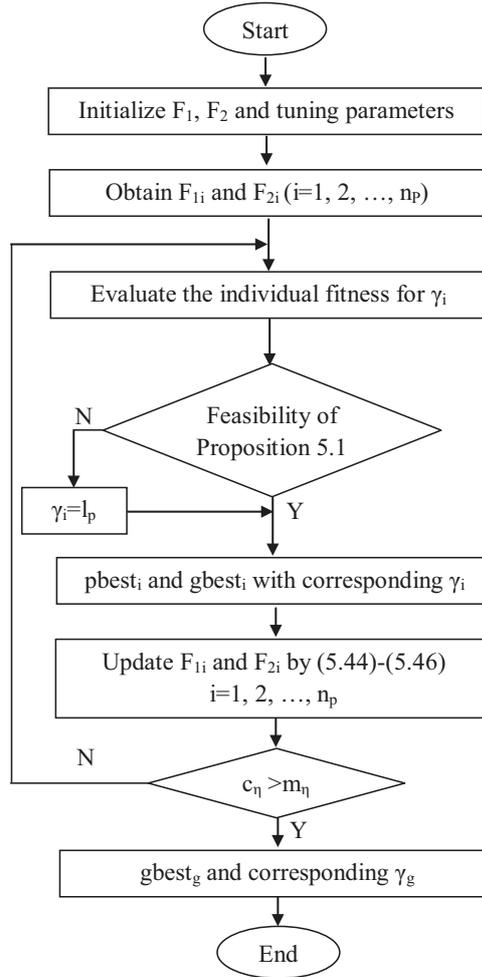


Figure 5.1: The flowchart of Algorithm 5.1

Algorithm 5.1 are provided as follows. For the system that can not be stabilized by a non-delayed static output feedback controller, but can be stabilized by a delayed static output feedback controller, one can obtain some rough estimates of  $[\alpha_i, \beta_i]$  referring to a pole placement theory [85]; while for the system that can be stabilized by a non-delayed static output feedback controller, one can estimate  $[\alpha_i, \beta_i]$  by some traditional control design strategies in a non-delayed feedback setting ([9], [27]).

**Remark 5.6.** The flowchart of Algorithm 5.1 is shown in Figure 5.1. It can be seen from Figure 5.1 that the evolutionary process will end when the search process converges in a given  $H_\infty$  tracking performance or will repeat for  $n_p$  particles until the maximum number of iterations  $m_\eta$  is reached. If the search process is ended by

$l_p$  and the conditions of Proposition 5.1 are not satisfied for given  $l_p$ ,  $F_{1g}^k$  and  $F_{2g}^k$  ( $k \in \mathbb{N}$ ), which means that no optimal solution is found, then one can adjust the search spaces or initialize the tuning parameters for another new search.

**Remark 5.7.** Unlike the frequency domain method in [64] and [151], an LMI-based criterion in the time domain is proposed to judge whether the system (5.9) is asymptotically stable with a prescribed  $H_\infty$  tracking performance. By using the feasibility of the LMI-based criterion, the proposed method facilitates to indicate the potential of an evolutionary process in the PSO technique. Moreover, using the proposed method, it is not required to specify the unknown external disturbance  $\omega(t)$  and the reference input  $r(t)$  to formulate the transfer function of the augmented system (5.9).

## 5.4 An example

In this section, we will show the positive effect of a network-induced delay on network-based output tracking control of the harmonic oscillator (5.2).

Consider the oscillator (5.2) with parameters  $\omega_0 = \sqrt{2}$ ,  $\zeta = -\sqrt{2}/40$  and an external disturbance  $\omega(t)$ , which is represented by the following state-space form

$$\begin{cases} \dot{x}(t) = \begin{bmatrix} 0 & 1 \\ -2 & 0.1 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t) + \begin{bmatrix} 0 \\ 0.1 \end{bmatrix} \omega(t) \\ y(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} x(t). \end{cases} \quad (5.47)$$

And the reference model is given by

$$\begin{cases} \dot{x}_r(t) = -x_r(t) + 0.2r(t) \\ y_r(t) = 0.5x_r(t). \end{cases} \quad (5.48)$$

Notice that the system (5.47) can not be stabilized by a static output feedback controller without a time-delay, but can be stabilized by a delayed static output feedback controller. So it is not possible to fulfil the output tracking control task by the controller  $\hat{u}(t) = f_1 y(t) + f_2 y_r(t)$ , where  $f_i$  ( $i = 1, 2$ ) are output feedback gains. To achieve a stable and satisfactory tracking effect, we intentionally insert a network

that induces communication delays ( $\tau_k^{sc}$  and  $\tau_k^{ca}$ ,  $\forall k \in \mathbb{Z}$ ) between the system (5.47) and the controller  $\hat{u}(t)$ . Following the process of (5.6) and (5.8), the input of the system (5.47) in a network environment can be described by

$$\begin{cases} u(t) = f_1 y(t - \tau(t)) + f_2 y_r(t - \tau(t)) \\ t \in [kh + \tau_k, (k+1)h + \tau_{k+1}), k \in \mathbb{Z} \end{cases} \quad (5.49)$$

where  $\tau(t) = t - kh$ ,  $\tau_k = \tau_k^{sc} + \tau_k^{ca}$ ,  $\forall k \in \mathbb{Z}$ .

Using (5.47), (5.48) and (5.49), we obtain the system (5.9) with the following matrices

$$\begin{aligned} \bar{A} &= \begin{bmatrix} 0 & 1 & 0 \\ -2 & 0.1 & 0 \\ 0 & 0 & -2 \end{bmatrix}, \quad \bar{B} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \quad \bar{E} = \begin{bmatrix} 0 & 0 \\ 0.1 & 0 \\ 0 & 0.2 \end{bmatrix}, \\ \bar{F} &= [f_1 \quad 0 \quad 0.5f_2], \quad \bar{C} = [1 \quad 0 \quad -0.5]. \end{aligned}$$

First, we show how to schedule network-induced delays  $\tau_k$  ( $\forall k \in \mathbb{N}$ ) to improve tracking control performance. Given  $f_1 = 1.2$  and  $f_2 = 0.6$ , one can see that the eigenvalues of the matrix  $\bar{A} + \bar{B}\bar{F}$  are  $0.5000 \pm 0.8930i$  and  $-1.0000$ , which means that the system (5.9) is unstable if there is no network between the system (5.47) and the controller  $\hat{u}(t)$ . For given  $\gamma = 1$ ,  $h = 0.05s$ ,  $\tau_{min}^{ini} = 0.4501s$ ,  $\tau_{max}^{ini} = 0.5595s$ ,  $\delta_1 = \delta_2 = 0.0001$ ,  $N = 1$  and  $W = 1$ , using steps above Proposition 5.2, one can obtain  $(\tau_{min}^p, \tau_{max}^p - h) = (0.3489s, 0.6571s)$ . Then purposefully producing network-induced delays  $0.3490s \leq \tau_k \leq 0.6570s$  ( $\forall k \in \mathbb{N}$ ) in the static feedback control can ensure a given  $H_\infty$  tracking performance  $\gamma = 1$ . In addition, applying Proposition 5.1 for different  $N$ ,  $\tau_{min}^{ini}$  and  $\tau_{max}^{ini}$ , we obtain the interval  $(\tau_{min}^p, \tau_{max}^p - h)$ , which is listed in Table 5.1. From Table 5.1, one can clearly see that the interval  $(\tau_{min}^p, \tau_{max}^p - h)$  is magnified as  $N$  increases, which shows that the conservatism of Proposition 5.1 is reduced significantly by increasing  $N$ . Depending on the range of  $\tau_k$  ( $\forall k \in \mathbb{N}$ ) produced in the feedback control, one can choose the corresponding delay interval in Table 5.1. For example, if  $\tau_m > 1s$ , one can schedule  $\tau_k$  ( $\forall k \in \mathbb{N}$ ) into the interval shown in the last row of Table 5.1.

Table 5.1:  $(\tau_{min}^p, \tau_{max}^p - h)$  for different  $N$  and given  $h = 0.05s$

N=1	N=2	N=3
(0.1839, 0.2347)	(0.1814, 0.2372)	(0.1810, 0.2376)
(0.3489, 0.7071)	(0.2940, 0.8020)	(0.2755, 0.8205)
(1.2172, 1.2674)	(1.1414, 1.3432)	(1.1228, 1.3618)

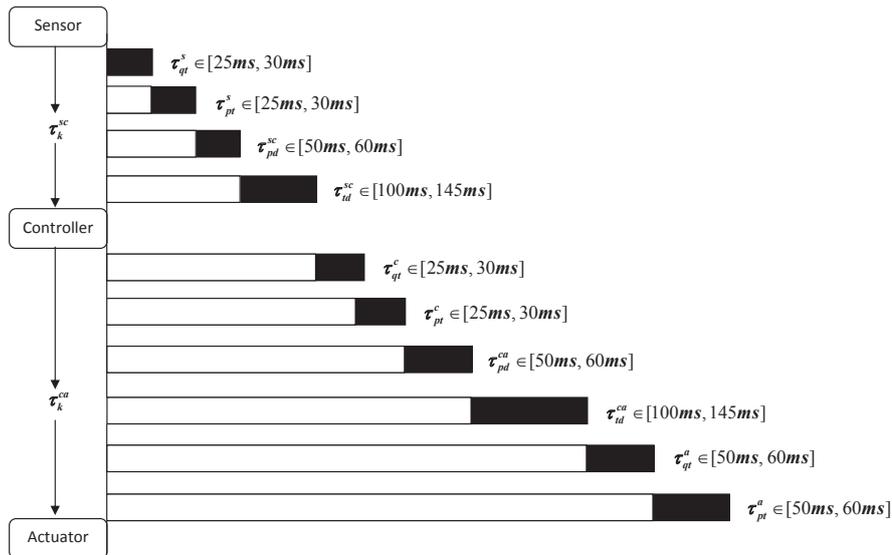


Figure 5.2: Ranges for the delays in the feedback control loop

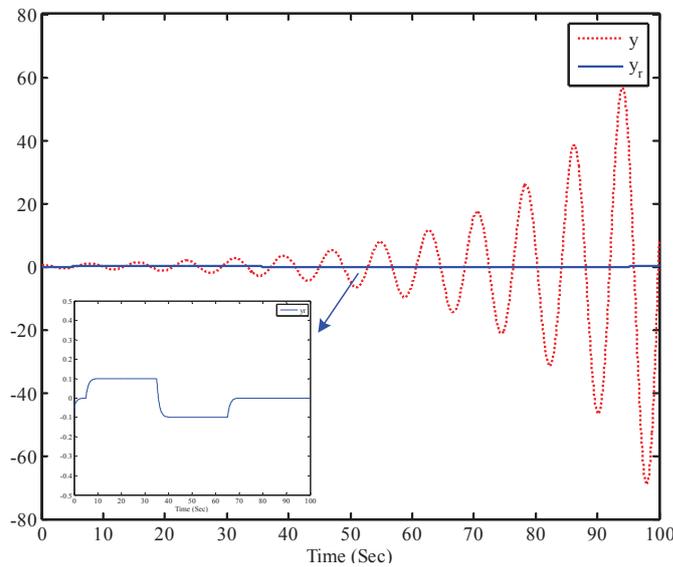


Figure 5.3: The outputs of the system (5.47)-(5.48) in Case I

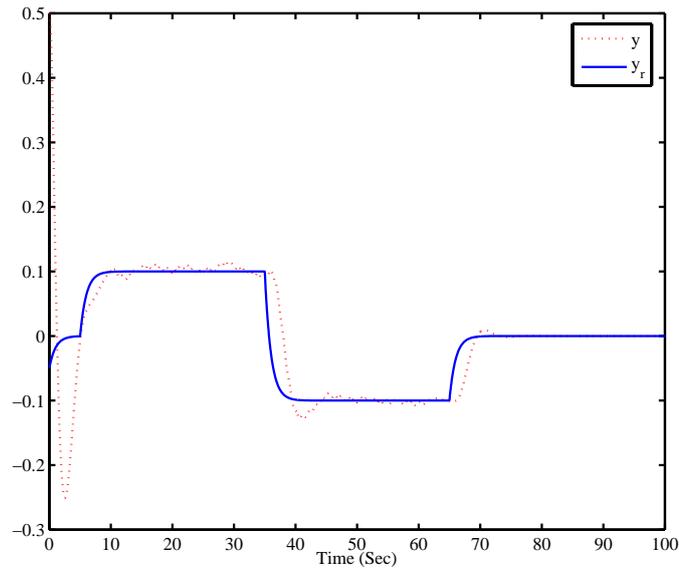


Figure 5.4: The outputs of the system (5.47)-(5.48) in Case II

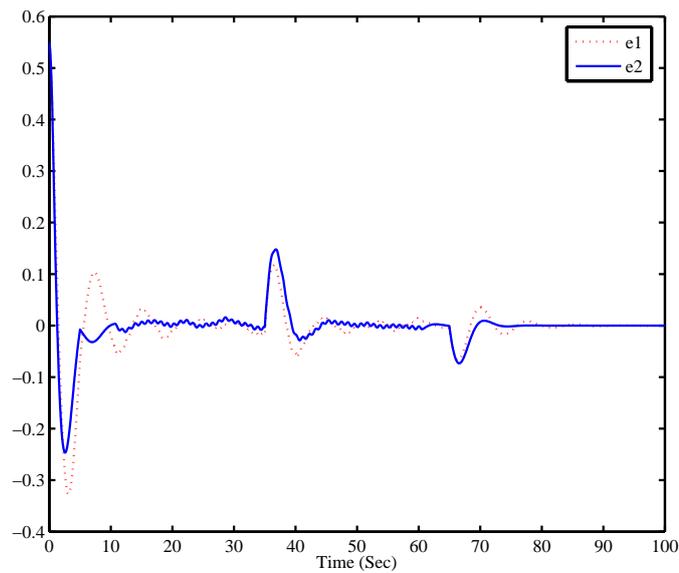


Figure 5.5: The comparison for the output tracking errors of the system (5.47)-(5.48) in Case II and Case III

Second, we design the network-based tracking controller by using Algorithm 5.1 for given  $\tau_m$  and  $\tau_M$ . The search spaces of  $f_i$  are roughly set to be  $[\alpha_i, \beta_i] = [-2, 2]$  ( $i = 1, 2$ ) and the tuning parameters are initialized to be  $n_p = 20$ ,  $c_i = 2$ ,  $\omega_M = 0.7000$ ,  $\omega_m = 0.4000$ ,  $m_\eta = 20$ ,  $\nu_i^{min} = 0$  and  $\nu_i^{max} = 0.4000$  ( $i = 1, 2$ ), which satisfy

the convergence of a PSO algorithm. Given  $\tau_m = 0.5000s$ ,  $\tau_M = 0.7000s$ ,  $N = 1$  and  $W = 1$ , use Algorithm 5.1 to obtain the minimum  $H_\infty$  tracking performance  $\gamma_{min} = 0.3259$  and the corresponding gains  $f_1 = 1.3542$  and  $f_2 = 0.6514$ . By inserting a network that induces network delays  $\tau_k$  ( $\forall k \in \mathbb{N}$ ) satisfy  $0.5000s \leq \tau_k \leq 0.6500s$  between the system (5.47) and the controller  $\hat{u}(t)$  with  $f_1 = 1.3542$  and  $f_2 = 0.6514$ , an  $H_\infty$  tracking performance  $\gamma_{min} = 0.3259$  can be ensured.

Third, we demonstrate the positive effects of network-induced delays on the tracking control performance. In simulation, the sampling period is  $h = 0.05s$  and the initial states of the system (5.47)-(5.48) are chosen as  $x(t_0) = [0.5 \ 0]^T$ ,  $x_r(t_0) = -0.1$ . It is assumed that

$$\omega(t) = \begin{cases} \sin(6t), & 10s \leq t \leq 60s, \\ 0, & \text{otherwise,} \end{cases}$$

$$r(t) = \begin{cases} 1, & 5s \leq t \leq 35s, \\ -1, & 35s < t \leq 65s, \\ 0, & \text{otherwise.} \end{cases}$$

We depict the output responses of the systems (5.47)-(5.48) controlled by the controller  $\hat{u}(t)$  with  $f_1 = 1.3542$  and  $f_2 = 0.6514$  in the following two cases, i.e., Case 1: no network connection and Case 2: network-induced delays  $\tau_k$  ( $\forall k \in \mathbb{N}$ ) vary in  $[0.5000s, 0.6500s]$ , where the specific delay ranges are shown in Fig. 5.2. Fig. 5.3 shows that the output of the system (5.47) is unstable if there is no network connecting the system (5.47) with the controller (??). Fig. 5.4 and Fig. 5.5 show the output and the tracking error of the system (5.47)-(5.48) in Case 2, respectively. It can be seen from Fig. 5.4 and Fig. 5.5 that network-induced delays are purposefully introduced to improve the tracking control performance.

Last, one can conclude that it is impossible to achieve the objective of output tracking control for the oscillator (5.47) by using the non-delayed static output feedback controller. However, purposefully producing appropriate network-induced delays in the feedback control loop can achieve a stable and satisfactory tracking control for the oscillator (5.47).

## 5.5 Summary

We have studied network-based static output feedback tracking control for a system that can not be stabilized by a static output feedback controller without a time-delay, but can be stabilized by a delayed static output feedback controller. For such a system, we have shown that purposefully producing a network-induced delay in the feedback control loop can achieve a stable and satisfactory tracking control. A new discontinuous complete Lyapunov-Krasovskii functional has been constructed to establish a delay-dependent criterion for  $H_\infty$  tracking performance analysis. We have proposed a novel control design algorithm by applying a particle swarm optimization technique with feasibility of the obtained criterion. The effectiveness of the proposed method has been illustrated by an example.



## Chapter 6

# Network-based fuzzy static output feedback tracking control using asynchronous constraints

### 6.1 Introduction

Consider the following nonlinear control system

$$\begin{cases} \dot{x}(t) = f(x(t))x(t) + g_1(x(t))u(t) + g_2(x(t))\omega(t) \\ y(t) = Cx(t) \end{cases} \quad (6.1)$$

where  $x(t) \in \mathbb{R}^n$ ,  $u(t) \in \mathbb{R}^m$  and  $y(t) \in \mathbb{R}^l$  are the state, the input and the output, respectively;  $\omega(t) \in \mathbb{R}^v$  denotes the external disturbance and  $\omega(t) \in L_2[t_0, \infty)$ ;  $C$  is a constant matrix;  $f(x(t))$  and  $g_i(x(t))$  ( $i = 1, 2$ ) are continuous functions over the compact region  $\mathbb{D}$ , and  $f(0) = 0$ ,  $g_i(0) = 0$  ( $i = 1, 2$ ). It is assumed that the system (6.1) can not be stabilized by a non-delayed static output feedback controller, but can be stabilized by a delayed static output feedback controller. Such a typical system is the Van der Pol's oscillator. It is shown in [6] and [141] that, for the Van der Pol's oscillator, a static output feedback (or position feedback) without a time-delay can only affect the frequency, but can not change the amplitude of oscillations; however, a delayed static output feedback can be alternatively used in the control of the oscillator.

Using the fuzzy modeling method in [124], the nonlinear system (6.1) on the compact region  $\mathbb{D}$  can be represented by a T-S fuzzy model, in which the fuzzy rule

is of the following form

*Plant Rule  $\mathcal{R}^i$* : IF  $\theta_1(t)$  is  $M_{i1}$  and  $\theta_2(t)$  is  $M_{i2}$  and  $\dots$  and  $\theta_g(t)$  is  $M_{ig}$ , THEN

$$\begin{cases} \dot{x}(t) = A_i x(t) + B_i u(t) + E_i \omega(t) \\ y(t) = C x(t) \end{cases} \quad (6.2)$$

where  $i = 1, 2, \dots, r$  and  $r$  denotes the number of IF-THEN rules;  $\theta_i(t)$  ( $i = 1, 2, \dots, g$ ) are the premise variables that are functions of the output  $y(t)$ ;  $M_{ij}$  ( $i = 1, 2, \dots, r$ ;  $j = 1, 2, \dots, g$ ) are the fuzzy sets;  $A_i$ ,  $B_i$  and  $E_i$  ( $i = 1, 2, \dots, r$ ) are system matrices of appropriate dimensions.

By using a center average defuzzifier, a product fuzzy inference and a singleton fuzzifier, the global dynamic of the fuzzy system (6.2) is inferred as follows

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^r \mu_i [A_i x(t) + B_i u(t) + E_i \omega(t)] \\ y(t) = C x(t) \end{cases} \quad (6.3)$$

where

$$\mu_i = \mu_i(\theta(t)) \geq 0, \quad i = 1, 2, \dots, r, \quad \theta(t) = [\theta_1^T(t), \theta_2^T(t), \dots, \theta_g^T(t)]^T, \quad \sum_{i=1}^r \mu_i = 1.$$

The objective of static output feedback tracking control is to drive the output  $y(t)$  via a static output feedback controller to follow the output  $y_r(t)$  as close as possible.  $y_r(t)$  is generated by the following reference model

$$\begin{cases} \dot{x}_r(t) = A_m x_r(t) + B_m r(t) \\ y_r(t) = C_m x_r(t) \end{cases} \quad (6.4)$$

where  $x_r(t) \in \mathbb{R}^{\bar{n}}$ ,  $r(t) \in \mathbb{R}^{\bar{v}}$  and  $y_r(t) \in \mathbb{R}^{\bar{l}}$  are the state, the energy bounded input and the output, respectively;  $A_m$ ,  $B_m$  and  $C_m$  are system matrices of appropriate dimensions, and  $A_m$  is a Hurwitz matrix.

In this chapter, the output tracking control is implemented by the following fuzzy static output feedback controller, which shares same fuzzy sets with the fuzzy model (6.2) in premise parts

*Control Rule  $\mathcal{R}^i$* : IF  $\theta_1(t)$  is  $M_{i1}$  and  $\theta_2(t)$  is  $M_{i2}$  and  $\dots$  and  $\theta_g(t)$  is  $M_{ig}$ , THEN

$$u(t) = F_{1i} y(t) + F_{2i} y_r(t), \quad i = 1, 2, \dots, r. \quad (6.5)$$

Analogous to (6.3), the fuzzy controller is given by

$$u(t) = \sum_{i=1}^r \mu_i [F_{1i}Cx(t) + F_{2i}C_mx_r(t)]. \quad (6.6)$$

For the nonlinear system under consideration, it is not possible to achieve the output tracking control objective by using a non-delayed fuzzy static output feedback controller but possible by using a delayed fuzzy static output feedback controller. To develop a delayed control input, we intentionally insert a communication network between the system (6.1) and the fuzzy controller (6.6). Notice that the insertion of the network in the feedback control system has several advantages such as system flexibility, low cost of installation and maintenance, and remote execution of tracking control [36]. Meanwhile, the use of the network leads to a network-induced delay in the feedback control loop. Usually, the network-induced delay is regarded as a main source of *system instability and tracking performance degradation* (see, [15], [28], [46] and the references therein). However, in this study, we will investigate *whether purposefully introducing a network-induced delay in the feedback control loop can provide a stable and satisfactory tracking control*. More specifically, the measurement  $y(kh)$  and  $y_r(kh)$  ( $\forall k \in \mathbb{Z}$ ) is transmitted in a single packet at each sampling instant  $kh$ , where  $h$  is the sampling period and  $\mathbb{Z}$  is the set of nonnegative integers. Due to the existence of the sensor-to-controller communication delay  $\tau_k^{sc}$  ( $\forall k \in \mathbb{Z}$ ), the fuzzy controller (6.6) in a network environment can be described by

$$\begin{cases} u(t^+) = \sum_{i=1}^r \mu_i^k [F_{1i}Cx(kh) + F_{2i}C_mx_r(kh)] \\ t \in \{kh + \tau_k^{sc}\}_{k=0}^{\infty}, \forall k \in \mathbb{Z} \end{cases} \quad (6.7)$$

where  $u(t^+) = \lim_{\delta \rightarrow t+0} u(\delta)$ ,  $\lim_{\delta \rightarrow t+0}$  is a limit taken from the left,  $\mu_i^k = \mu_i(\theta(kh)) \geq 0$  ( $i = 1, 2, \dots, r$ ),  $\sum_{i=1}^r \mu_i^k = 1$ ,  $\forall k \in \mathbb{Z}$ ,  $F_{1i}$  and  $F_{2i}$  ( $i = 1, 2, \dots, r$ ) are output feedback gains to be determined. Then the control signal  $u(t^+)$  is transmitted in a single packet over a controller-to-actuator channel. In the presence of the controller-to-actuator delay  $\tau_k^{ca}$  ( $\forall k \in \mathbb{Z}$ ), the update input of the actuator is

$$u(t) = \sum_{i=1}^r \mu_i^k [F_{1i}Cx(kh) + F_{2i}C_mx_r(kh)], \quad t \in \{kh + \tau_k\}_{k=0}^{\infty} \quad (6.8)$$

where  $\tau_k$  ( $\forall k \in \mathbb{Z}$ ) is the network-induced delay that lump the sensor-to-controller delay and the controller-to-actuator delay together and  $\tau_k = \tau_k^{sc} + \tau_k^{ca}$ . Defining  $\tau(t) = t - kh$  for  $t \in [kh + \tau_k, (k + 1)h + \tau_{k+1})$  ( $\forall k \in \mathbb{Z}$ ),  $\tau_m = \min_{k \in \mathbb{Z}} \{\tau_k\}$  and  $\tau_M = h + \max_{k \in \mathbb{Z}} \{\tau_k\}$ , we have

$$0 < \tau_m \leq \tau(t) \leq \tau_M, \quad t \in [kh + \tau_k, (k + 1)h + \tau_{k+1}). \quad (6.9)$$

Obviously,  $\tau(t)$  is discontinuous at the time instant  $kh + \tau_k$  and piecewise-linear with  $\dot{\tau}(t) = 1$  for  $t \neq kh + \tau_k$ . From (6.9) and the definitions of  $\tau(t)$ ,  $\tau_m$  and  $\tau_M$ , one can see that the network-induced delay  $\tau_k$  satisfies  $0 < \tau_m \leq \tau_k \leq \tau_M - h$  ( $\forall k \in \mathbb{Z}$ ), where  $\tau_m$  is both the lower bound of the network-induced delay and the lower bound of the sawtooth delay. The actuator holds the available data until next update. Then the input of the system (6.2) is

$$u(t) = \sum_{i=1}^r \mu_i^k [F_{1i} C x(t - \tau(t)) + F_{2i} C_m x_r(t - \tau(t))] \quad (6.10)$$

for  $t \in [kh + \tau_k, (k + 1)h + \tau_{k+1})$  ( $\forall k \in \mathbb{Z}$ ).

Using (6.3), (6.4) and (6.10), one obtains the following augmented system

$$\begin{cases} \dot{\xi}(t) = \sum_{i=1}^r \sum_{j=1}^r \mu_i \mu_j^k [\bar{A}_i \xi(t) + \bar{B}_i \bar{F}_j \xi(t - \tau(t)) + \bar{E}_i \bar{\omega}(t)] \\ e(t) = \bar{C} \xi(t), \quad t \in [kh + \tau_k, (k + 1)h + \tau_{k+1}) \end{cases} \quad (6.11)$$

where

$$\begin{aligned} \xi(t) &= [x^T(t) \quad x_r^T(t)]^T, \quad \bar{\omega}(t) = [\omega^T(t) \quad r^T(t)]^T, \quad e(t) = y(t) - y_r(t), \\ \bar{A}_i &= \begin{bmatrix} A_i & 0 \\ 0 & A_m \end{bmatrix}, \quad \bar{B}_i = \begin{bmatrix} B_i \\ 0 \end{bmatrix}, \quad \bar{E}_i = \begin{bmatrix} E_i & 0 \\ 0 & B_m \end{bmatrix}, \\ \bar{C} &= [C \quad -C_m], \quad \bar{F}_i = [F_{1i} C \quad F_{2i} C_m], \quad i = 1, 2, \dots, r. \end{aligned}$$

The initial condition of the system (6.11) is supplemented as  $\xi(t) = \phi(t)$ ,  $t \in [t_0 - \tau_M, t_0]$ , where  $\phi(t)$  is a continuous function on  $[t_0 - \tau_M, t_0]$ ,  $\phi(t_0) = [x_0^T \quad x_{r0}^T]^T$ , and  $x_0 = x(t_0)$  and  $x_{r0} = x_r(t_0)$  are initial states of the system (4.2) and the reference model (4.3), respectively. We choose the following  $H_\infty$  tracking performance

$$\int_{t_0}^{t_f} e^T(t) U e(t) dt \leq V(t_0) + \gamma^2 \int_{t_0}^{t_f} \bar{\omega}^T(t) \bar{\omega}(t) dt \quad (6.12)$$

where  $t_f$  is the terminal time,  $\gamma > 0$  is the desired tracking performance level,  $U > 0$  is the weighting matrix, and  $V(t_0)$  is the energy function of initial states.

The purpose of this chapter is to design the network-based fuzzy static output feedback controller (6.7) such that the system (6.11) is asymptotically stable with a prescribed  $H_\infty$  tracking performance, which means that

- 1) the system (6.11) with  $\bar{\omega}(t) = 0$  is asymptotically stable;
- 2) the output tracking error  $e(t)$  satisfies the  $H_\infty$  tracking performance (6.12), for all nonzero  $\bar{\omega}(t) \in \mathcal{L}_2[t_0, \infty)$ .

Notice that the system (6.11) with  $\tau(t) = 0$  is unstable whereas the system (6.11) with  $\tau(t)$  satisfying (6.9) may be stable. From a stability point of view, there exist a lower delay bound  $\tau_{min} > 0$  and an upper delay bound  $\tau_{max} > 0$  such that the system (6.11) is asymptotically stable with a prescribed  $H_\infty$  tracking performance. It is worth pointing out that  $\tau_m$  is both the lower bound of network-induced delays and the lower bound of the sawtooth delay, and therefore  $\tau_m > 0$  is essential for network-based static output feedback tracking control. Then one can see that the intentional introduction of an appropriate network-induced delay between the system (6.3) and the fuzzy static output feedback controller (6.6) can produce a stable and satisfactory tracking control.

There are some available results on network-based state feedback tracking control for a linear system [15], [28], [133], [137] and [148]. However, in many practical situations, the system is nonlinear and it is physically difficult to measure all the state variables of a nonlinear system. This fact demands the research on network-based output feedback tracking control for a nonlinear system. Motivated by (1) a T-S fuzzy model can be a universal approximator of any smooth nonlinear control systems on a compact region ([100], [124]); (2) a static output feedback controller can be easily implemented with low cost compared with a dynamic output feedback controller ([24], [102]); and (3) the positive effect of the network-induced delay on

fuzzy static output feedback tracking control has not been investigated, we will focus on network-based static output feedback tracking control for the nonlinear system (6.1) described by the T-S fuzzy system (6.3).

To further show the motivations and contributions of this study, we review some existing results on fuzzy modeling and control of a network-based T-S fuzzy system [54], [57], [98], [128], [156] and [157]. As mentioned in Chapter 4, there exist the following limitations in these references

- It is impossible to practically implement the network-based fuzzy controller depending on continuous premise variables in [57], [98] and [156].
- It is technically wrong to use a routine relaxation method for a traditional T-S fuzzy system to analyze stability and design a network-based controller for a T-S fuzzy system in [128] and [157].
- It is very conservative to design a network-based linear tracking controller for a T-S fuzzy system in [54], without using the routine relaxation method.

Moreover, in the above references [54], [57], [98], [128], [156] and [157], the delay-dependent criteria are derived by using simple Lyapunov-Krasovskii functionals (LKFs), where the definition of simple LKF can be found in [40]. The simple LKFs can not be applied to analyze  $H_\infty$  tracking performance for the system (6.11) since they require the system (6.11) with  $\tau(t) = 0$  to be stable, which is clearly not satisfied. Furthermore, the knowledge of fuzzy membership functions is not considered in these works [54], [57], [98], [128], [156] and [157], which may lead to some conservative results. To design an essentially nonlinear network-based fuzzy static output feedback controller by taking the knowledge of fuzzy membership functions into account, we introduce the following asynchronous constraints

$$|\mu_l - \mu_l^k| \leq \delta_l, \quad l = 1, 2, \dots, r \quad (6.13)$$

where  $\delta_l$  ( $l = 1, 2, \dots, r$ ) are some given positive constants.

The following lemma is introduced to achieve the upper bounds  $\delta_i$  ( $i = 1, 2, \dots, r$ )

**Lemma 6.1.** *Given the membership functions  $\mu_i(\theta(t))$  ( $i = 1, 2, \dots, r$ ), the universe of discourse  $\mathbb{D}$  and known positive scalars  $\tau_m$  and  $\tau_M$ . The following inequalities hold for any  $\tau(t) \in [\tau_m, \tau_M]$*

$$|\mu_i(\theta(t)) - \mu_i(\theta(t - \tau(t)))| \leq \delta_i, \quad i = 1, 2, \dots, r$$

where  $\delta_i = \min\{1, \epsilon_i \rho \tau_M\}$  ( $i = 1, 2, \dots, r$ ), if the following conditions hold

(i) *The fuzzy membership functions  $\mu_i(\theta(t))$  of the premise variable  $\theta(t)$  are Lipschitz continuous functions with known Lipschitz constants  $\epsilon_i$  ( $i = 1, 2, \dots, r$ ) on the compact region  $\mathbb{D}$ , i.e.,*

$$|\mu_i(\theta(t)) - \mu_i(\theta(t - \tau(t)))| \leq \epsilon_i |\theta(t) - \theta(t - \tau(t))|, \quad i = 1, 2, \dots, r.$$

(ii) *The derivative of the premise variable  $\theta(t)$  is bounded, i.e.,  $\|\dot{\theta}(t)\| \leq \rho$  for any  $t \in [kh + \tau_k, (k+1)h + \tau_{k+1}]$  ( $k = 0, 1, \dots$ ), where  $\rho$  is a known positive scalar.*

## 6.2 Performance analysis of static output feedback tracking control

In this section, we will derive a new criterion for  $H_\infty$  tracking performance analysis by using a complete LKF method. For simplicity of presentation, let

$$\eta^T(t) = [\xi^T(t) \quad \dot{\xi}^T(t) \quad \xi^T(t - \tau(t)) \quad \xi^T(t - \bar{\tau}(t)) \quad \xi^T(t - \tau_m) \quad \xi^T(t - \tau_M)],$$

$$e_1 = [I_{p \times p} \quad 0 \quad 0 \quad 0 \quad 0 \quad 0]_{p \times 6p},$$

$$e_2 = [0 \quad I_{p \times p} \quad 0 \quad 0 \quad 0 \quad 0]_{p \times 6p},$$

$$e_3 = [0 \quad 0 \quad I_{p \times p} \quad 0 \quad 0 \quad 0]_{p \times 6p},$$

$$e_4 = [0 \quad 0 \quad 0 \quad I_{p \times p} \quad 0 \quad 0]_{p \times 6p},$$

$$e_5 = [0 \quad 0 \quad 0 \quad 0 \quad I_{p \times p} \quad 0]_{p \times 6p},$$

$$e_6 = [0 \quad 0 \quad 0 \quad 0 \quad 0 \quad I_{p \times p}]_{p \times 6p}$$

where  $e_i$  ( $i = 1, 2, 3, 4, 5, 6$ ) are  $p \times 6p$  matrices;  $I_{p \times p}$  denotes a  $p \times p$  identity matrix; the others in  $e_j$  ( $j = 1, 2, \dots, 6$ ) are  $p \times p$  zero matrices;  $p$  is the dimension of  $\xi(t)$ . Then a delay-dependent criterion for  $H_\infty$  tracking performance analysis of the system (6.11) is given by the following proposition.

**Proposition 6.1.** *Given positive scalars  $\gamma$ ,  $\tau_m$ ,  $\tau_M$ ,  $\delta_i$  ( $i = 1, 2, \dots, r$ ), and gain matrices  $F_{1i}$  and  $F_{2i}$  ( $i = 1, 2, \dots, r$ ) and a weighting matrix  $U > 0$ , the system (6.11) is asymptotically stable with a given  $H_\infty$  tracking performance if there exist symmetric matrices  $P > 0$ ,  $Z_i > 0$  ( $i = 1, 2, 3, 4, 5$ ),  $S_i > 0$  ( $i = 0, 1, \dots, N$ ), and matrices  $X_1$ ,  $X_2$ ,  $M_i$  ( $i = 1, 2, \dots, r$ ),  $Y_i$  ( $i = 1, 2, 3$ ),  $Q_i$ ,  $R_{ij}$  ( $i, j = 0, 1, \dots, N$ ) such that the following linear matrix inequalities (LMIs) hold*

$$\begin{bmatrix} P & * \\ \tilde{Q}^T & \tilde{S} + \tilde{R} \end{bmatrix} > 0 \quad (6.14)$$

$$\Xi_{ii}^l < 0, \quad l = 1, 2, \quad i = 1, 2, \dots, r \quad (6.15)$$

$$\Xi_{ij}^l + \Xi_{ji}^l < 0, \quad l = 1, 2, \quad i, j = 1, 2, \dots, r, \quad i \leq j \quad (6.16)$$

$$\mathcal{E}_0 \Omega_{ij}^l \mathcal{E}_0^T + M_i > 0, \quad l = 1, 2, \quad i, j = 1, 2, \dots, r \quad (6.17)$$

where

$$\begin{aligned} \tilde{Q}^T &= (Q_0, Q_1, \dots, Q_N)^T, \\ \tilde{S} &= \text{diag}\{v^{-1}S_0, v^{-1}S_1, \dots, v^{-1}S_N\}, \\ \tilde{R} &= \begin{bmatrix} R_{00} & R_{01} & \cdots & R_{0N} \\ R_{10} & R_{11} & \cdots & R_{1N} \\ \vdots & \vdots & \ddots & \vdots \\ R_{N0} & R_{N1} & \cdots & R_{NN} \end{bmatrix}, \\ \Xi_{ij}^1 &= \begin{bmatrix} \Xi_{ij}^{11} & * & * & * \\ \Xi_{ij}^{21} & \Xi_{ij}^{22} & * & * \\ \tau Y_1 & 0 & -\tau Z_3 & * \\ \tau Y_3 & 0 & 0 & -\tau Z_4 \end{bmatrix}, \\ \Xi_{ij}^2 &= \begin{bmatrix} \Xi_{ij}^{11} & * & * \\ \Xi_{ij}^{21} & \Xi_{ij}^{22} & * \\ \tau Y_2 & 0 & -\tau Z_3 \end{bmatrix}, \end{aligned}$$

$$\Xi_{ij}^{11l} = \begin{bmatrix} \sum_{k=1}^r \delta_k \bar{\mathcal{E}}_0 (\mathcal{E}_0 \Omega_{ik}^l \mathcal{E}_0^T + M_i) \bar{\mathcal{E}}_0^T + \Omega_{ij}^l & * \\ \bar{E}_i^T (X_1 e_1 + X_2 e_2) & -2\gamma^2 I \end{bmatrix},$$

$$\Xi^{21} = \begin{bmatrix} \Gamma_1^{sT} & \Gamma_2^{sT} & 0 & 0 & 0 & \Gamma_3^{sT} & 0 \\ \Gamma_1^{aT} & \Gamma_2^{aT} & 0 & 0 & 0 & \Gamma_3^{aT} & 0 \end{bmatrix},$$

$$\Xi^{22} = \text{diag} \{-R_d - S_d, -3S_d\},$$

$$R_d = \begin{bmatrix} R_{d11} & R_{d12} & \cdots & R_{d1N} \\ R_{d21} & R_{d22} & \cdots & R_{d2N} \\ \vdots & \vdots & \ddots & \vdots \\ R_{dN1} & R_{dN2} & \cdots & R_{dNN} \end{bmatrix},$$

$$R_{dij} = v(R_{ij} - R_{i-1,j-1}) \quad (i, j = 1, 2, \dots, N),$$

$$S_d = \text{diag}\{S_1 - S_0, S_2 - S_1, \dots, S_N - S_{N-1}\},$$

$$\begin{aligned} \Omega_{ij}^1 = & e_1^T (X_1^T \bar{A}_i + \bar{A}_i^T X_1 + 2\bar{C}^T U \bar{C} + Q_N + Q_N^T + S_N + Z_1 - Z_2 - Z_5) e_1 \\ & + e_1^T (P - X_1^T + \bar{A}_i^T X_2) e_2 + e_1^T X_1^T \bar{B}_i \bar{F}_j e_3 + e_1^T Z_5 e_4 + e_1^T Z_2 e_5 - e_1^T Q_0 e_6 \\ & + e_2^T (P - X_1 + X_2^T \bar{A}_i) e_1 + e_3^T (\bar{B}_i \bar{F}_j)^T X_1 e_1 + e_4^T Z_5 e_1 + e_5^T Z_2 e_1 - e_6^T Q_0 e_1 \\ & + e_2^T (\tau_m^2 Z_2 + \tau Z_3 - X_2 - X_2^T) e_2 - e_4^T Z_5 e_4 - e_5^T (Z_1 + Z_2) e_5 - e_6^T S_0 e_6 \\ & + e_2^T X_2^T \bar{B}_i \bar{F}_j e_3 + e_3^T (\bar{B}_i \bar{F}_j)^T X_2 e_2 + (e_5 - e_3)^T Y_1 + Y_1^T (e_5 - e_3) \\ & + Y_2^T (e_3 - e_6) + (e_3 - e_6)^T Y_2 + (e_1 - e_4)^T Y_3 + Y_3^T (e_1 - e_4), \end{aligned}$$

$$\Omega_{ij}^2 = \Omega_{ij}^1 + \tau \Omega, \quad \tau = \tau_M - \tau_m,$$

$$\mathcal{E}_0 = [e_1^T \ e_2^T \ e_3^T]^T, \quad \bar{\mathcal{E}}_0 = [e_1^T \ e_2^T \ e_3^T],$$

$$\Omega = e_1^T Z_5 e_2 + e_2^T Z_5 e_1 + e_2^T Z_4 e_2 - e_2^T Z_5 e_4 - e_4^T Z_5 e_2,$$

$$\Gamma_i^s = [\Gamma_{i1}^s \ \Gamma_{i2}^s \ \cdots \ \Gamma_{iN}^s] \quad (i = 1, 2, 3, 4),$$

$$\Gamma_i^a = [\Gamma_{i1}^a \ \Gamma_{i2}^a \ \cdots \ \Gamma_{iN}^a] \quad (i = 1, 2, 3, 4),$$

$$\Gamma_{1n}^s = Q_{n-1} - Q_n + \frac{v}{2} (R_{n,N}^T + R_{n-1,N}^T),$$

$$\Gamma_{1n}^a = \frac{v}{2} (R_{n-1,N}^T - R_{n,N}^T),$$

$$\Gamma_{2n}^s = \frac{v}{2} (Q_n + Q_{n-1}),$$

$$\Gamma_{2n}^a = \frac{v}{2} (Q_{n-1} - Q_n),$$

$$\begin{aligned}\Gamma_{3n}^s &= \frac{v}{2}(-R_{n,0}^T - R_{n-1,0}^T), \\ \Gamma_{3n}^a &= \frac{v}{2}(R_{n,0}^T - R_{n-1,0}^T), \\ n &= 1, 2, \dots, r, \quad v = \tau_M/N.\end{aligned}$$

*Proof:* We first analyze the asymptotic stability for the system (6.11) with  $\bar{\omega}(t) = 0$  by using a complete LKF method. The complete LKF is constructed as follows

$$V(t, \xi_t(\theta), \dot{\xi}_t(\theta)) = V_1(t, \xi_t(\theta), \dot{\xi}_t(\theta)) + V_2(t, \xi_t(\theta), \dot{\xi}_t(\theta)) + V_3(t, \xi_t(\theta), \dot{\xi}_t(\theta)) \quad (6.18)$$

where  $\xi_t(\theta) = \xi(t+\theta)$  and  $\dot{\xi}_t(\theta) = \dot{\xi}(t+\theta)$ ,  $\forall \theta \in [-\tau_M, 0]$ , the space of functions  $\xi_t(\theta)$  and  $\dot{\xi}_t(\theta)$  is denoted by  $\mathbb{W}$  with the norm  $\|\xi\|_{\mathbb{W}} = \sup_{\theta \in [-\tau_M, 0]} \{\|\xi_t(\theta)\|, \|\dot{\xi}_t(\theta)\|\}$  and

$$\begin{aligned}V_1(t, \xi_t(\theta), \dot{\xi}_t(\theta)) &= \frac{1}{2} \xi^T(t) P \xi(t) + \xi^T(t) \int_{-\tau_M}^0 Q(\theta) \xi(t+\theta) d\theta \\ &\quad + \frac{1}{2} \int_{-\tau_M}^0 \int_{-\tau_M}^0 \xi^T(t+\theta) R(\theta, s) \xi(t+s) d\theta ds \\ &\quad + \frac{1}{2} \int_{-\tau_M}^0 \xi^T(t+\theta) S(\theta) \xi(t+\theta) d\theta, \\ V_2(t, \xi_t(\theta), \dot{\xi}_t(\theta)) &= \frac{1}{2} \int_{-\tau_m}^0 \xi^T(t+\theta) Z_1 \xi(t+\theta) d\theta \\ &\quad + \frac{\tau_m}{2} \int_{-\tau_m}^0 \int_s^0 \dot{\xi}^T(t+\theta) Z_2 \dot{\xi}(t+\theta) d\theta ds \\ &\quad + \frac{\tau_M - \tau_m}{2} \int_{-\tau_M}^{-\tau_m} \int_s^0 \dot{\xi}^T(t+\theta) Z_3 \dot{\xi}(t+\theta) d\theta ds, \\ V_3(t, \xi_t(\theta), \dot{\xi}_t(\theta)) &= \frac{\tau_M - \tau(t)}{2} \int_{-\bar{\tau}(t)}^0 \xi^T(t+\theta) Z_4 \dot{\xi}(t+\theta) d\theta \\ &\quad + \frac{\tau_M - \tau(t)}{2} (x(t) - x(t - \bar{\tau}(t)))^T Z_5 (x(t) - x(t - \bar{\tau}(t))),\end{aligned}$$

$$\bar{\tau}(t) = \tau(t) - \tau_k, \quad t \in [kh + \tau_k, (k+1)h + \tau_{k+1}), \quad \forall k \in \mathbb{Z},$$

$$Z_i \in \mathbb{R}^{p \times p}, \quad Z_i = Z_i^T (i=1, 2, 3, 4, 5),$$

$$P \in \mathbb{R}^{p \times p}, \quad P = P^T,$$

$$Q : [-\tau_M, 0] \rightarrow \mathbb{R}^{p \times p}, \quad Q(\theta) = Q^T(\theta),$$

$$S : [-\tau_M, 0] \rightarrow \mathbb{R}^{p \times p}, \quad S(\theta) = S^T(\theta),$$

$$R : [-\tau_M, 0] \times [-\tau_M, 0] \rightarrow \mathbb{R}^{p \times p}, \quad R(\theta, s) = R^T(s, \theta).$$

Using Lemma 5.2 in Chapter 5, we can obtain  $\varepsilon_1 \|\xi(t)\|^2 \leq V(t, \xi_t(\theta), \dot{\xi}_t(\theta)) \leq \varepsilon_2 \|\xi\|_{\mathbb{W}}^2$  if (6.14) and  $Z_i > 0$  ( $i = 1, 2, 3, 4, 5$ ) hold. Let  $V(t) = V(t, \xi_t(\theta), \dot{\xi}_t(\theta))$  and taking the derivative of  $V(t)$  yields

$$\begin{aligned} \dot{V}(t) = & \eta^T(t) e_2^T \left( \int_{-\tau_M}^0 Q(\theta) \xi(t+\theta) d\theta + P e_1 \eta(t) \right) + \eta^T(t) e_1^T \int_{-\tau_M}^0 Q(\theta) \dot{\xi}(t+\theta) d\theta \\ & + \int_{-\tau_M}^0 \dot{\xi}^T(t+\theta) S(\theta) \xi(t+\theta) d\theta + \int_{-\tau_M}^0 \int_{-\tau_M}^0 \dot{\xi}^T(t+\theta) R(\theta, s) \xi(t+s) d\theta ds \\ & + \frac{1}{2} \eta^T(t) [e_1^T Z_1 e_1 - (e_1 - e_4)^T Z_5 (e_1 - e_4) + e_2^T (\tau_m^2 Z_2 + \tau Z_3) e_2 - e_5^T Z_1 e_5] \eta(t) \\ & + \frac{\tau_M - \tau(t)}{2} \eta^T(t) [2(e_1 - e_4)^T Z_5 e_2 + e_2^T Z_4 e_2] \eta(t) - \frac{\tau_m}{2} \int_{t-\tau_m}^t \dot{\xi}^T(s) Z_2 \dot{\xi}(s) ds \\ & - \frac{1}{2} \int_{t-\tau_M}^{t-\tau_m} \dot{\xi}^T(s) Z_3 \dot{\xi}(s) ds - \frac{1}{2} \int_{t-\bar{\tau}(t)}^t \dot{\xi}^T(s) Z_4 \dot{\xi}(s) ds. \end{aligned} \quad (6.19)$$

For the system (6.11) with  $\bar{\omega}(t) = 0$ , the following equation hold for matrices  $X_1$  and  $X_2$  with appropriate dimensions

$$\sum_{i=1}^r \sum_{j=1}^r \mu_i \mu_j^k \eta^T(t) (e_1^T X_1^T + e_2^T X_2^T) (\bar{A}_i e_1 + \bar{B}_i \bar{F}_j e_3 - e_2) \eta(t) = 0 \quad (6.20)$$

where  $t \in [kh + \tau_k, (k+1)h + \tau_{k+1})$  ( $\forall k \in \mathbb{Z}$ ).

Using Jensen integral inequality, we have

$$- \tau_m \int_{t-\tau_m}^t \dot{\xi}^T(s) Z_2 \dot{\xi}(s) ds \leq -\eta^T(t) (e_1 - e_5)^T Z_2 (e_1 - e_5) \eta(t). \quad (6.21)$$

Applying Lemma 2.2 in Chapter 2 with  $\mathcal{E} = e_5 - e_3$ ,  $\psi = \eta(t)$  and  $Z = Y_1$  to the term  $- \int_{t-\tau(t)}^{t-\tau_m} \dot{\xi}^T(s) Z_3 \dot{\xi}(s) ds$ , we obtain

$$\begin{aligned} - \int_{t-\tau(t)}^{t-\tau_m} \dot{\xi}^T(s) Z_3 \dot{\xi}(s) ds \leq & \eta^T(t) [(e_5 - e_3)^T Y_1 + Y_1^T (e_5 - e_3)] \eta(t) \\ & + (\tau(t) - \tau_m) \eta^T(t) Y_1^T Z_3^{-1} Y_1 \eta(t). \end{aligned} \quad (6.22)$$

Similarly, the following inequalities hold

$$\begin{aligned} - \int_{t-\tau_M}^{t-\tau(t)} \dot{\xi}^T(s) Z_3 \dot{\xi}(s) ds \leq & \eta^T(t) [(e_3 - e_6)^T Y_2 + Y_2^T (e_3 - e_6)] \eta(t) \\ & + (\tau_M - \tau(t)) \eta^T(t) Y_2^T Z_3^{-1} Y_2 \eta(t) \end{aligned} \quad (6.23)$$

$$\begin{aligned} - \int_{t-\bar{\tau}(t)}^t \dot{\xi}^T(s) Z_4 \dot{\xi}(s) ds \leq & \eta^T(t) [(e_1 - e_4)^T Y_3 + Y_3^T (e_1 - e_4)] \eta(t) \\ & + (\tau(t) - \tau_m) \eta^T(t) Y_3^T Z_4^{-1} Y_3 \eta(t). \end{aligned} \quad (6.24)$$

Integrating by parts in (6.19) and applying the discretization scheme in [34] and (6.20)-(6.24), we have

$$\begin{aligned} \dot{V}(t) \leq & \frac{1}{2} \sum_{i=1}^r \sum_{j=1}^r \mu_i \mu_j^k \eta^T(t) \Omega_{ij}(t) \eta(t) + \frac{1}{2} \int_0^1 \tilde{\xi}^T(\alpha) S_d \tilde{\xi}(\alpha) d\alpha \\ & + \eta^T(t) \int_0^1 [\Gamma^s + (1 - 2\alpha)\Gamma^a] \tilde{\xi}(\alpha) d\alpha + \frac{1}{2} \int_0^1 \tilde{\xi}^T(\alpha) d\alpha R_d \int_0^1 \tilde{\xi}(\alpha) d\alpha \end{aligned} \quad (6.25)$$

for  $t \in [kh + \tau_k, (k + 1)h + \tau_{k+1})$  ( $\forall k \in \mathbb{Z}$ ), where

$$\begin{aligned} \tilde{\xi}^T(\alpha) &= [\tilde{\xi}^T(\alpha_1) \tilde{\xi}^T(\alpha_2) \cdots \tilde{\xi}^T(\alpha_N)]^T, \\ \tilde{\xi}^T(\alpha_i) &= \xi^T(t - iv + \alpha v), \quad i = 1, 2, \dots, N, \\ \Omega_{ij}(t) &= \Omega_{ij}^1 - 2e_1^T \bar{C}^T U \bar{C} e_1 + \Omega^1(t) + \Omega^2(t), \\ \Omega^1(t) &= (\tau(t) - \tau_m)(Y_1^T Z_3^{-1} Y_1 + Y_3^T Z_4^{-1} Y_3), \\ \Omega^2(t) &= (\tau_M - \tau(t))(\Omega + Y_2^T Z_3^{-1} Y_2). \end{aligned}$$

Applying Proposition 5.21 in [34] to the left side of (6.25), we obtain

$$\dot{V}(t) \leq \frac{1}{2} \sum_{i=1}^r \sum_{j=1}^r \mu_i \mu_j^k \begin{bmatrix} \eta(t) \\ \zeta(\alpha) \end{bmatrix}^T \bar{\Xi}_{ij}(t) \begin{bmatrix} \eta(t) \\ \zeta(\alpha) \end{bmatrix} \quad (6.26)$$

for  $t \in [kh + \tau_k, (k + 1)h + \tau_{k+1})$  ( $\forall k \in \mathbb{Z}$ ), where

$$\begin{aligned} \zeta(\alpha) &= \int_0^1 \tilde{\xi}(\alpha) d\alpha, \\ \bar{\Xi}_{ij}(t) &= \begin{bmatrix} \Omega_{ij}(t) & * \\ \bar{\Xi}^{21} & \bar{\Xi}^{22} \end{bmatrix}, \\ \bar{\Xi}^{21} &= \begin{bmatrix} \Gamma_1^{sT} & \Gamma_2^{sT} & 0 & 0 & 0 & \Gamma_3^{sT} \\ \Gamma_1^{aT} & \Gamma_2^{aT} & 0 & 0 & 0 & \Gamma_3^{aT} \end{bmatrix}. \end{aligned}$$

Let

$$\bar{\Xi}^l(t) = \begin{bmatrix} \Omega^l(t) & * \\ \bar{\Xi}^{21} & \bar{\Xi}^{22} \end{bmatrix}, \quad l = 1, 2.$$

Clearly,  $\bar{\Xi}_{ij}(t)$  is a convex combination of  $\bar{\Xi}^1(t)$  and  $\bar{\Xi}^2(t)$  on  $\tau(t) \in [\tau_m, \tau_M]$ . By using the convex combination technique and Schur complement, we can see that the

inequality  $\sum_{i=1}^r \sum_{j=1}^r \mu_i \mu_j^k \bar{\Xi}_{ij}(t) < 0$  can be ensured by

$$\begin{bmatrix} \sum_{i=1}^r \sum_{j=1}^r \mu_i \mu_j^k \bar{\Omega}_{ij}^1 & * & * & * \\ \bar{\Xi}^{21} & \bar{\Xi}^{22} & * & * \\ \tau Y_1 & 0 & -\tau Z_3 & * \\ \tau Y_3 & 0 & 0 & -\tau Z_4 \end{bmatrix} < 0 \quad (6.27)$$

$$\begin{bmatrix} \sum_{i=1}^r \sum_{j=1}^r \mu_i \mu_j^k \bar{\Omega}_{ij}^2 & * & * \\ \bar{\Xi}^{21} & \bar{\Xi}^{22} & * \\ \tau Y_2 & 0 & -\tau Z_3 \end{bmatrix} < 0 \quad (6.28)$$

where  $\bar{\Omega}_{ij}^1 = \Omega_{ij}^1 - 2e_1^T \bar{C}^T U \bar{C} e_1$  and  $\bar{\Omega}_{ij}^2 = \bar{\Omega}_{ij}^1 + \tau \Omega$ .

Notice that  $\sum_{i=1}^r (\mu_i^k - \mu_i) = 0$  and  $|\mu_i^k - \mu_i| \leq \delta_i$  for  $i = 1, 2, \dots, r$ . If the LMI (6.17) holds, we have

$$\begin{aligned} & \sum_{i=1}^r \sum_{j=1}^r \mu_i \mu_j^k \bar{\Omega}_{ij}^l \\ &= \sum_{i=1}^r \sum_{j=1}^r \mu_i \mu_j \bar{\Omega}_{ij}^l + \sum_{i=1}^r \sum_{j=1}^r \mu_i (\mu_j^k - \mu_j) \bar{\Omega}_{ij}^l \\ &\leq \sum_{i=1}^r \sum_{j=1}^r \mu_i \mu_j \left[ \bar{\Omega}_{ij}^l + \sum_{k=1}^r \delta_k \bar{\mathcal{E}}_0 (\mathcal{E}_0 \bar{\Omega}_{ik}^l \mathcal{E}_0^T + M_i) \bar{\mathcal{E}}_0^T \right]. \end{aligned} \quad (6.29)$$

Then it is easy to see that (6.27)-(6.28) are implied by the LMIs (6.15)-(6.16). It follows from (6.26) that  $\dot{V}(t) \leq -\varepsilon_3 \|\xi(t)\|^2$  for  $t \in [kh + \tau_k, (k+1)h + \tau_{k+1})$ ,  $\forall k \in \mathbb{Z}$ . is satisfied, which means that the system (6.11) is asymptotically stable.

We now consider  $H_\infty$  tracking performance of the system (6.11). Taking the time derivative of the LKF (6.18), we have

$$\begin{aligned} \dot{V}(t) &\leq \frac{1}{2} \sum_{i=1}^r \sum_{j=1}^r \mu_i \mu_j^k \begin{bmatrix} \tilde{\eta}(t) \\ \zeta(\alpha) \end{bmatrix}^T \Xi_{ij}(t) \begin{bmatrix} \tilde{\eta}(t) \\ \zeta(\alpha) \end{bmatrix} \\ &\quad + \gamma^2 \omega^T(t) \omega(t) - \xi^T(t) \bar{C}^T U \bar{C} \xi(t) \end{aligned} \quad (6.30)$$

for  $t \in [kh + \tau_k, (k + 1)h + \tau_{k+1})$  ( $\forall k \in \mathbb{Z}$ ), where

$$\begin{aligned}\tilde{\eta}^T(t) &= [\eta^T(t) \ \omega^T(t)], \\ \Xi_{ij}(t) &= \begin{bmatrix} \Xi_{ij}^{11}(t) & * \\ \Xi_{21} & \Xi_{22} \end{bmatrix}, \\ \Xi_{ij}^{11}(t) &= \begin{bmatrix} \Omega_{ij}(t) + 2e_1^T \bar{C}^T U \bar{C} e_1 & * \\ \bar{E}_i^T (X_1 e_1 + X_2 e_2) & -2\gamma^2 I \end{bmatrix}.\end{aligned}$$

By using the convex combination technique and Schur complement, we obtain that

$\sum_{i=1}^r \sum_{j=1}^r \mu_i \mu_j^k \Xi_{ij}(t) < 0$  holds if the following inequalities are satisfied

$$\begin{bmatrix} \sum_{i=1}^r \sum_{j=1}^r \mu_i \mu_j^k \bar{\Xi}_{ij}^1 & * & * & * \\ \Xi_{21} & \Xi_{22} & * & * \\ \tau Y_1 & 0 & -\tau Z_3 & * \\ \tau Y_3 & 0 & 0 & -\tau Z_4 \end{bmatrix} < 0 \quad (6.31)$$

$$\begin{bmatrix} \sum_{i=1}^r \sum_{j=1}^r \mu_i \mu_j^k \bar{\Xi}_{ij}^2 & * & * \\ \Xi_{21} & \Xi_{22} & * \\ \tau Y_2 & 0 & -\tau Z_3 \end{bmatrix} < 0 \quad (6.32)$$

where

$$\bar{\Xi}_{ij}^l = \begin{bmatrix} \Omega_{ij}^l & * \\ \bar{E}_i^T (X_1 e_1 + X_2 e_2) & -2\gamma^2 I \end{bmatrix}, \quad l = 1, 2.$$

Using (6.30), we can see that (6.31)-(6.32) can be ensured by (6.15)-(6.17). It follows from (6.28) that  $\dot{V}(t) + e^T(t)Ue(t) - \gamma^2 \bar{\omega}^T(t)\bar{\omega}(t) < 0$  for  $t \in [kh + \tau_k, (k + 1)h + \tau_{k+1})$ ,  $\forall k \in \mathbb{Z}$ . Then the  $H_\infty$  tracking performance (6.12) is ensured for the system (6.11) on  $[t_0, \infty)$ , which completes the proof.

**Remark 6.1.** It can be seen from the above proof that the inherent piecewise-linear time-varying delay information  $\dot{\tau}(t) = 1$  on  $[kh + \tau_k, (k + 1)h + \tau_{k+1})$  ( $\forall k \in \mathbb{Z}$ ) and the knowledge about premise variables and membership functions ( $\rho$  and  $\epsilon_i$ ,  $i = 1, 2, \dots, r$ ) are utilized to establish a less conservative delay-dependent criterion. Notice that for the asynchronous T-S fuzzy system (6.11), the  $H_\infty$  performance criterion may

be generally cast as negativity of fuzzy summations in the form

$$\varphi^T(t) \left( \sum_{i=1}^r \sum_{j=1}^r \mu_i \mu_j^k \Xi_{ij} \right) \varphi(t) < 0, \quad \forall \varphi(t) \neq 0. \quad (6.33)$$

Then the inequality (6.33) can be ensured by  $\Xi_{ij} < 0$  ( $i, j = 1, 2, \dots, r$ ), which is conservative, see [93]. In this study,  $\sum_{i=1}^r (\mu_i^k - \mu_i) = 0$  and  $|\mu_i - \mu_i^k| \leq \delta_i$  are fully utilized to introduce some free-weighting matrices  $M_i$  ( $i = 1, 2, \dots, r$ ), which can be used to reduce the conservatism.

**Remark 6.2.** From a stability point of view, there exist a lower delay bound  $\tau_{min}$  and an upper delay bound  $\tau_{max}$  such that the system (6.11) is asymptotically stable with a prescribed  $H_\infty$  tracking performance  $\gamma$ . By scheduling the network-induced delay  $\tau_k$  ( $\forall k \in \mathbb{Z}$ ) and choosing an appropriate sampling period  $h$ , we make the interval  $[\tau_m, \tau_M]$  fall into  $(\tau_{min}, \tau_{max})$  to ensure the system (6.11) with a given  $H_\infty$  tracking performance  $\gamma$ . The steps mentioned above Proposition 5.2 in Chapter 5 can be used to determine appropriate delay bounds  $\tau_m$  and  $\tau_M$  such that the  $H_\infty$  tracking performance  $\gamma$  is ensured for the system (6.11).

For given  $\tau_m > 0$ ,  $\tau_M > 0$ ,  $U > 0$ ,  $F_{1i}$  and  $F_{2i}$  ( $i = 1, 2, \dots, r$ ), one can employ Proposition 6.1 to determine the minimum  $\gamma$ , which can be obtained by solving the following optimization problem:

$$\begin{aligned} & \text{minimize} && \gamma \\ & \text{subject to} && P > 0, \quad Z_i > 0 \quad (i=1, 2, 3, 4, 5), \quad S_i > 0 \quad (i=0, 1, \dots, N), \\ & && \text{and LMIs (6.14) – (6.17)}. \end{aligned}$$

### 6.3 Fuzzy static output feedback tracking control design

In this section, we are interested in designing the network-based fuzzy static output feedback tracking controller (6.7) for the nonlinear system (6.1) described by (6.3).

Notice that in Proposition 6.1, constant matrices  $C$ ,  $C_r$  and gain matrices  $F_{1i}$ ,  $F_{2i}$  are coupled into  $\bar{F}_i$  ( $i = 1, 2, \dots, r$ ), which makes it difficult to design a static output feedback tracking controller. Similar to the control design method in Chapter 3, we will propose a design algorithm of a fuzzy static output feedback tracking controller by applying a particle swarm optimization (PSO) technique with feasibility of Proposition 6.1. The control design algorithm is as follows

**Algorithm 6.1.**

Step 1. Initialization

- 1.1 Randomly initialize a group with  $n_p$  particles. Each particle consists of members  $f_{ij}(0)$  in  $F_{1\rho j}^0$  and  $F_{2\rho j}^0$ ; and  $f_{ij}(0)$  lies in the range  $[\alpha_j, \beta_j]$ , where  $i = 1, 2, \dots, n_p$ ,  $\rho = 1, 2, \dots, r$  and  $j = 1, 2, \dots, 2mlr$ ;
- 1.2 Initialize parameters  $c_1, c_2, \omega_M, \omega_m, m_\eta, \nu_j^{min}$  and  $\nu_j^{max}$ , where  $j = 1, 2, \dots, 2mlr$ ;
- 1.3 Initialize the velocity of  $n_p$  particles and  $\nu_j^{min} \leq \nu_{ij}(0) \leq \nu_j^{max}$ , where  $i = 1, 2, \dots, n_p$  and  $j = 1, 2, \dots, 2mlr$ , and set  $k = 0$ ;
- 1.4 Initialize the fitness value  $\gamma_i^0 = l_p$ , where  $l_p$  is a positive constant and  $i = 1, 2, \dots, n_p$ . Solve the minimization problem at the end of Section 6.2 to obtain the minimum  $\gamma_i^0$  for given  $F_{1\rho j}^0$  and  $F_{2\rho j}^0$ ,  $i \in \{1, 2, \dots, n_p\}$  and  $\rho = 1, 2, \dots, r$ ;
- 1.4.1 Assign the minimum  $\gamma_i^0$  to  $\gamma_{ip}^0$  and  $f_{ij}(0)$  to  $\text{pbest}_{ij}$ , respectively, where  $\gamma_{ip}^0$  is the fitness value of the particle  $\text{pbest}$ ,  $i = 1, 2, \dots, n_p$ , and  $j = 1, 2, \dots, 2mlr$ , and set  $k = 0$ ;
- 1.4.2 Assign  $\min_i \{\gamma_i^0 \mid i \in \{1, 2, \dots, n_p\}\}$  to  $\gamma_g^0$  and  $f_{gj}(0)$  to  $\text{gbest}_j(0)$ , respectively, where  $\gamma_g^0$  is the fitness value of the particle  $\text{gbest}$ , and  $j = 1, 2, \dots, 2mlr$  and set  $k = 0$ ;

Step 2. Fitness evaluation of particles

- 2.1 Obtain  $F_{1\rho i}^k$  and  $F_{2\rho i}^k$  from  $f_{ij}(k)$  in  $n_p$  particles, where  $i = 1, 2, \dots, n_p$ ,  $\rho = 1, 2, \dots, r$ ,  $j = 1, 2, \dots, 2mlr$ , and  $k \geq 1$ ,  $k \in \mathbb{N}$ ;
- 2.2 Solve the minimization problem at the end of Section 6.2 to obtain the minimum  $\gamma_i^k$  for given  $F_{1\rho i}^k$  and  $F_{2\rho i}^k$ , where  $i \in \{1, 2, \dots, n_p\}$ ,  $\rho = 1, 2, \dots, r$ , and  $k \geq 1$ ,  $k \in \mathbb{N}$ ;
- 2.3 Record the previous best particles and their fitness values. If  $\gamma_i^k < \gamma_{ip}$ , then assign  $\gamma_i^k$  to  $\gamma_{ip}$  and  $f_{ij}(k)$  to  $\text{pbest}_{ij}$ , respectively, where  $i = 1, 2, \dots, n_p$ ,  $j = 1, 2, \dots, 2mlr$ , and  $k \geq 1$ ,  $k \in \mathbb{N}$ ;
- 2.4 Record the global best particle and its fitness value. If  $\gamma_g^{k-1} > \min\{\gamma_i^k\}_{i=1}^{n_p}$ , then assign  $\min_i\{\gamma_i^k \mid i \in \{1, 2, \dots, n_p\}\}$  to  $\gamma_g^k$  and  $f_{gj}(k)$  to  $\text{gbest}_j(k)$ , respectively; otherwise, assign  $\gamma_g^{k-1}$  to  $\gamma_g^k$  and store the corresponding particle, where  $j = 1, 2, \dots, 2mlr$ , and  $k \geq 1$ ,  $k \in \mathbb{N}$ . If  $|\gamma_g^k - \gamma_g^{k-1}| \leq \varepsilon$  is satisfied within  $m_\eta$  iterations, where  $\varepsilon > 0$  is a sufficiently small constant, then exit, and  $k \geq 1$ ,  $k \in \mathbb{N}$ ; otherwise, go to Step 2.5;
- 2.5 Update the velocity of  $n_p$  particles by (3.43) and (3.45) in Chapter 3. If  $\nu_{ij}(k) < \nu_j^{\min}$  or  $\nu_{ij}(k) > \nu_j^{\max}$ , then randomly generate  $\nu_{ij}(k)$  satisfying  $\nu_j^{\min} \leq \nu_{ij}(k) \leq \nu_j^{\max}$ , where  $i = 1, 2, \dots, n_p$ ,  $j = 1, 2, \dots, 2mlr$ , and  $k \geq 1$ ,  $k \in \mathbb{N}$ ;
- 2.6 Update the position of  $n_p$  particles by (3.44) in Chapter 3. If  $f_{ij}(k) < \alpha_j$  or  $f_{ij}(k) > \beta_j$ , then randomly generate  $f_{ij}(k)$  satisfying  $\alpha_j \leq f_{ij}(k) \leq \beta_j$ , where  $i = 1, 2, \dots, n_p$ ,  $j = 1, 2, \dots, 2mlr$ , and  $k \geq 1$ ,  $k \in \mathbb{N}$ ;
- 2.7 If  $k > m_\eta$ , where  $m_\eta$  is the maximum number of iterations, then exit; otherwise, set  $k = k + 1$  and go to Step 2.1;

Step 3. Obtain the minimum  $\gamma_g > 0$  and the corresponding  $F_1$  and  $F_2$  from the global best particle.

**Remark 6.3.** Algorithm 6.1 is a novel evolutionary algorithm which utilizes the particle swarm optimization technique with feasibility of an LMI-based  $H_\infty$  tracking performance criterion. This algorithm can effectively avoid some ideal assumptions and linearization techniques such as equality or inequality constraints and iteration algorithm in delay-dependent static output feedback control design.

## 6.4 An example

In this section, we will show the effectiveness of the proposed design method by performing network-based static output feedback tracking control for the following nonlinear system

$$\begin{cases} \dot{x}_1(t) = (1 + 0.1 \sin^2(x_1(t)))x_2(t) \\ \dot{x}_2(t) = 0.1x_2(t)(\cos^2(x_1(t)) - \sin^2(x_1(t))) - 2x_1(t) + u(t) + 0.1\omega(t) \\ y(t) = x_1(t) \end{cases} \quad (6.35)$$

where  $y(t)$  is the output and  $\omega(t)$  is the external disturbance.

Consider the following reference model

$$\begin{cases} \dot{x}_r(t) = -x_r(t) + 0.2r(t) \\ y_r(t) = 0.5x_r(t). \end{cases} \quad (6.36)$$

Suppose that the operating domain of the system (6.35) is  $\mathbb{D}_1 = \{x(t) : |x_1(t)| \leq 0.5\pi, |x_2(t)| \leq 0.5\}$ . Choose the premise variable as  $\theta(t) = y(t)$ . Then the nonlinear system (6.35) can be exactly expressed as

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^2 \mu_i(\theta(t)) [A_i x(t) + B_i u(t) + E_i \omega(t)] \\ y(t) = Cx(t) \end{cases} \quad (6.37)$$

where

$$\begin{aligned} A_1 &= \begin{bmatrix} 0 & 1 \\ -2 & 0.1 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad E_1 = \begin{bmatrix} 0 \\ 0.1 \end{bmatrix}, \\ A_2 &= \begin{bmatrix} 0 & 1.1 \\ -2.1 & 0 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad E_2 = \begin{bmatrix} 0 \\ 0.1 \end{bmatrix} \\ C &= [1 \quad 0], \quad \mu_1 = \cos^2(x_1(t)), \quad \mu_2 = \sin^2(x_1(t)), \end{aligned}$$

Table 6.1: The maximum  $\tau_M$  for given  $\tau_m = 0.30s$  and different  $N$ 

N	1	2	3
$\tau_M(s)$	0.5000	0.6403	0.7006

It is easy to see that  $|x_1(t) - x_1(kh)| \leq \int_{t-\tau(t)}^t |\dot{x}_1(s)| ds \leq 1.1\tau_M$  and  $|\mu_i - \mu_i^k| \leq |x_1(t) - x_1(kh)| = 1.1\tau_M$  ( $i = 1, 2$ ) hold on the compact region  $\mathbb{D}_1$ . Then we have  $\delta_i = \min\{1, 1.1\tau_M\}$  ( $i = 1, 2$ ).

Notice that there do not exist output feedback gains  $f_i$  such that  $A_i + B_i f_i C$  ( $i = 1, 2$ ) are Hurwitz, which means that the system (6.35) can not be stabilized by a fuzzy static output feedback controller. So it is impossible to fulfill the output tracking control task by using the following controller

$$u(t) = \sum_{i=1}^2 \mu_i [f_{1i}y(t) + f_{2i}y_r(t)] \quad (6.38)$$

where  $f_{1i}$  and  $f_{2i}$  ( $i = 1, 2$ ) are the output feedback gains to be determined. To achieve a stable and satisfactory tracking control, we intentionally insert a network that induces the communication delays ( $\tau_k^{sc}$  and  $\tau_k^{ca}$ ,  $\forall k \in \mathbb{Z}$ ) between the system (6.35) and the controller (6.38). Then the input of the system (6.35) in a network environment can be described by

$$\begin{cases} u(t) = \sum_{i=1}^2 \mu_i [f_{1i}y(t-\tau(t)) + f_{2i}y_r(t-\tau(t))] \\ t \in [kh + \tau_k, (k+1)h + \tau_{k+1}), k \in \mathbb{Z} \end{cases} \quad (6.39)$$

where  $\tau(t) = t - kh$ ,  $\tau_k = \tau_k^{sc} + \tau_k^{ca}$ ,  $\forall k \in \mathbb{Z}$ .

We now design the network-based fuzzy static output feedback tracking controller by using Algorithm 6.1. The search spaces of  $f_k^i$  ( $i = 1, 2, \dots, n_p$ ) are roughly set to be  $[\alpha_k, \beta_k] = [-2, 2]$  ( $k = 1, 2, 3, 4$ ) and the tuning parameters are initialized to be  $n_p = 20$ ,  $c_1 = c_2 = 2$ ,  $\omega_M = 0.7$ ,  $\omega_m = 0.4$ ,  $m_\eta = 100$  and  $v_k^{max} = 0.4$  ( $k = 1, 2, 3, 4$ ), which satisfy the convergence of a PSO algorithm [18]. For given  $\tau_m = 0.30s$ ,  $\tau_M = 0.50s$ ,  $N = 1$  and  $U = 1$ , applying Algorithm 6.1, we obtain the

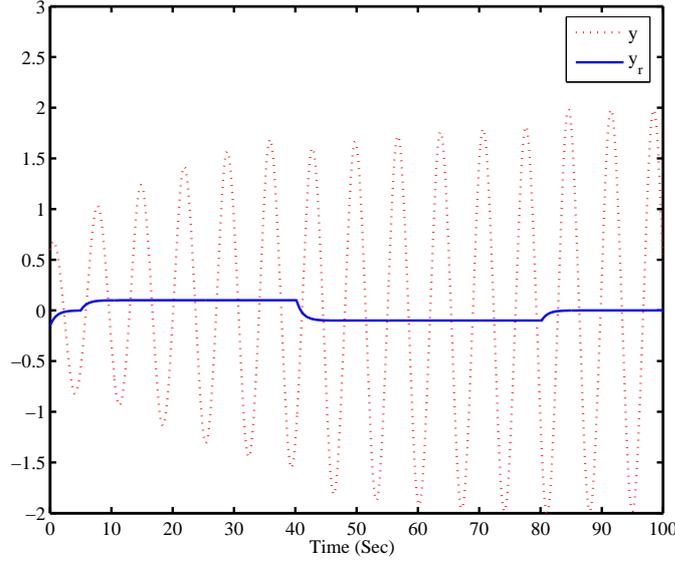


Figure 6.1: The outputs of the system (6.35)-(6.36) in Case I

minimum  $H_\infty$  tracking performance  $\gamma_{min} = 0.8812$  and the corresponding output feedback gains  $f_{11} = 1.3242$ ,  $f_{12} = 0.8121$ ,  $f_{21} = 1.2163$  and  $f_{22} = 0.5619$ . Then an  $H_\infty$  tracking performance  $\gamma_{min} = 0.8812$  is ensured by the obtained controller in the presence of  $\tau_k$  ( $\forall k \in \mathbb{Z}$ ) satisfying  $0.30s \leq \tau_k \leq 0.50s - h$ , where  $h$  ( $0 < h \leq 0.20s$ ) is a sampling period. Using Proposition 6.1 with  $f_{11} = 1.3242$ ,  $f_{12} = 0.8121$ ,  $f_{21} = 1.2163$  and  $f_{22} = 0.5619$ ,  $\gamma_{min} = 0.8812$  and  $\tau_m = 0.30s$ , the maximum  $\tau_M$  for different  $N$  is solved and listed in Table 5.1. From Table 5.1, we can clearly see that the interval  $[0.30s, \tau_M]$  is magnified as  $N$  increases, which means that the conservatism of the stability criterion is reduced significantly by increasing  $N$ . On the other hand, if there is no available knowledge of membership functions, using Proposition 6.1 with  $\tau_m = 0.30s$ ,  $\tau_M = 0.50s$  and the obtained output feedback gains, we can obtain the minimum  $H_\infty$  tracking performance  $\gamma_{min} = 1.3619$ , which is much larger than  $\gamma_{min} = 0.8812$ . Then we can conclude that a better  $H_\infty$  tracking performance can be achieved by using the criterion involving the knowledge of membership functions.

Next, we show the positive effect of the network-induced delay on the tracking performance. In simulation, the sampling period is  $h = 0.05s$  and the initial states

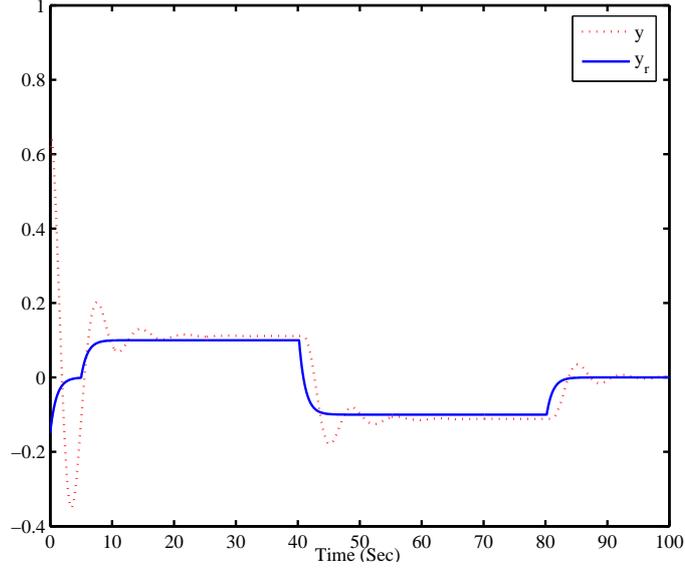


Figure 6.2: The outputs of the system (6.35)-(6.36) in Case II

are chosen as  $x(t_0) = [0.5 \ 0]^T$ ,  $x_r(t_0) = -0.3$ . It is assumed that

$$\omega(t) = \begin{cases} \sin(6t), & 10s \leq t \leq 60s \\ 0, & \text{otherwise,} \end{cases}$$

$$r(t) = \begin{cases} 1, & 5s \leq t \leq 35s \\ -1, & 35s < t \leq 65s \\ 0, & \text{otherwise.} \end{cases}$$

We depict the output responses of the system (6.35)-(6.36) in the following two cases, i.e., Case I: there is no network between the system (6.35) and the controller (6.38) and Case II: the network-induced delay  $\tau_k$  ( $\forall k \in \mathbb{Z}$ ) varies in  $[0.30s, 0.45s]$ . Figure 6.1 and Figure 6.2 shows the outputs of the system (6.35)-(6.36) in Case I and Case II, respectively. The output tracking error  $e(t)$  in Case II is shown by Figure 6.3. In Figure 6.1, the output of the system (6.35) is unstable when there is no network connecting the system (6.35) with the obtained controller. However, if we purposefully introduce a network-induced delay in Case II in the feedback control loop, the output of the system (6.35) can track the output of the reference model (6.36) very well, which can be clearly seen in Figure 6.2 and Figure 6.3. Then we can conclude that the purposeful introduction of a network-induced delay in the feedback control loop can produce a stable and satisfactory tracking control.

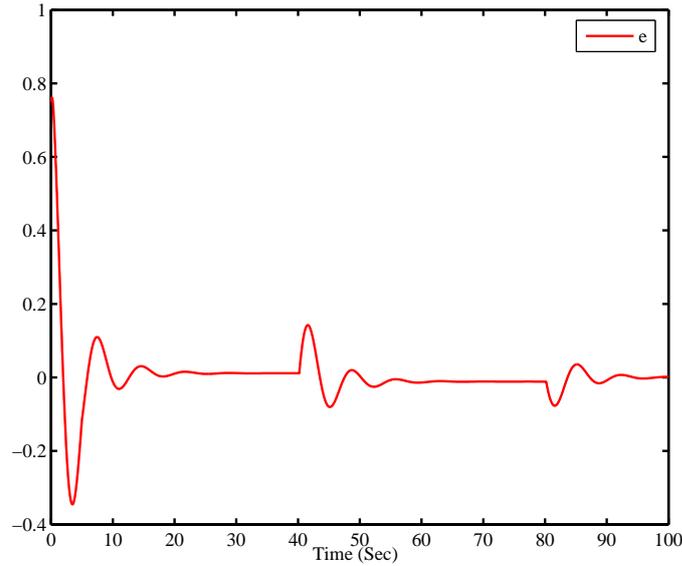


Figure 6.3: The output tracking error of the system (6.35)-(6.36) in Case II

## 6.5 Summary

In this chapter, we have dealt with network-based fuzzy static output feedback tracking control for a nonlinear system that can not be stabilized by a fuzzy static output feedback controller without a time-delay, but can be stabilized by a delayed fuzzy static output feedback controller. We have shown that a stable and satisfactory tracking control can be achieved by intentionally inserting a communication network that leads to a network-induced delay in the feedback control loop. The network-based tracking control system has been described by an asynchronous T-S fuzzy system with an interval time-varying sawtooth delay. A new discontinuous complete Lyapunov-Krasovskii functional has been constructed to establish a delay-dependent criterion for  $H_\infty$  tracking performance analysis. To reduce the conservatism of the obtained criterion, we have proposed a new relaxation method by utilizing the asynchronous constraints on membership functions to introduce some free-weighting matrices. A novel control design algorithm has been proposed by applying a particle swarm optimization technique with feasibility of the derived criterion. The effectiveness of the design method has been shown by an example.

# Chapter 7

## Conclusions and future work

### 7.1 Conclusions

This dissertation has been concerned with the network-based output tracking control for continuous-time systems. The systems under consideration are linear systems and nonlinear systems described by T-S fuzzy models, respectively. The systems have been classified as Case I: the systems can be stabilized by non-delayed static output feedback controllers; and Case II: the systems can not be stabilized by non-delayed static output feedback controllers, but can be stabilized by delayed static output feedback controllers.

For the systems in Case I, the negative effects of network-induced delays and/or packet dropouts on network-based output tracking control have been considered. More specifically, in this dissertation, the following problems have been studied.

- *Network-based state feedback tracking control for a linear system has been studied by taking network-induced delays and packet dropouts into account. Some delay-dependent criteria for  $H_\infty$  tracking performance analysis and controller design have been established by using a new discontinuous simple Lyapunov-Krasovskii functional and a generalized Jensen integral inequality that combines a convex delay analysis method. Two examples have shown that the obtained criteria are less conservative and can ensure a better  $H_\infty$  tracking performance over some existing ones.*

- *Network-based output tracking control for a linear system via an observer-based controller has been considered.* The network-based tracking control system has been modeled as a system with two different interval time-varying delays by taking into consideration the asynchronous inputs of the linear system and the controller due to the effects of network-induced delays and packet dropouts in the controller-to-actuator channel. A new delay-dependent criterion has been established such that the system with two delays is exponentially stable with a prescribed  $H_\infty$  tracking performance. Since a separation principle can not be applied to design the controller, a particle swarm optimization algorithm has been presented to search for the minimum  $H_\infty$  tracking performance and the corresponding gains. Network-based output tracking control of a mobile robot has shown the validity of the proposed method.
- *Network-based fuzzy state feedback tracking control for a nonlinear system via a T-S fuzzy model has been considered by using the knowledge of membership functions.* Using a fuzzy controller that depends on available sampled-data measurement of feedback states and premise variables, the network-based nonlinear control system has been represented by an asynchronous T-S fuzzy system with an interval time-varying sawtooth delay induced by sampling behaviors, network-induced delays and packet dropouts. Since a routine relaxation method for  $H_\infty$  performance analysis and controller design of a traditional T-S fuzzy system can not be used for that of the asynchronous fuzzy system, a new relaxation method has been proposed by using asynchronous constraints. Using the proposed relaxation method and a discontinuous simple Lyapunov-Krasovskii functional, some delay-dependent criteria for  $H_\infty$  tracking performance analysis and tracking controller design have been established in terms of linear matrix inequalities. Network-based tracking control of the Duffing forced-oscillation system has been demonstrated in simulation.

For the systems in Case II, the positive effects of network-induced delays on network-based output tracking control have been investigated. More specifically, the following problems have been considered.

- *For a linear system in Case II, the positive effect of a network-induced delay on network-based output tracking control has been investigated.* By intentionally inserting a network between the system and a static output feedback controller, a network-induced delay has been purposefully introduced in the feedback control loop to produce a stable and satisfactory tracking control. A new discontinuous complete Lyapunov-Krasovskii functional has been constructed to derive a delay-dependent criterion for  $H_\infty$  tracking performance analysis. By applying a particle swarm optimization technique with feasibility of the obtained criterion, a novel control design algorithm has been proposed to search for the minimum  $H_\infty$  tracking performance and the corresponding gain. The effectiveness of the proposed method has been shown by performing network-based output tracking control of a damped harmonic oscillator.
- *For a nonlinear system in Case II, the positive effect of a network-induced delay on network-based output tracking control has been investigated.* For such a system, it is impossible to achieve a stable tracking control by a non-delayed fuzzy static output feedback controller; however, by intentionally inserting a network between the nonlinear system and a fuzzy static output feedback controller, we have purposefully introduced a network-induced delay in the feedback control loop to produce a stable and satisfactory tracking control. Taking sampling behaviors and the network-induced delay into account, the network-based tracking control system has been modeled as an asynchronous T-S fuzzy system. By using a new discontinuous complete Lyapunov-Krasovskii functional and the asynchronous constraints on membership functions, a delay-dependent  $H_\infty$  tracking performance criterion has been established. The fuzzy tracking

control design algorithm has been proposed by applying a particle swarm optimization technique with feasibility of the obtained criterion. The effectiveness of the proposed method has been illustrated by an example.

## 7.2 Future work

Network-based output tracking control for systems has received increasing attention in recent years. The current research has been focused on some fundamental issues in network-based output tracking control for continuous-time systems, such as modeling, stability analysis and controller design of network-based tracking control systems in the presence of network-induced delays and/or packet dropouts. In addition, there are several topics associated with network-based output tracking control for the future research work. Some related topics are listed below.

- *The effects of some network-induced constraints such as quantization errors, time-varying samplings and communication constraints on network-based output tracking control will be investigated.* The effects of network-induced delays and/or packet dropouts on system stability and tracking performance have been investigated in this dissertation. Besides network-induced delays and packet dropouts, there exist some network-induced constraints such as quantization errors, time-varying samplings and communication constraints. It is important to investigate the effects of these network-induced constraints simultaneously on network-based output tracking control.
- *Network-based output tracking control for a continuous-time system via a dynamic output feedback controller will be investigated.* Notice that a static output feedback tracking controller can be simply implemented with low cost. However, it can not be applied in some particular cases, for example, if Kimura-Davison conditions are not satisfied. In this case, a dynamic output feedback controller is required to perform the network-based output tracking control.

Different from static output feedback control, a two-channel NCS via dynamic output feedback control is not equivalent to a one-channel NCS due to network-induced delays and packet dropouts in the controller-to-actuator channel. It is a challenging issue to design a network-based dynamic output feedback controller for the two-channel NCS.

- *A network-based nonlinear fuzzy tracking controller will be designed for a nonlinear system in a T-S fuzzy model when the asynchronous constraints on fuzzy membership functions are unknown or partly unknown.* In this dissertation, the fuzzy controller shares same fuzzy sets and premise variables with the T-S fuzzy model and the asynchronous constraints are available. It is of importance to investigate the network-based output tracking control for a T-S fuzzy system with unknown or partly unknown asynchronous constraints.
- *Network-based fuzzy output tracking control for a fuzzy-model-based nonlinear system via an observer-based controller is an problem to be addressed.* There exist two main difficulties related to the research problem. One is how to model the nonlinear network-based control system by taking the fire mechanisms of both the T-S fuzzy model and the fuzzy observer-based controller into account. The other is how to design the fuzzy observer-based tracking controller in a network environment when a separation principle does not work.
- *Some less conservative  $H_\infty$  tracking performance criteria with few matrix variables and tracking control design methods will be considered.* In this dissertation, the obtained criteria involve some free-weighting matrices, which cost much computational complexity. It is worthwhile to seek new methods to reduce the conservatism of the criteria without using many matrix variables. In addition, the proposed control design algorithm employs a standard particle swarm optimization technique, which leaves room for improvements.

- *Network-based output tracking control for a discrete-time system will be considered.* In this dissertation, the controlled plant is a continuous-time system and the reference model is a linear system. When the plant is a discrete-time system or the reference model is a nonlinear system, the problem of network-based output tracking control needs further investigation.

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