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2. These two cases correspond to the second loop gain being broken and amplified, respectively.

Numerical simulations are given in Figs. 2–3, which correspond to $N_2 = 0$ and 2, respectively. In these figures, the dashed, solid and dash-dotted lines denote the timing responses of norms of states and controls by LQR and VSC designs with $\eta = 0.7$ and 2.2, respectively. The dotted line and starred line denote the norm of the sliding vector by VSC design with $\eta = 0.7$ and 2.2, respectively. From these figures, system states are observed convergent to zero for all the two cases, which agree with the theoretic results. When u_2 fails to operate, the system is found to have a linear uncontrollable mode $\lambda = 0$ and the associated closed-loop dynamics of y_1 decouples from the others as $\dot{y}_1 = -y_1^3$. It means that y_1 will approach zero but the convergence rate will be progressively smaller as $|y_1|$ gets smaller. This is why the norm of system states in Fig. 2 converges to zero very slowly. In addition, since in this example $\Gamma(\mathbf{x}) = 0$, u_1^* is then the main force to make system states reach the sliding surface. It follows that the first reaching time of system states to the sliding surface depends only on the choice of control parameter η . The first reaching time observed from $\|s\|$ in Figs. 2–3 for $\eta = 0.7$ and $\eta = 2.2$ are around $t_{\text{reach}} \approx 1.2$ and 0.4. This implies that the larger the value of η is, the shorter the first reaching time t_{reach} is. These phenomena can also be told from the abrupt change of the control magnitude, where the VSC reliable system is driven mainly by the equivalent part of $\mathbf{u}$ after the first reaching time. This example verifies that the control parameters of the VSC reliable design can be tuned as large as possible to increase the response while fulfilling the maximum control magnitude constraint.

IV. CONCLUDING REMARKS

Variable structure type stabilizing control laws have been proposed in this note to study reliable control issues. This approach can alleviate the computational burden for solving the HJ equation or inequality. In addition, the regions of acceptable change in control gain magnitude that guarantees system's stability can be made larger than those obtained by LQR reliable design by the choice of control parameters. As a matter of fact, the control parameters can be tuned as large as possible in practical applications to promote the responding performances while fulfilling the maximum control magnitude constraint.

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REFERENCES

- [1] M. Bodson and J. E. Groszkiewicz, "Multivariable adaptive algorithms for reconfigurable flight control," *IEEE Trans. Control Syst. Technol.*, vol. 5, no. 2, pp. 217–229, Mar. 1997.
- [2] J. D. Boskovic and R. K. Mehra, "A decentralized scheme for accommodation of multiple simultaneous actuator failures," in *Proc. Amer. Control Conf.*, 2002, pp. 5098–5103.
- [3] M. L. Corradini and G. Parlangeli, "A fault tolerant control system for the output stabilization of SISO plants with actuator uncertain hysteresis nonlinearities," in *Proc. IEEE Conf. Decision and Control*, 2003, pp. 4044–4049.
- [4] R. A. Decarlo, S. H. Zak, and G. P. Matthews, "Variable structure control of nonlinear multivariable systems: A tutorial," *Proc. IEEE*, vol. 76, no. 3, pp. 212–232, Mar. 1988.
- [5] Y. Diao and K. M. Passino, "Stable fault-tolerant adaptive fuzzy/neural control for a turbine engine," *IEEE Trans. Control Syst. Technol.*, vol. 9, no. 3, pp. 494–509, May 2001.
- [6] J. Huang and C.-F. Lin, "Numerical approach to computing nonlinear control laws," *J. Guid., Control, Dyna.*, vol. 18, pp. 989–994, 1995.
- [7] E. M. Jafarov and R. Tasaltin, "Robust sliding-mode control for the uncertain MIMO aircraft model F-18," *IEEE Trans. Aerospace Electron. Syst.*, vol. 36, no. 4, pp. 1127–1141, Oct. 2000.
- [8] J. Jiang and Q. Zhao, "Design of reliable control systems possessing actuator redundancies," *J. Guid., Control, Dyna.*, vol. 23, pp. 706–710, 2000.
- [9] Y.-W. Liang, D.-C. Liaw, and T.-C. Lee, "Reliable Control of Nonlinear Systems," *IEEE Trans. Autom. Control*, vol. 45, no. 4, pp. 706–710, Apr. 2000.
- [10] F. Liao, J. L. Wang, and G.-H. Yang, "Reliable robust flight tracking control: An LMI approach," *IEEE Trans. Control Syst. Technol.*, vol. 10, no. 1, pp. 76–89, Jan. 2002.
- [11] D.-C. Liaw, Y.-W. Liang, and C.-C. Cheng, "Nonlinear control for missile terminal guidance," *J. Dyna. Syst., Measure., Control*, vol. 122, pp. 663–668, 2000.
- [12] R. J. Veillette, J. V. Medanic, and W. R. Perkins, "Design of reliable control systems," *IEEE Trans. Autom. Control*, vol. 37, no. 3, pp. 290–304, Mar. 1992.
- [13] R. J. Veillette, "Reliable linear-quadratic state-feedback control," *Automatica*, vol. 31, pp. 137–143, 1995.
- [14] M. Vidyasagar and N. Viswanadham, "Reliable stabilization using a multi-controller configuration," *Automatica*, pp. 599–602, 1985.
- [15] G.-H. Yang, J. L. Wang, and Y. C. Soh, "Reliable guaranteed cost control for uncertain nonlinear systems," *IEEE Trans. Autom. Control*, vol. 45, no. 11, pp. 2188–2192, Nov. 2000.

Comments and Remarks on "On Improved Delay-Dependent Robust Control for Uncertain Time-Delay Systems"

Qing-Long Han

Abstract—The purpose of this note is to correct some statements and numerical examples' results in the above paper. A few remarks are also given to clarify the facts that for systems with *small* delay, the results in Han *et al.* (2003) are much less conservative than those in Kwon and Park (2004); for systems with *non-small* delay, the criterion in Kwon and Park (2004) fails to make any conclusion, while the criterion in Han *et al.* (2003) can be applicable to these kinds of systems.

I. COMMENTS AND REMARKS

Consider the following system [1]:

$$\begin{cases} \dot{x}(t) = (A + \Delta A)x(t) + (A_1 + \Delta A_1)x(t-h) + (B + \Delta B)u(t) \\ x(s) = \phi(s), s \in [-h, 0] \end{cases} \quad (1)$$

where ΔA , ΔA_1 , and ΔB are uncertain matrices satisfying

$$\Delta A = D_1 F_1(t) E_1, \Delta A_1 = D_2 F_2(t) E_2, \Delta B = D_3 F_3(t) E_3$$

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$$-X + F_{22} < 0 \quad (3)$$

$$\begin{bmatrix} -X & hY_1^T \\ * & -X \end{bmatrix} < 0 \quad (4)$$

$$\begin{bmatrix} F_{11} & F_{12} & F_{13} \\ * & F_{22} & F_{23} \\ * & * & F_{33} \end{bmatrix} > 0 \quad (5)$$

$$\begin{bmatrix} \Xi_{11} & \Xi_{12} & \Xi_{13} & 0 & 0 & 0 & 0 & \Pi \\ * & -h^{-1}(\alpha - 1)X & \Xi_{23} & D_1 & D_2 & 0 & 0 & 0 \\ * & * & -W + hF_{33} & 0 & 0 & \varepsilon_2 X E_2^T & \varepsilon_5 X E_5^T & 0 \\ * & * & * & -\varepsilon_4 I & 0 & 0 & 0 & 0 \\ * & * & * & * & -\varepsilon_5 I & 0 & 0 & 0 \\ * & * & * & * & * & -\varepsilon_2 I & 0 & 0 \\ * & * & * & * & * & * & -\varepsilon_5 I & 0 \\ * & * & * & * & * & * & * & \Gamma \end{bmatrix} < 0 \quad (6)$$

where D_i, E_i ($i = 1, 2, 3$) are known real constant matrices of appropriate dimensions, and $F_i(t) \in \mathbf{R}^{k_i \times l_i}$ are unknown matrices satisfying $F_i^T(t)F_i(t) \leq I$ ($i = 1, 2, 3$). For system (1), Kwon and Park obtained a stabilization result, i.e., [1, Th. 1]. By the theorem, one can easily derive a stability criterion for the following system:

$$\begin{cases} \dot{x}(t) = (A + \Delta A)x(t) + (A_1 + \Delta A_1)x(t - h) \\ x(s) = \phi(s), \quad s \in [-h, 0]. \end{cases} \quad (2)$$

Corollary 1: For given $h > 0$ and $\alpha > 1$, system (2) is robustly stable if there exist positive-definite matrices X, W, F_{11}, F_{22} , and F_{33} , positive scalars $\varepsilon_1, \varepsilon_2, \varepsilon_4$, and ε_5 , and matrices Y_1, F_{12}, F_{13} , and F_{23} such that where

$$\begin{aligned} \Xi_{11} &= AX + XA^T + Y_1 + Y_1^T + BY + Y^T B \\ &\quad + W + hF_{11} \\ \Xi_{12} &= XA^T + Y_1^T + Y^T B + F_{12} \\ \Xi_{13} &= A_1X - Y_1 + hF_{13} \\ \Xi_{23} &= A_1X - Y_1 + F_{23} \\ \Pi &= [D_1 \quad D_2 \quad \varepsilon_1 X E_1^T \quad \varepsilon_4 X E_1^T \quad \alpha h Y_1^T] \\ \Gamma &= \text{diag}\{-\varepsilon_1 I \quad -\varepsilon_2 I \quad -\varepsilon_1 I \quad -\varepsilon_4 I \quad -\alpha h X\}. \end{aligned}$$

Remark 1: It should be pointed out that positive scalars ε_i ($i = 1, 2, 3, 4, 5, 6$) in [1] are regarded as variables, *not* tuning parameters since [1, Th. 1] states that “... there *exist* ..., positive scalars ε_i ($i = 1, 2, 3, 4, 5, 6$) and ...”. Therefore, the statement

The inequalities of Theorem 1 and Corollary 1 are LMIs with tuning parameters

in [1, Rem. 2] is **incorrect** because [1, eq. (7)] and (6), as shown at the top of the page, in this note are **not** LMIs concerning the variables X, Y , and ε_i ($i = 1, 2, 3, 4, 5, 6$) due to the terms $\varepsilon_1 X E_1^T, \varepsilon_2 X E_2^T, \varepsilon_3 Y^T E_3^T, \varepsilon_4 X E_1^T, \varepsilon_5 X E_5^T, \varepsilon_6 Y^T E_3^T$ appeared in [1, eq. (7)]; the similar case occurred in (6). One way to solve this problem is to set ε_i ($i = 1, 2, 3, 4, 5, 6$) as given parameters.

Remark 2: The stability criterion in [2] is much less conservative than Corollary 1. The reason is that for the nominal system of (2), it has been proven that the existence of a more general quadratic form Lyapunov–Krasovskii functional, which is used in [2], is necessary and sufficient for the stability [3] while Kwon and Park [1] used the **simple** Lyapunov–Krasovskii functional whose existence is **only** sufficient for the stability. Therefore, the statements such as

TABLE I
COMPARISON OF h USING DIFFERENT METHODS.

Methods	Li and de Souza [4]	Kwon and Park (2004) [1]
h	0.2716	0.2827
Methods	Moon et al (2001) [5]	Fridman and Shaked (2002) [6]
h	0.4500	0.5865

Three numerical examples are given to show the superiority of the present results to those in other literature

in the Introduction, and

By comparing our results with other ones through numerical examples, we showed the derived criterion is less conservative than those in the literature.

in the Conclusion in [1], are **incorrect**.

Remark 3: The computation results in Examples 1, 2, and 4 are **incorrect**. We give the correct results for these examples as follows.

Example 1: Consider the uncertain system

$$\dot{x}(t) = (A + \Delta A)x(t) + (A_1 + \Delta A_1)x(t - h) + Bu(t)$$

with the system matrices

$$\begin{aligned} A &= \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \quad A_1 = \begin{pmatrix} -2 & -0.5 \\ 0 & -1 \end{pmatrix} \quad D_1 = 0.2I \\ E_1 &= I, \quad D_2 = D_1 \quad E_2 = E_1 \quad D_3 = 0 \quad E_3 = 0. \end{aligned}$$

Using the parameters $\alpha = 1.7$ and $\varepsilon_1 = 4.4449 \times 10^8, \varepsilon_2 = 4.7163 \times 10^8, \varepsilon_3 = 0, \varepsilon_4 = 5.0158 \times 10^8, \varepsilon_5 = 5.3096 \times 10^8$, and $\varepsilon_6 = 0$ selected by Kwon and Park [1] and applying [1, Th. 1], the delay bound is computed as $\bar{h} = 0.2827$ and the controller gain matrix is $K = YX^{-1} = (-0.9433 \quad -6.3209)$. Table I shows the comparison of the obtained result with some existing ones. From this table, it is clear to see that the result in [1] only slightly improves the result in [4], but fails to improve the recent results in [5] and [6].

Example 2: Consider the uncertain time-delay system borrowed from [2]

$$\dot{x}(t) = (A + \Delta A)x(t) + (A_1 + \Delta A_1)x(t - h)$$

$$\text{where } A = \begin{pmatrix} -2 & 0 \\ 0 & -0.9 \end{pmatrix} A_1 = \begin{pmatrix} -1 & 0 \\ -1 & -1 \end{pmatrix} \\ D_1 = \beta I \quad E_1 = I \quad D_2 = D_1 \quad E_2 = E_1.$$

For this example, we select $\alpha = 5.4$ and $\varepsilon_1 = \varepsilon_2 = \varepsilon_4 = \varepsilon_5 = 1$. For $\beta = 0$, using Corollary 1 we can only obtain the delay bound $\bar{h} = 3.5824$ while Han *et al.* [2] gave the delay bound as $\bar{h} = 6.171$. For $\beta = 0.2$, Corollary 1 gives the delay bound $\bar{h} = 0.4357$ which is much more conservative than $\bar{h} = 3.133$ derived in [2]. The delay bound $\bar{h} = 0.4357$ is also conservative compared with $\bar{h} = 0.4437$ in [7], $\bar{h} = 2.39$ in [8]. Therefore, the statement

It is easy to see that the result in this note significantly improve the ones in the literature

in [1, Ex. 2] is **incorrect**.

Example 4: Consider the nominal unforced time-delay system

$$\dot{x}(t) = \begin{pmatrix} -1 & 0.5 \\ 0.5 & -1 \end{pmatrix} x(t) + \begin{pmatrix} -2 & 2 \\ -2 & -2 \end{pmatrix} x(t-h).$$

Selecting $\alpha = 1.15$ and $\varepsilon_1 = \varepsilon_2 = \varepsilon_4 = \varepsilon_5 = 1$ and using Corollary 1 gives the delay bound as $\bar{h} = 0.3343$ which is much conservative than $\bar{h} = 0.3645$ provided in Han *et al.* (2003) [2]. However, the delay bound $\bar{h} = 0.3343$ is less conservative than the results in [4], [9], and [10].

Remark 4: For linear systems with *nonsmall* delay, which means that the system is stable with some *nonzero* delay, but is unstable without the delay, the criterion in [1] using the simple Lyapunov–Krasovskii functional is *no longer* valid to judge the stability problem since it requires that the corresponding systems *without* delay need to be stable. This is the serious limitation for such a criterion. However, the criterion in [2] has successfully overcome the limitation.

REFERENCES

- [1] O. M. Kwon and J. H. Park, "On improved delay-dependent robust control for uncertain time-delay systems," *IEEE Trans. Autom. Control*, vol. 49, no. 11, pp. 1991–1995, Nov. 2004.
- [2] Q.-L. Han, K. Gu, and X. Yu, "An improved estimate of the robust stability bound of time-delay systems with norm-bounded uncertainty," *IEEE Trans. Autom. Control*, vol. 48, no. 9, pp. 1629–1634, Sep. 2003.
- [3] W. Huang, "Generalization of Liapunov's theorem in a linear delay system," *J. Math. Anal. Appl.*, vol. 142, no. 1, pp. 83–94, 1989.
- [4] X. Li and C. D. de Souza, "Delay-dependent robust stability and stabilization of uncertain linear time-delay systems: A linear matrix inequality approach," *IEEE Trans. Autom. Control*, vol. 42, no. 8, pp. 1144–1148, Aug. 1997.
- [5] Y. S. Moon, P. Park, W. H. Kwon, and Y. S. Lee, "Delay-dependent robust stabilization of uncertain state-delayed systems," *Int. J. Control*, vol. 74, no. 14, pp. 1447–1455, 2001.
- [6] E. Fridman and U. Shaked, "An improved stabilization method for linear time-delay systems," *IEEE Trans. Autom. Control*, vol. 47, no. 11, pp. 1931–1937, Nov. 2002.
- [7] C. E. de Souza and X. Li, "Delay-dependent robust H_∞ control of uncertain linear state-delayed systems," *Automatica*, vol. 35, pp. 1313–1321, 1997.
- [8] Y. He, M. Wu, J. H. She, and G. P. Liu, "Delay-dependent robust stability criteria for uncertain neutral systems with mixed delays," *Syst. Control Lett.*, vol. 51, pp. 57–65, 2004.
- [9] E. Fridman, "New Lyapunov–Krasovskii functionals for stability of linear retarded and neutral type systems," *Syst. Control Lett.*, vol. 43, pp. 309–319, 2001.
- [10] C. H. Lien and J. D. Chen, "Discrete-delay-independent and discrete-delay-dependent criteria for a class of neutral systems," *J. Dyna. Syst. Meas. Control*, vol. 125, pp. 33–41, 2003.

Authors' Reply

O. M. Kwon and Ju H. Park

We thank Q.-L. Han [2] for pointing some corrections in our paper. Han [2] has made several remarks on the results [1] and have pointed errors therein summarized as follows.

1) The statement

The inequalities of Theorem 1 and Corollary 1 are LMIs with tuning parameters

in [1, Rem. 2] is incorrect.

2) The simulation results of [1, Ex. 1, 2, 4] are incorrect.

3) For linear systems with *nonsmall* delay, the criterion in [1] is no longer valid to judge the stability problem.

Reply to Comment 1: In [1, Rem. 2], the authors have stated that

the inequalities of Theorem 1 and Corollary 1 are LMIs with tuning parameters

in the sense that the criteria are LMIs when the tuning parameters α and ε_i are preselected by any procedure.

Inequality (7) in Theorem 1 can be easily modified to LMI concerning all the variables, which is described later.

Reply to Comment 2: We agree that the simulation results in [1, Ex. 1, 2, 4] are incorrect. In the process of applying [1, Th. 1] to the three examples, we found that there are errors in simulation codes. This leads to unexpected results in the examples. Now, in order to correct the result on Examples 1, 2, and 4, we reformulate [1, Th. 1] to LMIs with respect to all the solution variables and resimulate the examples. The reformulated LMI is obtained as follows.

By exchanging the position of ε_i with ε_i^{-1} on the right-hand sides of (16), inequality (7) is modified to the following LMI:

$$\begin{bmatrix} \Xi_1 \bar{E}_{12} & \Xi_{13} & \alpha h Y_1^T Y^T E_3^T X E_1^T Y^T E_3^T & 0 & 0 \\ * & \Xi_{22} & \Xi_{23} & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & -W + h F_{33} & 0 & 0 & 0 & 0 & X E_2^T X E_2^T \\ * & * & * & -\alpha h X & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & -\varepsilon_3 I & 0 & 0 & 0 & 0 \\ * & * & * & * & * & -\varepsilon_4 I & 0 & 0 & 0 \\ * & * & * & * & * & * & -\varepsilon_6 I & 0 & 0 \\ * & * & * & * & * & * & * & -\varepsilon_2 I & 0 \\ * & * & * & * & * & * & * & * & -\varepsilon_5 I \end{bmatrix} < 0 \quad (1)$$

where

$$\begin{aligned} \Xi_{11} &= AX + XA^T + Y_1 + Y_1^T + BY + Y^T B^T + W + h F_{11} \\ &\quad + \varepsilon_1 D_1 D_1^T + \varepsilon_2 D_2 D_2^T + \varepsilon_3 D_3 D_3^T, \\ \Xi_{12} &= XA^T + Y_1^T + Y^T B^T + F_{12}, \\ \Xi_{13} &= A_1 X - Y_1 + h F_{13}, \\ \Xi_{22} &= -h^{-1}(\alpha - 1)X + \varepsilon_4 D_1 D_1^T + \varepsilon_5 D_2 D_2^T + \varepsilon_6 D_3 D_3^T, \\ \Xi_{23} &= XA_1 - Y_1^T + F_{23}. \end{aligned}$$

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