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On Designing Fuzzy Controllers for a Class of Nonlinear Networked Control Systems

Xiefu Jiang and Qing-Long Han

Abstract—This paper is concerned with controller design for a class of nonlinear networked control systems. These systems are approximated by uncertain linear networked Takagi–Sugeno (T–S) models with both network-induced delay and data packet dropout. Sufficient conditions are derived for the existence of a fuzzy controllers. Then, an iterative algorithm for the controller design is proposed. A control problem of a flexible-joint robot arm system is studied to show the effectiveness of the iterative algorithm.

Index Terms—Controller design, data packet dropout, fuzzy, networked control systems, network-induced delay, nonlinear systems, Takagi–Sugeno (T–S) model.

I. INTRODUCTION

THE TREND of modern manufacturing plants and distributed industrial processes/control systems is to integrate computing, communication, and control into different levels of factory operations and information processes. In traditional control systems, system components such as sensors, controllers, and actuators are connected together through point-to-point cables [1]. However, expanding physical setups and functionality are pushing the limits of the point-to-point architecture [2]. Recently, as a result of the vast progress in communication technology, as network technology becomes cheaper and more reliable than fixed point-to-point connections, even in small-localized systems, more and more control systems will operate over networks, especially for control systems with a large number of distributed sensors and actuators. Murray [3], for example, foresees sensor, actuator, diagnostic, and command and coordination signals all traveling over a network. Compared with point-to-point architectures, several advantages of the network architectures include: reduced system wiring, plug and play devices, increased system agility, and ease of system diagnosis and maintenance. Therefore, modeling, analysis, and control of

networked control systems (NCSs) have recently emerged as a topic of much interest to community. Researchers from a variety of disciplines have become interested in the existing potential and challenges of NCSs (see, for example, [1], [2], [4]–[7], and references therein).

Due to the introduction of the network, two of the major challenges in NCSs still to be fully addressed are the effects of both network-induced delay and data packet dropout on the system performance, since both of them can degrade the performance of control systems designed without considering them and even destabilize the systems. Therefore, the challenging question which remains to be answered is how to quantitatively evaluate the effects of network-induced delay and data packet dropout. The answer to this question will significantly enhance the applications of NCSs in many manufacturing plants and distributed industrial processes/control systems, resulting in improved reliability, efficiency, and productivity. Some methodologies have been formulated based on several types of network behaviors and configurations in conjunction with different ways to treat the delay problem. For example, Halevi and Ray [8] proposed a methodology as the augmented deterministic discrete-time model to control a linear plant over a periodic delay network. Nilsson [9] proposed the optimal stochastic control method to control an NCS on random delay networks. Walsh *et al.* [10] considered a linear continuous plant and a continuous controller. They introduced the notation of maximum allowable transfer interval (MATI), which supposes that successive sensor messages are separated by at most MATI seconds. Gökas [11] used a modified Padé approximation and considered the network delay as an uncertainty, and designed a networked controller in the frequency domain using the robust control theory. Zhang *et al.* [7] used a hybrid system technique to study the stability of an NCS under the network-induced delay. However, most of the aforementioned papers fall into the case of constant time-delay. Existing constant time-delay control methodologies [12], [13] may not be directly suitable for controlling a system over the network, since network delays are usually time-varying, especially on the Internet. Therefore, to handle time-varying network-induced delays in a closed-loop control system over a network, a more advanced methodology is required.

In terms of the data packet dropout issue in the NCSs, because of the uncertainties and noise in the communication channels, there exist unavoidable errors in the transmitted data packet or even loss. If this happens, the corrupted data packet is dropped and the receiver (controller or actuator) uses the data packet that it received most recently. In addition, data packet dropout may occur when one packet, say, a sampled value from the sensor, reaches the destination later than its successors. In such

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a situation, the old data packet is dropped, and its successive data packet is used instead. However, very limited work has been done in examining the data packet dropout effect on the NCSs. In [14], the performance of the system as measured by the gain was expressed as a function of packet loss. Zhang *et al.* [7] modeled the NCSs with data packet dropouts as asynchronous switched systems. Ling and Lemmon [15] proposed optimal dropout compensation.

From this brief review, one can see that most of the existing results are concerned with the analysis and synthesis of linear NCSs [5]–[7]. For nonlinear NCSs, little attention has been received due to the systems' complexity and only a few results are available. Walsh *et al.* [16] proposed a two-step design approach to use standard control methodologies and to choose the network protocol and bandwidth in order to ensure important closed-loop properties are preserved when a computer network is inserted into the feedback loop. Nešić and Teel [17] addressed the input-to-state stability and L_p stability of nonlinear NCSs and did not consider how to design the controller. Based on this fact, some new methods and approaches should be developed for designing controllers for nonlinear NCSs, which motivates this paper.

In this paper, we will investigate the effects of network-induced delay and data packet dropout for a class of nonlinear networked control systems. Different from the methodology proposed by Walsh *et al.* [16], we will employ the Takagi–Sugeno (T–S) model approach [18]. More specifically, the nonlinear networked control systems will be approximated by uncertain linear networked T–S models. Then, we will give a sufficient condition for the existence of a fuzzy controller under consideration of both network-induced delay and data packet dropout. Based on this condition, we will propose an iterative algorithm for obtaining the controller. Finally, the proposed method on the application example of the control of a flexible-joint robot arm through a network is illustrated to show the effectiveness of the iterative algorithm.

Notation. $\mathbb{R}^n$ denotes the n -dimensional Euclidean space. $\mathbb{R}^{m \times n}$ is the set of all $m \times n$ real matrices. The notation $P > 0$ ($P \geq 0$) means that P is symmetric and positive definite (positive semidefinite). For symmetric matrices P and Q , the notation $P > Q$ ($P \geq Q$) means that matrix $P - Q$ is positive definite (positive semidefinite). I is an identity matrix of appropriate dimensions. $\lambda_i(A)$ is the i th eigenvalue of a real matrix A . For a real symmetric matrix P , $\lambda_{\max}(P)$ [respectively, $\lambda_{\min}(P)$] denotes the maximum (respectively, minimum) eigenvalue of the matrix P . $\|\cdot\|$ stands for the Euclidean vector norm or the induced matrix 2-norm as appropriate. $\text{diag}\{\dots\}$ denotes the block-diagonal matrix.

II. PROBLEM STATEMENT

Consider the following nonlinear system

$$\dot{x}(t) = f(x(t)) + g(x(t))u(t) \quad (1)$$

where $x(t) \in \mathbb{R}^n$ is the state vector and $u(t) \in \mathbb{R}^p$ is the input vector. We assume that $f(x(t))$ and $g(x(t))$ are sufficiently smooth on a domain $\mathbb{D} \subset \mathbb{R}^n$, and $f(0) = 0$, where “sufficiently smooth” means that all the partial derivatives are defined and

continuous. The initial condition of the system (1) is given by

$$x(t_0) = x_0. \quad (2)$$

Throughout this paper, we assume that systems (1) and (2) are controlled through a network and the system state is available for feedback.

From [18], the nonlinear system (1) can be represented by some simple local linear dynamic systems with their linguistic description as

Plant Rule R^i :

IF $z_1(t)$ is M_{i1} , $z_2(t)$ is M_{i2} , $\dots$, $z_g(t)$ is M_{ig}

THEN $\dot{x}(t) = A_i x(t) + B_i u(t)$ (3)

where $i = 1, 2, \dots, r$ and r is the number of IF–THEN rules; $z_1(t), z_2(t), \dots, z_g(t)$ are the premise variables of (3) and M_{ij} ($i = 1, 2, \dots, r$; $j = 1, 2, \dots, g$) are the fuzzy sets corresponding to $z_j(t)$ and the plant rules; and A_i and B_i are known parameter matrices of appropriate dimensions.

By using a center average defuzzifier, product inference, and a singleton fuzzifier, the global dynamics of the T–S fuzzy systems (3) are described by

$$\dot{x}(t) = \sum_{i=1}^r h_i(z(t)) [A_i x(t) + B_i u(t)] \quad (4)$$

where

$$h_i(z(t)) = \frac{\mu_i(z(t))}{\sum_{i=1}^r \mu_i(z(t))}, \mu_i(z(t)) = \prod_{j=1}^g M_{ij}(z_j(t))$$

$$z(t) = [z_1(t), z_2(t), \dots, z_g(t)]$$

in which $M_{ij}(z_j(t))$ is the grade of membership of $z_j(t)$ in M_{ij} . Then, it can be seen that

$$\mu_i(z(t)) \geq 0, i = 1, 2, \dots, r, \sum_{i=1}^r \mu_i(z(t)) > 0, \forall t \geq t_0.$$

Therefore

$$h_i(z(t)) \geq 0, i = 1, 2, \dots, r, \sum_{i=1}^r h_i(z(t)) = 1, \forall t \geq t_0. \quad (5)$$

From (1) and (4), we have

$$\begin{aligned} \dot{x}(t) &= \sum_{i=1}^r h_i(z(t)) [A_i x(t) + B_i u(t)] \\ &+ \left(f(x(t)) - \sum_{i=1}^r h_i(z(t)) A_i x(t) \right) \\ &+ \left(g(x(t)) u(t) - \sum_{i=1}^r h_i(z(t)) B_i u(t) \right) \\ &\triangleq \sum_{i=1}^r h_i(z(t)) [A_i x(t) + B_i u(t)] + \Delta f + \Delta g \end{aligned} \quad (6)$$

where $\Delta f + \Delta g$ is the approximation error between the nonlinear model (1) and the global fuzzy model (4).

Fig. 1. Networked control system.

In the presence of the control network, which is shown in Fig. 1, data transfers between the controller and the remote system, e.g., sensors and actuators in a distributed control system, will induce networked delay in addition to the controller proceeding delay. As mentioned in the introduction, there are essentially three kinds of delay: (1) communication delay τ^{sc} between the sensor and the controller; (2) computational delay τ^c in the controller; and (3) communication delay τ^{ca} between the controller and the actuator. Since the computational delay τ^c is usually very small, it is omitted in this paper.

First, since there exists the communication delay τ^{sc} between the sensor and the controller, which is shown in Fig. 1, the following state feedback T-S fuzzy-model-based control law is employed for the system (6) by utilizing the idea of parallel distributed compensation (PDC) [19], [20] in which the same fuzzy sets with the fuzzy model are shared for the designed fuzzy controller in the premise parts

$$\begin{aligned} R^i: & \text{IF } z_1(t) \text{ is } M_{i1}, z_2(t) \text{ is } M_{i2}, \dots, z_g(t) \text{ is } M_{ig} \\ \text{THEN } & u(t^+) = K_i x(t - \tau_k^{sc}) \\ & t \in \{kh + \tau_k^{sc}, k = 1, 2, \dots\} \end{aligned} \quad (7)$$

where $u(t^+) = \lim_{\xi \rightarrow t+0} u(\xi)$, K_i ($i = 1, 2, \dots, r$) are the controller gains of (7) to be determined, and h is the sampling period. Analogous to (4), the defuzzified output of the controller rules is given by

$$\begin{aligned} u(t^+) &= \sum_{i=1}^r h_i(z(t)) K_i x(t - \tau_k^{sc}) \\ & t \in \{kh + \tau_k^{sc}, k = 1, 2, \dots\}. \end{aligned} \quad (8)$$

In this paper, we assume that $u(t) = 0$ before the first control signal reaches the plant. For the sake of simplicity, $h_i(z(t))$ is denoted by h_i in the following.

Second, substituting (8) into (6) yields the closed-loop global fuzzy system of (6) by considering the communication delay τ^{ca} between the controller and the actuator.

$$\dot{x}(t) = \sum_{i=1}^r \sum_{j=1}^r h_i h_j [A_i x(t) + B_i K_j x(kh)] + \Delta f + \Delta g \quad (9)$$

for $t \in [kh + \tau_k, (k+1)h + \tau_{k+1})$, where $\tau_k = \tau_k^{sc} + \tau_k^{ca}$, $k = 1, 2, \dots$, $\Delta g = \sum_{i=1}^r \sum_{j=1}^r h_i h_j (g(x(t)) - B_i) K_j x(kh)$. Considering the data packet dropout, the closed-loop global

Fig. 2. Time diagram for data packets.

fuzzy system (9) can be modified as

$$\dot{x}(t) = \sum_{i=1}^r \sum_{j=1}^r h_i h_j [A_i x(t) + B_i K_j x(i_k h)] + \Delta f + \Delta g \quad (10)$$

where $\Delta g = \sum_{i=1}^r \sum_{j=1}^r h_i h_j (g(x(t)) - B_i) K_j x(i_k h)$ for $t \in [i_k h + \tau_k, i_{k+1} h + \tau_{k+1})$, $i_k, k = 1, 2, 3, \dots$ are some integers, and $\{i_1, i_2, i_3, \dots\} \subset \{0, 1, 2, 3, \dots\}$. The time-delay $\tau_k = \tau_k^{sc} + \tau_k^{ca}$ denotes the time from the instant $i_k h$ when sensor nodes sample sensor data from a plant to the instant when actuators transfer data to the plant (as shown in Fig. 2). Obviously, $\bigcup_{k=1}^{\infty} [i_k h + \tau_k, i_{k+1} h + \tau_{k+1}) = [t_0, \infty)$, $t_0 \geq 0$.

Suppose that there exist known real constant matrices D_i , E_i , and E_{ib} ($i = 1, 2, \dots, r$) of appropriate dimensions such that

$$\Delta f = \sum_{i=1}^r h_i D_i F_i(t) E_i x(t) \quad (11)$$

and

$$\Delta g = \sum_{i=1}^r \sum_{j=1}^r h_i h_j D_i F_i(t) E_{ib} K_j x(i_k h) \quad (12)$$

where $F_i(t) \in \mathbb{R}^{l \times m}$ ($i = 1, 2, \dots, r$) are unknown time-varying matrix functions with Lebesgue measurable elements satisfying

$$F_i^T(t) F_i(t) \leq I, i = 1, 2, \dots, r. \quad (13)$$

Then, (10) can be rewritten as

$$\dot{x}(t) = \sum_{i=1}^r \sum_{j=1}^r h_i h_j [A_i(t) x(t) + B_i(t) K_j x(i_k h)] \quad (14)$$

for $t \in [i_k h + \tau_k, i_{k+1} h + \tau_{k+1})$, $k = 1, 2, \dots$, where $A_i(t) = A_i + \Delta A_i(t)$ and $B_i(t) = B_i + \Delta B_i(t)$ ($i = 1, 2, \dots, r$). $\Delta A_i(t)$ and $\Delta B_i(t)$ are unknown real matrices of appropriate dimensions representing time-varying parameter uncertainties of the system (14) and satisfying

$$[\Delta A_i(t) \quad \Delta B_i(t)] = D_i F_i(t) [E_i \quad E_{ib}], i = 1, 2, \dots, r. \quad (15)$$

The following assumptions and definitions are required throughout the paper.

Assumption 1: [5], [6] The sensor is clock-driven; the controller and actuator are event-driven.

Assumption 2: [5], [6] There exist two constants $\tau_m \geq 0$ and $\eta > 0$ such that

$$\begin{cases} (i_{k+1} - i_k)h + \tau_{k+1} \leq \eta \\ \tau_k \geq \tau_m, k = 1, 2, \dots \end{cases} \quad (16)$$

Remark 1: Since $x(i_k h) = x(t - (t - i_k h))$, defining $\tau(t) = t - i_k h$, $t \in [i_k h + \tau_k, i_{k+1} h + \tau_{k+1})$, $k = 1, 2, \dots$, we rewrite (14) as

$$\dot{x}(t) = \sum_{i=1}^r \sum_{j=1}^r h_i h_j [A_i(t)x(t) + B_i(t)K_j x(t - \tau(t))] \quad (17)$$

where $\tau(t)$ is piecewise-linear with derivative $\dot{\tau}(t) = 1$ for $t \neq t_k$ and is discontinuous at the points $t = t_k$, $k = 1, 2, \dots$. It is clear that $\tau_k \leq \tau(t) \leq (i_{k+1} - i_k)h + \tau_{k+1} \leq \eta$ for $t \in [i_k h + \tau_k, i_{k+1} h + \tau_{k+1})$, $k = 1, 2, \dots$. We supplement the initial condition of the state $x(t)$ on $[t_0 - \eta, t_0]$ as

$$x(t) = \phi(t), t \in [t_0 - \eta, t_0] \quad (18)$$

with $\phi(t_0) = x_0$, where $\phi(t)$ is a continuous function on $[t_0 - \eta, t_0]$.

Definition 1: System (17) is said to be robustly exponentially stable if there exist constants $\alpha > 0$ and $\beta > 0$ such that $\|x(t)\| \leq \alpha \sup_{t_0 - \eta \leq s \leq t_0} \|\phi(s)\| e^{-\beta(t-t_0)}$ for $t \geq t_0$ for all admissible uncertainties satisfying (15).

From Remark 1, it is clear to see that system (14) is equivalent to the delay system (17). In the following, system (14) being robustly exponentially stable means that the delay system (17) is robustly exponentially stable.

To end this section, we introduce the following lemmas which are useful in deriving the sufficient condition of the existence of fuzzy controllers.

Lemma 1: For two matrices $Z \in \mathbb{R}^{n \times m}$, $G \in \mathbb{R}^{n \times m}$, and a symmetric positive definite matrix $P \in \mathbb{R}^{n \times n}$, we have

$$-G^T P^{-1} G \leq Z^T P Z - G^T Z - Z^T G. \quad (19)$$

Equality holds if and only if $Z = P^{-1} G$.

Proof: It is easy to see that

$$(PZ - G)^T P^{-1} (PZ - G) \geq 0.$$

Rearranging this matrix inequality yields (19). The equality holds if and only if $PZ = G$, i.e., $Z = P^{-1} G$. $\square$

Lemma 2: [21] For any constant matrix $W \in \mathbb{R}^{n \times n}$, $W = W^T > 0$, scalar $\gamma > 0$, and vector function $\dot{x} : [-\gamma, 0] \rightarrow \mathbb{R}^n$ such that the following integration is well defined, then

$$\begin{aligned} & -\gamma \int_{-\gamma}^0 \dot{x}^T(t + \xi) W \dot{x}(t + \xi) d\xi \\ & \leq \begin{pmatrix} x^T(t) & x^T(t - \gamma) \end{pmatrix} \begin{pmatrix} -W & W \\ W & -W \end{pmatrix} \begin{pmatrix} x(t) \\ x(t - \gamma) \end{pmatrix}. \end{aligned}$$

III. MAIN RESULT

Notice that the system described by (13)–(15) can be rewritten as [22], [23]

$$\dot{x}(t) = \sum_{i=1}^r \sum_{j=1}^r h_i h_j [A_i x(t) + B_i K_j x(i_k h) + D_i v_{ij}(t)] \quad (20)$$

subject to uncertain feedback

$$v_{ij}(t) = F_i(t) g_{ij}(t), \quad (21)$$

$$g_{ij}(t) = E_i x(t) + E_{ib} K_j x(i_k h) \quad (22)$$

for $t \in [i_k h + \tau_k, i_{k+1} h + \tau_{k+1})$, $k = 1, 2, \dots$. In view of (13), (21) and (22), we have

$$\begin{aligned} v_{ij}^T(t) v_{ij}(t) & \leq [E_i x(t) + E_{ib} K_j x(i_k h)]^T \\ & \quad \times [E_i x(t) + E_{ib} K_j x(i_k h)]. \end{aligned} \quad (23)$$

The following proposition gives a sufficient condition of the existence of the fuzzy controller of system (14).

Proposition 1: For given scalars τ_m and η , system (14) is robustly exponentially stable for network-induced delay and data packet dropout satisfying (16) and all admissible uncertainties by (13) and (15), if there exist a scalar $\tilde{\varepsilon} > 0$, some matrices $X > 0$, $\tilde{Q} > 0$, $R > 0$, $S > 0$, and Y_j ($j = 1, 2, \dots, r$) of appropriate dimensions such that

$$\begin{cases} \Xi_{ii} < 0, & \text{for } 1 \leq i \leq r \\ \Xi_{ij} + \Xi_{ji} < 0, & \text{for } 1 \leq i < j \leq r \end{cases} \quad (24)$$

simultaneously hold, where

$$\Xi_{ij} = \begin{bmatrix} \Gamma_{11} & \Gamma_{12} & \Gamma_{13} & \tilde{\varepsilon} D_i & \Gamma_{15} & \Gamma_{16} & \Gamma_{17} \\ \Gamma_{12}^T & \Gamma_{22} & 0 & 0 & \Gamma_{25} & \Gamma_{26} & \Gamma_{27} \\ \Gamma_{13}^T & 0 & \Gamma_{33} & 0 & \Gamma_{25} & \Gamma_{26} & \Gamma_{27} \\ \tilde{\varepsilon} D_i^T & 0 & 0 & -\tilde{\varepsilon} I & \Gamma_{45} & \Gamma_{46} & 0 \\ \Gamma_{15}^T & \Gamma_{25}^T & \Gamma_{25}^T & \Gamma_{45}^T & -R^{-1} & 0 & 0 \\ \Gamma_{16}^T & \Gamma_{26}^T & \Gamma_{26}^T & \Gamma_{46}^T & 0 & -S^{-1} & 0 \\ \Gamma_{17}^T & \Gamma_{27}^T & \Gamma_{27}^T & 0 & 0 & 0 & -\tilde{\varepsilon} I \end{bmatrix}$$

with

$$\delta = \eta - \tau_m$$

$$\Gamma_{11} = A_i X + B_i Y_j + (A_i X + B_i Y_j)^T$$

$$\Gamma_{12} = B_i Y_j$$

$$\Gamma_{13} = -\tilde{Q} + B_i Y_j$$

$$\Gamma_{15} = \tau_m (A_i X + B_i Y_j)^T$$

$$\Gamma_{16} = \delta (A_i X + B_i Y_j)^T$$

$$\Gamma_{17} = (E_i X + E_{ib} Y_j)^T$$

$$\Gamma_{22} = -X S X$$

$$\Gamma_{25} = \tau_m (B_i Y_j)^T$$

$$\Gamma_{26} = \delta (B_i Y_j)^T$$

$$\begin{aligned}\Gamma_{27} &= (E_{ib}Y_j)^T \\ \Gamma_{33} &= -\tilde{Q} - XRX \\ \Gamma_{45} &= \tau_m \tilde{\varepsilon} D_i^T \\ \Gamma_{46} &= \delta \tilde{\varepsilon} D_i^T.\end{aligned}$$

Moreover, the controller gains of (8) are given by $K_j = Y_j X^{-1}$ for $j = 1, 2, \dots, r$.

Proof: See the Appendix. $\square$

Notice that (24) is not a linear matrix inequality (LMI), so it cannot be solved by MATLAB LMI Toolbox. Using Lemma 1, (24) is implied by

$$\begin{cases} \Phi(i, i) < 0, \text{ for } 1 \leq i \leq r \\ \Phi(i, j) + \Phi(j, i) < 0, \text{ for } 1 \leq i < j \leq r \end{cases} \quad (25)$$

where

$$\Phi(i, j) = \begin{bmatrix} \hat{\Gamma}_{11} & \hat{\Gamma}_{12} & \hat{\Gamma}_{13} & \tilde{\varepsilon} D_i & \hat{\Gamma}_{15} & \hat{\Gamma}_{16} & \hat{\Gamma}_{17} \\ \hat{\Gamma}_{12}^T & \hat{\Gamma}_{22} & 0 & 0 & \hat{\Gamma}_{25} & \hat{\Gamma}_{26} & \hat{\Gamma}_{27} \\ \hat{\Gamma}_{13}^T & 0 & \hat{\Gamma}_{33} & 0 & \hat{\Gamma}_{25} & \hat{\Gamma}_{26} & \hat{\Gamma}_{27} \\ \tilde{\varepsilon} D_i^T & 0 & 0 & -\tilde{\varepsilon} I & \hat{\Gamma}_{45} & \hat{\Gamma}_{46} & 0 \\ \hat{\Gamma}_{15}^T & \hat{\Gamma}_{25}^T & \hat{\Gamma}_{25}^T & \hat{\Gamma}_{45}^T & -\tilde{R} & 0 & 0 \\ \hat{\Gamma}_{16}^T & \hat{\Gamma}_{26}^T & \hat{\Gamma}_{26}^T & \hat{\Gamma}_{46}^T & 0 & -\tilde{S} & 0 \\ \hat{\Gamma}_{17}^T & \hat{\Gamma}_{27}^T & \hat{\Gamma}_{27}^T & 0 & 0 & 0 & -\tilde{\varepsilon} I \end{bmatrix}$$

with

$$\begin{aligned}\hat{\Gamma}_{11} &= A_i X + B_i Y_j + (A_i X + B_i Y_j)^T \\ \hat{\Gamma}_{12} &= B_i Y_j \\ \hat{\Gamma}_{13} &= -\tilde{Q} + B_i Y_j \\ \hat{\Gamma}_{15} &= \tau_m (A_i X + B_i Y_j)^T \\ \hat{\Gamma}_{16} &= \delta (A_i X + B_i Y_j)^T \\ \hat{\Gamma}_{17} &= (E_i X + E_{ib} Y_j)^T \\ \hat{\Gamma}_{22} &= Z^T \tilde{S} Z - Z^T X - X Z \\ \hat{\Gamma}_{25} &= \tau_m (B_i Y_j)^T \\ \hat{\Gamma}_{26} &= \delta (B_i Y_j)^T \\ \hat{\Gamma}_{27} &= (E_{ib} Y_j)^T \\ \hat{\Gamma}_{33} &= \tilde{Z}^T \tilde{R} \tilde{Z} - \tilde{Z}^T X - X \tilde{Z} - \tilde{Q} \\ \hat{\Gamma}_{45} &= \tau_m \tilde{\varepsilon} D_i^T \\ \hat{\Gamma}_{46} &= \delta \tilde{\varepsilon} D_i^T\end{aligned}$$

for two given $n \times n$ matrices Z and $\tilde{Z}$, where $\tilde{R} = R^{-1}$, $\tilde{S} = S^{-1}$. (25) is equivalent to (24) if and only if $Z = \tilde{S}^{-1} X$ and $\tilde{Z} = \tilde{R}^{-1} X$.

One can clearly see that the nonlinear matrix inequality (24) is solvable for $\tau_m = \delta = 0$ if (24) is solvable for $\tau_m > 0$ and $\delta > 0$. For $\tau_m = \delta = 0$, by pre- and postmultiplying both sides of the nonlinear matrix inequality (24) with the diagonal matrix $\text{diag}\{I, I, I, I, I, X R, X S, I, I\}$ and its transpose, respec-

tively, one can obtain that (24) is equivalent to

$$\begin{cases} \tilde{\Phi}(i, i) < 0, & \text{for } 1 \leq i \leq r \\ \tilde{\Phi}(i, j) + \tilde{\Phi}(j, i) < 0, & \text{for } 1 \leq i < j \leq r \end{cases} \quad (26)$$

where

$$\tilde{\Phi}(i, j) = \begin{bmatrix} \Gamma_{11} & \Gamma_{12} & \Gamma_{13} & \tilde{\varepsilon} D_i & 0 & 0 & \Gamma_{17} \\ \Gamma_{12}^T & -\tilde{S} & 0 & 0 & 0 & 0 & \Gamma_{27} \\ \Gamma_{13}^T & 0 & \bar{\Gamma}_{33} & 0 & 0 & 0 & \Gamma_{27} \\ \tilde{\varepsilon} D_i^T & 0 & 0 & -\tilde{\varepsilon} I & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\tilde{R} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\tilde{S} & 0 \\ \Gamma_{17}^T & \Gamma_{27}^T & \Gamma_{27}^T & 0 & 0 & 0 & -\tilde{\varepsilon} I \end{bmatrix}$$

with $\bar{\Gamma}_{33} = -\tilde{Q} - \tilde{R}$, $\tilde{R} = X R X$, and $\tilde{S} = X S X$. If (26) has a feasible solution $(\tilde{\varepsilon}, X, \tilde{Q}, \tilde{R}, \tilde{S}, Y_j)$ for $j = 1, 2, \dots, r$, then (25) is solvable with a feasible solution $(\tilde{\varepsilon}, X, \tilde{Q}, \tilde{R} = X \tilde{R}^{-1} X, \tilde{S} = X \tilde{S}^{-1} X, Y_j)$ for $j = 1, 2, \dots, r$, $Z = X^{-1} \tilde{S}$, $\tilde{Z} = X^{-1} \tilde{R}$, and $\tau_m = \delta = 0$. Therefore, there exist $\tau_m > 0$ and $\delta > 0$ such that (25) is solvable for $Z = X^{-1} \tilde{S}$ and $\tilde{Z} = X^{-1} \tilde{R}$. If (26) is not solvable, then the original nonlinear matrix inequality (24) is not solvable for any $\tau_m > 0$ or $\delta > 0$.

Based on the aforesaid discussion, one can see that two constant matrices Z and $\tilde{Z}$ are introduced for solving nonlinear matrix inequality (24). As a result, they need to be given in advance for solving the LMI (25). In general, it is not easy to determine the matrices Z and $\tilde{Z}$ in advance before solving the LMI (25). By Lemma 1, if there exist a scalar $\tilde{\varepsilon} > 0$, some matrices $X > 0$, $\tilde{Q} > 0$, $\tilde{R} > 0$, $\tilde{S} > 0$, and Y_j of appropriate dimensions such that (25) holds for two given $n \times n$ matrices Z and $\tilde{Z}$, then (25) holds for $\tilde{\varepsilon} > 0$, $X > 0$, $\tilde{Q} > 0$, $\tilde{R} > 0$, $\tilde{S} > 0$, $Y_j (j = 1, 2, \dots, r)$ and $Z = \tilde{S}^{-1} X$, $\tilde{Z} = \tilde{R}^{-1} X$. Therefore, the following novel iterative algorithm is proposed for finding a maximum interval $[\tau_m^*, \tau_m^* + \delta]$ such that the closed-loop system (14) is robustly exponentially stable for any network-induced delay and data packet dropout satisfying (16) by solving the nonlinear matrix inequality (24), and the matrices Z and $\tilde{Z}$ do not need to be given in advance, where $\tau_m^* \geq 0$ is a given constant.

Algorithm 1: 1) For $\tau_m = \delta = 0$, find a feasible solution to the LMI (26)

$$(\tilde{\varepsilon}_0, X_0, \tilde{Q}_0, \tilde{R}_0, \tilde{S}_0, Y_{j0})$$

where $j = 1, 2, \dots, r$. Set $Z_1 = X_0^{-1} \tilde{S}_0$, $\tilde{Z}_1 = X_0^{-1} \tilde{R}_0$, and $k = 1$. If there is none, exit.

2) For two $n \times n$ matrices $Z = Z_1$, $\tilde{Z} = \tilde{Z}_1$, and a sufficiently small constant $\tau_m > 0$, find a maximum $\delta = \delta_1$ such that the LMI (25) is solvable with a feasible solution

$$(\tilde{\varepsilon}_1, X_1, \tilde{Q}_1, \tilde{R}_1, \tilde{S}_1, Y_{j1})$$

where $j = 1, 2, \dots, r$. Set $Z_2 = \tilde{S}_1^{-1} X_2$, $\tilde{Z}_2 = \tilde{R}_1^{-1} X_1$, and $k = k + 1$.

3) For the matrices Z_k , $\tilde{Z}_k$ and the given constant $\tau_m > 0$, find a maximum $\delta = \delta_k$ such that the LMI (25) is solvable with

Fig. 3. Flexible-joint robot arm system.

a feasible solution for $j = 1, 2, \dots, r$

$$(\tilde{\varepsilon}_k, X_k, \tilde{Q}_k, \tilde{R}_k, \tilde{S}_k, Y_{jk}).$$

4) If a stopping criterion is satisfied, exit. Otherwise, set $Z_{k+1} = \tilde{S}_k^{-1} X_k$, $\tilde{Z}_{k+1} = \tilde{R}_k^{-1} X_k$, and $k = k + 1$. If $\tau_m < \tau_m^*$, then return to Step 3) after increasing τ_m to some extent until $\tau_m = \tau_m^*$. Otherwise, return to Step 3) directly.

Remark 2: Steps 1), 2) and every Step 3) of the algorithm are simple LMI problems, and they can be solved directly by using the MATLAB LMI Toolbox. The stopping criterion can be chosen as a specified number of iterations or the minimum increment of δ .

The following proposition shows that the proposed algorithm is convergent.

Proposition 2: The sequence $\{\delta_k\}$ is monotone increasing and bounded upper by δ_M for a given $\tau_m > 0$, where $\delta_M > 0$ is determined by a given time-delay system. Thus, the sequence $\{\delta_k\}$ converges to some value $\delta_\infty \leq \delta_M$.

Proof: It is easy to see from Lemma 1 that (25) implies (24). For $k > 0$, given constant $\tau_m > 0$ and matrices $Z_k, \tilde{Z}_k$, if there exist $\tilde{\varepsilon}_k, X_k, \tilde{Q}_k, \tilde{R}_k, \tilde{S}_k, Y_{jk}$ such that (25) holds for δ_k , then (24) also holds. So (25) holds for $Z_{k+1} = \tilde{S}_k^{-1} X_k$, $\tilde{Z}_{k+1} = \tilde{R}_k^{-1} X_k$, $\tilde{\varepsilon}_k, X_k, \tilde{Q}_k, \tilde{R}_k, \tilde{S}_k, Y_{jk}$, and δ_k . Thus, for matrices $Z_{k+1}, \tilde{Z}_{k+1}$, and the given constant $\tau_m > 0$, we can find a maximum $\delta_{k+1} \geq \delta_k$ such that (25) is solvable with a feasible solution $\tilde{\varepsilon}_{k+1}, X_{k+1}, \tilde{Q}_{k+1}, \tilde{R}_{k+1}, \tilde{S}_{k+1}, Y_{j,k+1}$. Moreover, for a given time-delay system (1) and a given constant $\tau_m > 0$, there exists $\delta_M > 0$ such that $\delta_k \leq \delta_M$. $\square$

Remark 3: One can find from the aforesaid discussion that if the original nonlinear matrix inequality (24) is solvable for some $\tau_m^* > 0$ and $\delta^* > 0$, then a solution to (24) can be found for some $0 < \tau_m \leq \tau_m^*$ and $0 < \delta \leq \delta^*$ by the proposed iterative algorithm. If no solution can be found for any $\tau_m \geq 0$ or $\delta \geq 0$ by the proposed iterative algorithm, then the original nonlinear matrix inequality (24) is not solvable for any $\tau_m \geq 0$ or $\delta \geq 0$.

IV. NUMERICAL EXAMPLE

To illustrate the iterative algorithm for fuzzy controller design, we study the following control problem of a flexible-joint robot arm system, shown in Fig. 3 [24], which is controlled through a network.

Taking into account uncertainties, the flexible-joint robot arm model is considered as [24]

$$\begin{cases} (I_1 + \delta I_1) \ddot{\theta}_1 + (mgl + \delta m) \sin(\theta_1) + (k + \delta k)(\theta_1 - \theta_2) = 0 \\ (I_2 + \delta I_2) \ddot{\theta}_2 + (b + \delta b) \dot{\theta}_2 + (k + \delta k)(\theta_2 - \theta_1) = u + \delta u \end{cases} \quad (27)$$

where

$$|\delta I_1| \leq c \cdot I_1, |\delta_2| \leq c \cdot I_2, |\delta m| \leq c \cdot mgl$$

$$|\delta k| \leq c \cdot k, |\delta u| \leq c \cdot u, |\delta b| \leq c \cdot b.$$

Let $x_1 = \theta_1$, $x_2 = \dot{\theta}_1$, $x_3 = \theta_2$, and $x_4 = \dot{\theta}_2$. Then, the state equation of the system is

$$\dot{x}(t) = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \Upsilon_1 & 0 & \Upsilon_2 & 0 \\ 0 & 0 & 0 & 1 \\ \Upsilon_3 & 0 & -\Upsilon_3 & \Upsilon_4 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{u(t) + \delta u(t)}{I_2 + \delta I_2} \end{bmatrix} \quad (28)$$

where

$$\Upsilon_1 = -\frac{mgl + \delta m}{I_1 + \delta I_1} \cdot \frac{\sin x_1(t)}{x_1(t)} - \frac{k + \delta k}{I_1 + \delta I_1}$$

$$\Upsilon_2 = \frac{k + \delta k}{I_1 + \delta I_1}, \Upsilon_3 = \frac{k + \delta k}{I_2 + \delta I_2}, \Upsilon_4 = -\frac{b + \delta b}{I_2 + \delta I_2}$$

$$x(t) = [x_1(t), x_2(t), x_3(t), x_4(t)]^T.$$

The system parameters are taken to be $m = 0.01$ kg, $I_1 = I_2 = 1$ kgm², $k = 0.05$ N.m/rad, $l = 1$ m, $b = 0.007$ N.ms/rad, $c = 10\%$, and $g = 9.81$ m/s² for illustrative purposes [24], [25]. Then, the following T-S model is used to approximate the nonlinear system (28):

$$\left\{ \begin{array}{l} \text{Plant Rule } R^1: \text{ IF } x_1(t) \text{ is about } 0 \\ \quad \text{THEN } \dot{x}(t) = (A_1 + D_1 F_1 E_1) x(t) \\ \quad \quad \quad + (B_1 + D_1 F_1 E_{1b}) u(t) \\ \text{Plant Rule } R^2: \text{ IF } x_1(t) \text{ is about } \pm \pi/2 \\ \quad \text{THEN } \dot{x}(t) = (A_2 + D_2 F_2 E_2) x(t) \\ \quad \quad \quad + (B_2 + D_2 F_2 E_{2b}) u(t) \end{array} \right. \quad (29)$$

where

$$A_1 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -0.1481 & 0 & 0.05 & 0 \\ 0 & 0 & 0 & 1 \\ 0.05 & 0 & -0.05 & -0.007 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -0.1125 & 0 & 0.05 & 0 \\ 0 & 0 & 0 & 1 \\ 0.05 & 0 & -0.05 & -0.007 \end{bmatrix}$$

$$B_1 = B_2 = [0 \ 0 \ 0 \ 1]^T$$

Fig. 4. Trajectories of states x_1 , x_2 , x_3 , and x_4 .

$$D_1 = D_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$E_1 = E_2 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ \frac{2}{9} & 0 & \frac{2}{9} & 0 \\ 0 & 0 & 0 & 0 \\ \frac{2}{9} & 0 & \frac{2}{9} & \frac{2}{9} \end{bmatrix}, \quad E_{1b} = E_{2b} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{2}{9} \end{bmatrix}.$$

For this example, we assume that the nonlinear system (27) is controlled through a network, and $\tau_m = 0.01s$. By Algorithm 1, the closed-loop nonlinear system of the system (27) is exponentially stable for $\tau_k \in [0.01s, 0.8441s]$ under the fuzzy controller (7) with

$$K_1 = [0.5272 \quad 0.0604 \quad -0.2735 \quad -0.8792]$$

$$K_2 = [0.5333 \quad 0.0816 \quad -0.2696 \quad -0.8768].$$

The initial conditions are chosen as $x(0) = [1 \ 0 \ 0 \ 0]^T$. The sampling rate is chosen as $0.01s$, that is, $h = 0.01s$. The simulation results for the four states of the nonlinear system are shown in Fig. 4, from which one can find that the system states are in equilibrium after 40 s, while they need more than 2000 s from [25, Fig. 6]. Also, the peak value of Fig. 4 is much smaller than that of [25, Fig. 6].

V. CONCLUSION

We have addressed the problem of fuzzy controller design for a class of nonlinear networked control systems considering both time-varying network-induced delay and data packet dropout simultaneously. To the best of our knowledge, the problem has not been investigated in the open literature. We have employed uncertain networked Takagi–Sugeno models to approximate the nonlinear networked control systems. Then, we have derived some sufficient conditions for the existence of fuzzy controllers. Based on these conditions, we have proposed an iterative algorithm for achieving fuzzy controllers. In order to show the effectiveness of the iterative algorithm, we have studied the fuzzy control design of a flexible-joint robot arm through a network.

APPENDIX

PROOF OF PROPOSITION 1

Choose a functional as

$$\begin{aligned} V(t) = & x^T(t)Px(t) + \int_{t-\tau_m}^t x^T(s)Qx(s)ds \\ & + \tau_m \int_{-\tau_m}^0 ds \int_{t+s}^t \dot{x}^T(\theta)R\dot{x}(\theta)d\theta \\ & + \delta \int_{-\eta}^{-\tau_m} ds \int_{t+s}^t \dot{x}^T(\theta)S\dot{x}(\theta)d\theta \end{aligned} \quad (30)$$

where $P > 0$, $Q > 0$, $R > 0$, and $S > 0$.

Taking the derivative of $V(t)$ with respect to t along the trajectory of (20) yields

$$\begin{aligned} \dot{V}(t) = & \sum_{i=1}^r \sum_{j=1}^r h_i h_j 2x^T(t) P [A_i x(t) + B_i K_j x(i_k h) + D_i v_{ij}(t)] \\ & - x^T(t - \tau_m) Q x(t - \tau_m) + x^T(t) Q x(t) \\ & + \dot{x}^T(t) (\tau_m^2 R + \delta^2 S) \dot{x}(t) - \tau_m \int_{t-\tau_m}^t \dot{x}^T(s) R \dot{x}(s) ds \\ & - \delta \int_{t-\eta}^{t-\tau_m} \dot{x}^T(\theta) S \dot{x}(\theta) d\theta \end{aligned} \quad (31)$$

for $t \in [i_k h + \tau_k, i_{k+1} h + \tau_{k+1})$, $k = 1, 2, \dots$

We use Lemma 2 to obtain

$$\begin{aligned} -\tau_m \int_{t-\tau_m}^t \dot{x}^T(s) R \dot{x}(s) ds \\ \leq -[x(t) - x(t - \tau_m)]^T R [x(t) - x(t - \tau_m)] \end{aligned} \quad (32)$$

and

$$\begin{aligned} -\delta \int_{t-\eta}^{t-\tau_m} \dot{x}^T(\theta) S \dot{x}(\theta) d\theta \leq -\delta \int_{i_k h}^{t-\tau_m} \dot{x}^T(\theta) S \dot{x}(\theta) d\theta \\ \leq -[x(t - \tau_m) - x(i_k h)]^T S [x(t - \tau_m) - x(i_k h)] \end{aligned} \quad (33)$$

From (31)–(33), we have

$$\dot{V}(t) \leq \sum_{i=1}^r \sum_{j=1}^r h_i h_j \Upsilon_{ij} + \dot{x}^T(t) \Theta \dot{x}(t) \quad (34)$$

for $t \in [i_k h + \tau_k, i_{k+1} h + \tau_{k+1})$, $k = 1, 2, \dots$, where

$$\begin{aligned} \Theta = & \tau_m^2 R + \delta^2 S \\ \Upsilon_{ij} = & 2x^T(t) P [A_i x(t) + B_i K_j x(i_k h) + D_i v_{ij}(t)] \\ & + x^T(t) Q x(t) - x^T(t - \tau_m) Q x(t - \tau_m) \\ & - [x(t) - x(t - \tau_m)]^T R [x(t) - x(t - \tau_m)] \\ & - [x(t - \tau_m) - x(i_k h)]^T S [x(t - \tau_m) - x(i_k h)]. \end{aligned}$$

By the Schur complement, it is easy to see that the following inequality implies that the right of (34) is a negative definite function.

$$\begin{bmatrix} \sum_{i=1}^r \sum_{j=1}^r h_i h_j \Upsilon_{ij} & \dot{x}^T(t) \\ \dot{x}(t) & -\Theta^{-1} \end{bmatrix} < 0.$$

From (20), the aforesaid inequality is written as

$$\sum_{i=1}^r \sum_{j=1}^r h_i h_j \begin{bmatrix} \Upsilon_{ij} & \Gamma \\ \Gamma^T & -\Theta^{-1} \end{bmatrix} < 0 \quad (35)$$

where $\Gamma = [A_i x(t) + B_i K_j x(i_k h) + D_i v_{ij}(t)]^T$. It is clear that (35) is implied by

$$\begin{bmatrix} \Upsilon_{ij} & \Gamma \\ \Gamma^T & -\Theta^{-1} \end{bmatrix} < 0$$

for $i, j = 1, 2, \dots, r$. We use the Schur complement to obtain

$$\Upsilon_{ij} + \Gamma \Theta \Gamma^T < 0$$

which can be further written as

$$\xi^T \Omega_{ij} \xi < 0 \quad (36)$$

for $t \in [i_k h + \tau_k, i_{k+1} h + \tau_{k+1})$, $i, j = 1, 2, \dots, r, k = 1, 2, \dots$, where

$$\begin{aligned} \xi^T = & [x^T(t) \quad x^T(i_k h) \quad x^T(t - \tau_m) \quad v_{ij}^T(t)] \\ \Omega_{ij} = & \begin{bmatrix} (1,1) & (1,2) & R & (1,4) \\ (1,2)^T & (2,2) & S & (2,4) \\ R & S & (3,3) & 0 \\ (1,4)^T & (2,4)^T & 0 & D_i^T \Theta D_i \end{bmatrix} \end{aligned}$$

with

$$\begin{aligned} (1,1) &= P A_i + A_i^T P + Q - R + A_i^T \Theta A_i \\ (1,2) &= P B_i K_j + A_i^T \Theta B_i K_j \\ (1,4) &= P D_i + A_i^T \Theta D_i \\ (2,2) &= -S + (B_i K_j)^T \Theta B_i K_j \\ (2,4) &= (B_i K_j)^T \Theta D_i \\ (3,3) &= -Q - R - S. \end{aligned}$$

Note that (23), by $\mathcal{S}$ -procedure, (36) is implied by

$$\tilde{\Xi}_{ij} + \tilde{\Xi}_{ji} < 0 \quad (37)$$

for $\varepsilon > 0$ and $1 \leq i \leq j \leq r$, where

$$\tilde{\Xi}_{ij} = \Omega_{ij} + \varepsilon \begin{bmatrix} E_i^T E_i & E_i^T E_{ib} K_j & 0 & 0 \\ (E_{ib} K_j)^T E_i & (E_{ib} K_j)^T E_{ib} K_j & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -I \end{bmatrix}.$$

It follows that there exists a $\lambda > 0$ such that

$$\dot{V}(t) < -\lambda x^T(t) x(t) - \lambda x^T(i_k h) x(i_k h)$$

for $t \in [i_k h + \tau_k, i_{k+1} h + \tau_{k+1})$, $k = 1, 2, \dots$. Next, we prove the robust exponential stability of the system (14) if the matrix inequality (37) holds. Define a new functional as

$$\tilde{V}(t) = e^{\sigma t} V(t)$$

where $\sigma > 0$ is a constant to be determined. Then, using a similar method in [26], we obtain

$$\begin{aligned} \dot{\tilde{V}}(t) &= e^{\sigma t} \dot{V}(t) + \sigma e^{\sigma t} V(t) \\ &\leq -\lambda e^{\sigma t} \{x^T(t) x(t) + x^T(i_k h) x(i_k h)\} + \sigma e^{\sigma t} V(t) \\ &\leq \varphi_1(t) + \tau_m \sigma e^{\sigma t} \int_{-\tau_m}^t ds \int_{t+s}^t \dot{x}^T(\theta) R \dot{x}(\theta) d\theta \\ &\quad + \delta \sigma e^{\sigma t} \int_{-\eta}^{-\tau_m} ds \int_{t+s}^t \dot{x}^T(\theta) S \dot{x}(\theta) d\theta \end{aligned}$$

$$\begin{aligned}
&= \varphi_1(t) + \tau_m \sigma e^{\sigma t} \int_{t-\tau_m}^t (\theta - t + \tau_m) \dot{x}^T(\theta) R \dot{x}(\theta) d\theta \\
&\quad + \delta \sigma e^{\sigma t} \int_{t-\eta}^{t-\tau_m} (\theta - t + \eta) \dot{x}^T(\theta) S \dot{x}(\theta) d\theta \\
&\quad + \delta^2 \sigma e^{\sigma t} \int_{t-\tau_m}^t \dot{x}^T(\theta) S \dot{x}(\theta) d\theta \\
&\leq \varphi_1(t) + \tau_m^2 \sigma e^{\sigma t} \int_{t-\tau_m}^t \dot{x}^T(\theta) R \dot{x}(\theta) d\theta \\
&\quad + \delta^2 \sigma e^{\sigma t} \int_{t-\eta}^t \dot{x}^T(\theta) S \dot{x}(\theta) d\theta
\end{aligned}$$

where $\varphi_1(t) = -\lambda e^{\sigma t} \{x^T(t)x(t) + x^T(i_k h)x(i_k h)\} + \sigma e^{\sigma t} x^T(t)Px(t) + \sigma e^{\sigma t} \int_{t-\tau_m}^t x^T(s)Qx(s)ds$. Integrating both sides from $i_k h + \tau_k$ to t , we have

$$\begin{aligned}
&\tilde{V}(t) - \tilde{V}(i_k h + \tau_k) \\
&\leq -\lambda \int_{i_k h + \tau_k}^t e^{\sigma s} \{x^T(s)x(s) + x^T(i_k h)x(i_k h)\} ds \\
&\quad + \int_{i_k h + \tau_k}^t \sigma e^{\sigma s} x^T(s)Px(s) ds \\
&\quad + \int_{i_k h + \tau_k}^t ds \int_{s-\tau_m}^s \sigma e^{\sigma s} x^T(\theta)Qx(\theta) d\theta \\
&\quad + \tau_m^2 \int_{i_k h + \tau_k}^t ds \int_{s-\tau_m}^s \sigma e^{\sigma s} \dot{x}^T(\theta)R\dot{x}(\theta) d\theta \\
&\quad + \delta^2 \int_{i_k h + \tau_k}^t ds \int_{s-\eta}^s \sigma e^{\sigma s} \dot{x}^T(\theta)S\dot{x}(\theta) d\theta. \quad (38)
\end{aligned}$$

Since $V(x_t)$ is continuous on $[t_0, \infty)$, then from (38), it is easy to obtain

$$\begin{aligned}
&\tilde{V}(t) - \tilde{V}(t_0) \\
&\leq \varphi_2(t) + \sigma \lambda_{\max}(Q) \int_{t_0}^t ds \int_{s-\tau_m}^s e^{\sigma s} \|x(\theta)\|^2 d\theta \\
&\quad + \sigma \tau_m^2 \lambda_{\max}(R) \int_{t_0}^t ds \int_{s-\tau_m}^s e^{\sigma s} \|\dot{x}(\theta)\|^2 d\theta \\
&\quad + \sigma \delta^2 \lambda_{\max}(S) \int_{t_0}^t ds \int_{s-\eta}^s e^{\sigma s} \|\dot{x}(\theta)\|^2 d\theta \\
&\leq \varphi_2(t) + \sigma \lambda_{\max}(Q) \int_{t_0-\tau_m}^t d\theta \int_{\theta}^{\theta+\tau_m} e^{\sigma s} \|x(\theta)\|^2 ds \\
&\quad + \sigma \tau_m^2 \lambda_{\max}(R) \int_{t_0-\tau_m}^t d\theta \int_{\theta}^{\theta+\tau_m} e^{\sigma s} \|\dot{x}(\theta)\|^2 ds \\
&\quad + \sigma \delta^2 \lambda_{\max}(S) \int_{t_0-\eta}^t d\theta \int_{\theta}^{\theta+\eta} e^{\sigma s} \|\dot{x}(\theta)\|^2 ds \\
&\leq \varphi_2(t) + \sigma \tau_m \lambda_{\max}(Q) e^{\sigma \tau_m} \int_{t_0-\tau_m}^t e^{\sigma \theta} \|x(\theta)\|^2 d\theta \\
&\quad + \sigma \tau_m^3 \lambda_{\max}(R) e^{\sigma \tau_m} \int_{t_0-\tau_m}^t e^{\sigma \theta} \|\dot{x}(\theta)\|^2 d\theta
\end{aligned}$$

$$\begin{aligned}
&\quad + \sigma \delta^2 \eta \lambda_{\max}(S) e^{\sigma \eta} \int_{t_0-\eta}^t e^{\sigma \theta} \|\dot{x}(\theta)\|^2 d\theta \\
&\leq \varphi_2(t) + \sigma \tau_m \lambda_{\max}(Q) e^{\sigma \tau_m} \int_{t_0-\tau_m}^t e^{\sigma s} \|x(s)\|^2 ds \\
&\quad + \sigma \tau_m^3 \lambda_{\max}(R) e^{\sigma \tau_m} \alpha_1 \int_{t_0-\tau_m}^t e^{\sigma s} \|x(s)\|^2 ds \\
&\quad + \sigma \delta^2 \eta \lambda_{\max}(S) e^{\sigma \eta} \alpha_1 \int_{t_0-\eta}^t e^{\sigma s} \|x(s)\|^2 ds \\
&\quad + \sigma (\tau_m^3 \lambda_{\max}(R) e^{\sigma \tau_m} + \delta^2 \eta \lambda_{\max}(S) e^{\sigma \eta}) \alpha_2 \varphi(t) \\
&= N_1 \int_{t_0}^t e^{\sigma s} \|x(s)\|^2 ds \\
&\quad + (\sigma \tau_m \lambda_{\max}(Q) e^{\sigma \tau_m} \\
&\quad + \sigma \tau_m^3 \lambda_{\max}(R) e^{\sigma \tau_m} \alpha_1) \int_{t_0-\tau_m}^{t_0} e^{\sigma s} \|\phi(s)\|^2 ds \\
&\quad + \sigma \delta^2 \eta \lambda_{\max}(S) e^{\sigma \eta} \alpha_1 \int_{t_0-\eta}^{t_0} e^{\sigma s} \|\phi(s)\|^2 ds + N_2 \varphi(t) \quad (39)
\end{aligned}$$

where

$$\begin{aligned}
\varphi_2(t) &= -\lambda \int_{t_0}^t e^{\sigma s} \|x(s)\|^2 ds - \lambda \varphi(t) + \sigma \lambda_{\max}(P) \\
&\quad \times \int_{t_0}^t e^{\sigma s} \|x(s)\|^2 ds \\
\varphi(t) &= \sum_{i=1}^{k-1} \int_{i_i h + \tau_i}^{i_{i+1} h + \tau_{i+1}} e^{\sigma s} \|x(i_i h)\|^2 ds \\
&\quad + \int_{i_k h + \tau_k}^t e^{\sigma s} \|x(i_k h)\|^2 ds
\end{aligned}$$

$$\alpha_1 = \sum_{i=1}^r (\|A_i\| + \|D_i\| \cdot \|E_i\|)$$

$$\alpha_2 = \sum_{i,j=1}^r (\|B_i\| + \|D_i\| \cdot \|E_{ib}\|) \|K_j\|$$

$$N_1 = -\lambda + \sigma \lambda_{\max}(P) + \sigma \tau_m \lambda_{\max}(Q) e^{\sigma \tau_m} + \sigma (\tau_m^3 \lambda_{\max}(R) e^{\sigma \tau_m} + \delta^2 \eta \lambda_{\max}(S) e^{\sigma \eta}) \alpha_1$$

$$N_2 = -\lambda + \sigma (\tau_m^3 \lambda_{\max}(R) e^{\sigma \tau_m} + \delta^2 \eta \lambda_{\max}(S) e^{\sigma \eta}) \alpha_2.$$

It is clear to see that there exists a sufficiently small constant $\sigma > 0$ such that $N_1 < 0$ and $N_2 < 0$. Then, from (39), we have

$$\begin{aligned}
\tilde{V}(t) &\leq (\sigma \tau_m \lambda_{\max}(Q) e^{\sigma \tau_m} + \sigma \tau_m^3 \lambda_{\max}(R) e^{\sigma \tau_m} \alpha_1) \\
&\quad \times \int_{t_0-\tau_m}^{t_0} e^{\sigma s} \|\phi(s)\|^2 ds \\
&\quad + \sigma \delta^2 \eta \lambda_{\max}(S) e^{\sigma \eta} \alpha_1 \int_{t_0-\eta}^{t_0} e^{\sigma s} \|\phi(s)\|^2 ds \\
&\quad + e^{\sigma t_0} \left(x_0^T P x_0 + \int_{t_0-\tau_m}^{t_0} \phi^T(s) Q \phi(s) ds \right)
\end{aligned}$$

$$\begin{aligned}
& + \tau_m \lambda_{\max}(R) \alpha_1 \int_{-\tau_m}^0 ds \int_{t_0+s}^{t_0} \|\phi(\theta)\|^2 d\theta \\
& + \delta \lambda_{\max}(S) \alpha_1 \int_{-\eta}^{-\tau_m} ds \int_{t_0+s}^{t_0} \|\phi(\theta)\|^2 d\theta \Big).
\end{aligned}$$

In consequence,

$$\|x(t)\|^2 \leq \frac{M_1 + M_2}{\lambda_{\min}(P)} \sup_{t_0 - \eta \leq s \leq t_0} \|\phi(\theta)\|^2 e^{-\sigma(t-t_0)}$$

where

$$\begin{aligned}
M_1 &= \tau_m \lambda_{\max}(Q) e^{\sigma \tau_m} + \tau_m^3 \lambda_{\max}(R) e^{\sigma \tau_m} \alpha_1 \\
&+ \delta^2 \eta \lambda_{\max}(S) e^{\sigma \eta} \alpha_1 \\
M_2 &= \lambda_{\max}(P) + \tau_m \lambda_{\max}(Q) + \frac{1}{2} \tau_m^3 \lambda_{\max}(R) \alpha_1 \\
&+ \frac{1}{2} \delta^2 (\eta + \tau_m) \lambda_{\max}(S) \alpha_1.
\end{aligned}$$

So the system (14) is robustly exponentially stable if matrix inequality (37) holds. By the Schur complement, matrix inequality (37) is equivalent to

$$\hat{\Xi}_{ij} + \hat{\Xi}_{ji} < 0 \quad (40)$$

where

$$\hat{\Xi}_{ij} = \begin{bmatrix} \tilde{\Gamma}_{11} & \tilde{\Gamma}_{12} & R & PD_i & \tau_m A_i^T & \delta A_i^T & E_i^T \\ \tilde{\Gamma}_{12} & -S & S & 0 & \tilde{\Gamma}_{25} & \tilde{\Gamma}_{26} & \tilde{\Gamma}_{27} \\ R & S & \tilde{\Gamma}_{33} & 0 & 0 & 0 & 0 \\ D_i^T P & 0 & 0 & -\varepsilon I & \tau_m D_i^T & \delta D_i^T & 0 \\ \tau_m A_i & \tilde{\Gamma}_{25}^T & 0 & \tau_m D_i & -R^{-1} & 0 & 0 \\ \delta A_i & \tilde{\Gamma}_{26}^T & 0 & \delta D_i & 0 & -S^{-1} & 0 \\ E_i & \tilde{\Gamma}_{27}^T & 0 & 0 & 0 & 0 & \tilde{\Gamma}_{77} \end{bmatrix}$$

with

$$\begin{aligned}
\tilde{\Gamma}_{11} &= PA_i + A_i^T P + Q - R \\
\tilde{\Gamma}_{12} &= PB_i K_j \\
\tilde{\Gamma}_{25} &= \tau_m (B_i K_j)^T \\
\tilde{\Gamma}_{26} &= \delta (B_i K_j)^T \\
\tilde{\Gamma}_{27} &= (E_{ib} K_j)^T \\
\tilde{\Gamma}_{33} &= -Q - R - S \\
\tilde{\Gamma}_{77} &= -\varepsilon^{-1} I.
\end{aligned}$$

Pre- and postmultiplying both sides of the aforesaid matrix inequality (40) by

$$\begin{bmatrix} X & X & X & 0 & 0 & 0 & 0 \\ 0 & X & 0 & 0 & 0 & 0 & 0 \\ 0 & X & X & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \varepsilon^{-1} I & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & I & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & I & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & I \end{bmatrix}$$

and its transpose, respectively, and setting $Y_j = K_j X$, $\tilde{Q} = X Q X$, $\tilde{\varepsilon} = \varepsilon^{-1}$ yields (24). This completes the proof. $\square$

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