Robust H_∞ Filter Design of Uncertain Descriptor Systems with Discrete and Distributed Delays

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Abstract - The robust H_∞ filtering problem for a class of continuous-time uncertain linear descriptor systems with time-varying discrete and distributed delays is investigated. The time-delays are assumed to be constant and known. The uncertainties under consideration are norm-bounded, and possible time-varying, uncertainties. Sufficient condition for the existence of an H_∞ filter is expressed in terms of strict linear matrix inequalities (LMIs). Instead of using decomposition technique, a unified form of LMIs is proposed to show the exponential stability of the augmented systems. The condition for assuring the stability of the "fast" subsystem is implied from the unified form of LMIs, which is shown to be less conservative than the characteristic equation based conditions or matrix norm based conditions. The suitable filter is derived through a convex optimization problem. A numerical example is given to show the effectiveness of the method.

I. INTRODUCTION

Signal estimation has received significant attention in the past decades. Current efforts on this topic can be divided into two classes: namely Kalman filtering approach and H_∞ filtering approach.

In the Kalman filtering approach, the system's disturbances are assumed to be Gaussian noises with known statistics, see, for example, for linear systems [14], and for linear descriptor systems [1]. When the system's noise sources are assumed to be arbitrary signals with bounded energy (or average power), the H_∞

filtering approach provides a guaranteed noise attenuation level. One of its main advantages is the fact that it is insensitive to the exact knowledge of the statistics of the noise signals. Several methods are proposed to solve the H_∞ filtering problem [8, 17].

When there exist parameter uncertainties in the system's model, robust H_∞ filtering can provide a powerful signal estimation. It designs an asymptotically stable filter, based on an uncertain signal model, which ensures that the filtering error dynamics is asymptotically stable and that the L_2 -induced gain from the noise signals to the filtering error remains bounded by a prescribed level for all allowed uncertainties. Many results regarding robust H_∞ filtering are obtained, see, e.g. [12].

Time-delays are frequently encountered in practical systems such as engineering and biological systems. Their existence may induce instability, oscillation and poor performance [6]. Time-delays also arise in several signal processing such as multipath propagation, telemanipulation systems, data communication in high-speed internet, and network control systems. When one designs an H_∞ filter, the time-delay must be taken into account in order to make the system work in the expected performance. Otherwise, the system may collapse in the presence of time-delays. Recently there have been increasing interests in designing an H_∞ filter for time-delay systems. For example, in [11], an H_∞ filter design for precisely known systems with a single time delayed measurement was proposed. In [13], based on an algebraic Riccati matrix inequality approach, the robust H_∞ filtering was investigated for uncertain linear systems with delayed states and outputs. In [2], robust H_∞ filtering for uncertain linear systems with multiple time-varying state delays was considered

and delay-independent sufficient condition was given in the form of linear matrix inequalities (LMIs). In [3], based on a descriptor model transformation, a delay-dependent H_∞ filtering design was proposed for linear systems with constant time-delay. The filter which was obtained was of the Luenberger observer type. The results in [3] was extended to a system with time-varying delay and improved by employing Park's [10] inequality for the bounding of cross terms [5].

As is well known, one can use the time-delay model to describe the so-called "lossless propagation phenomena" [6]. These models can be further transformed to descriptor systems with time-delay, see, e.g. [9]. The descriptor systems with time-delay are systems of a more general type. It is of significance to consider the H_∞ filtering problem for these kinds of systems. To the best of authors' knowledge, the problem was only investigated in [4] where the approach was based on the decomposition technique, and the filter was of the Luenberger observer type. In [4], the uncertainties under consideration were polytopic ones. It is difficult to extend the results in [4] to other types of uncertainties such as norm-bounded ones. The distributed delay was not considered in [4].

This paper will be concerned with the robust H_∞ filtering for a class of uncertain linear descriptor systems with discrete and distributed delays. The uncertainties are norm-bounded ones. Sufficient condition for the existence of an H_∞ filter will be expressed in terms of strict LMIs. Instead of using decomposition technique, a unified form of LMIs will be proposed to show the exponential stability of the augmented systems. The condition for assuring the exponential stability of the "fast" subsystem will be implied from the unified form of LMIs, which is shown to be less conservative than the characteristic equation based conditions or matrix norm based conditions. The suitable filter will be derived through a convex optimization problem. A numerical example will be finally given to show the effectiveness of the method.

Notation: R^n denotes the n -dimensional Euclidean space, $R^{n \times m}$ is the set of $n \times m$ real matrices, I is the identity matrix of appropriate dimensions, $\|\cdot\|$ stands for either the Euclidean vector norm or its induced matrix 2-norm. The notation $X > 0$ (respectively, $X \geq 0$), for $X \in R^{n \times n}$ means that the matrix X is a real symmetric positive definite (respectively, positive semi-definite). C_0 denotes the set of all continuous functions from $[-\tau', 0]$ to R^n . $\lambda_{\max}(X)$ ($\lambda_{\min}(X)$) denotes the maximum (minimum) eigenvalue of the real symmetric matrix X . $\text{tr}(Y)$ denotes the trace of a matrix Y . $\text{Re}(s)$ denotes the real part of a complex num-

bers. "*" denotes the entries implied by symmetry of a matrix. For a vector function $g(t) \in L_2[0, \infty)$, its norm is defined as

$$\|g(t)\|_2 = \sqrt{\int_0^\infty \|g(t)\|^2 dt}$$

II. PRELIMINARIES

Consider the following uncertain descriptor system with discrete and distributed delays

$$\begin{aligned} E\dot{x}(t) &= (A + \Delta A(t))x(t) + (A_\tau + \Delta A_\tau(t))x(t - \tau) \\ &\quad + \int_{t-h}^t (A_h + \Delta A_h(s))x(s)ds + B_1w(t), \quad (1) \end{aligned}$$

$$\begin{aligned} y(t) &= (C + \Delta C(t))x(t) + (C_\tau + \Delta C_\tau(t))x(t - \tau) \\ &\quad + \int_{t-h}^t (C_h + \Delta C_h(s))x(s)ds + B_2w(t) \quad (2) \end{aligned}$$

$$z(t) = Lx(t), \quad (3)$$

$$x(t) = \varphi(t), \quad t \in [-\tau', 0], \quad \tau' = \max\{\tau, h\}, \quad (4)$$

where $x(t) \in R^n$ is the system state, $w(t) \in R^q$ is the external disturbance signal which belongs to $L_2[0, \infty)$, $y(t) \in R^r$ is the measurement, $z(t) \in R^p$ is the signal to be estimated. $\tau > 0$ and $h > 0$ are constants describing the magnitude of delay time. $\varphi(t) \in C_0$ denotes the initial function. $E, A, A_\tau, A_h, B_1, C, C_\tau, C_h, B_2$ and L are known constant matrices of appropriate dimensions. $\Delta A(t), \Delta A_\tau(t), \Delta A_h(t), \Delta C(t), \Delta C_\tau(t)$ and $\Delta C_h(t)$ denote the parameter uncertainties which satisfy

$$\begin{aligned} &\begin{bmatrix} \Delta A(t) & \Delta A_\tau(t) & \Delta A_h(t) \\ \Delta C(t) & \Delta C_\tau(t) & \Delta C_h(t) \end{bmatrix} \\ &= \begin{bmatrix} D_a \\ D_b \end{bmatrix} F(t) \begin{bmatrix} E_a & E_b & E_c \end{bmatrix}, \quad (5) \end{aligned}$$

where D_a, D_b, E_a, E_b and E_c are known matrices of appropriate dimensions, $F(t)$ is an unknown, piecewise continuous time-varying matrix which satisfies $\|F(t)\| \leq 1$. Throughout this paper, we assume that $\text{rank}(E) = q \leq n$.

First, we introduce a definition on regularity and non-impulsiveness of the system (1).

Definition 1: Descriptor system (1) (with $w(t) = 0$) is said to be regular and impulse-free if $(E, A + \Delta A(t))$ is regular and impulse-free.

Consider a linear filter with full order as

$$E \frac{d\bar{x}(t)}{dt} = A_f \bar{x}(t) + K_f y(t), \quad (6)$$

$$\bar{z}(t) = L \bar{x}(t), \quad (7)$$

$$\bar{x}(0) = 0, \quad (8)$$

where $\bar{x}$ is the state estimate and, the constant matrices A_f and K_f are filter parameters to be determined.

To begin with the study of the state estimation problem, we define the state error variable as

$$e(t) = x(t) - \bar{x}(t). \quad (9)$$

Then, from (1),(2) and (6), $e(t)$ satisfies the following dynamics

$$\begin{aligned} E\dot{e}(t) = & A_f e(t) + [A - A_f + \Delta A(t) \\ & - K_f (C + \Delta C(t)) x(t)] \\ & [+A_\tau - K_f C_\tau + \Delta A_\tau(t) \\ & - K_f \Delta C_\tau(t) x(t - \tau)] \\ & + \int_{t-h}^t [A_h - K_f C_h + \Delta A_h(s) \\ & - K_f \Delta C_h(s) x(s)] ds \\ & + (B_1 - K_f B_2) w(t). \end{aligned} \quad (10)$$

From (1), (3) and (10), we have the following augmented system

$$\begin{aligned} E_f \dot{x}_f(t) = & (A_{af} + \Delta A_{af}(t)) x_f(t) \\ & + (A_{\tau f} + \Delta A_{\tau f}(t)) x_f(t - \tau) \\ & + \int_{t-h}^t (A_{hf} + \Delta A_{hf}(s)) x_f(s) ds \\ & + B_f w(t), \end{aligned} \quad (11)$$

$$z_f(t) = L_f x_f(t), \quad (12)$$

$$x_f(t) = \begin{bmatrix} \varphi(t) \\ \varphi(t) \end{bmatrix}, t \in [-\tau', 0], \quad (13)$$

where $z_f(t)$ is the estimation error,

$$\begin{aligned} x_f &= \begin{bmatrix} x(t) \\ e(t) \end{bmatrix}, E_f = \begin{bmatrix} E & 0 \\ 0 & E \end{bmatrix}, \\ A_{af} &= \begin{bmatrix} A & 0 \\ A - A_f - K_f C & A_f \end{bmatrix}, \\ A_{\tau f} &= \begin{bmatrix} A_\tau & 0 \\ A_\tau - K_f C_\tau & 0 \end{bmatrix}, \\ A_{hf} &= \begin{bmatrix} A_h & 0 \\ A_h - K_f C_h & 0 \end{bmatrix}, \\ B_f &= \begin{bmatrix} B_1 \\ B_1 - K_f B_2 \end{bmatrix}, L_f = [0 \quad L], \\ \Delta A_{af}(t) &= \begin{bmatrix} \Delta A(t) & 0 \\ \Delta A(t) - K_f \Delta C(t) & 0 \end{bmatrix}, \\ \Delta A_{\tau f}(t) &= \begin{bmatrix} \Delta A_\tau(t) & 0 \\ \Delta A_\tau(t) - K_f \Delta C_\tau(t) & 0 \end{bmatrix}, \\ \Delta A_{hf}(t) &= \begin{bmatrix} \Delta A_h(t) & 0 \\ \Delta A_h(t) - K_f \Delta C_h(t) & 0 \end{bmatrix}. \end{aligned} \quad (14)$$

From (5), $\Delta A_{af}(t)$, $\Delta A_{\tau f}(t)$ and $\Delta A_{hf}(t)$ can be expressed as

$$\begin{bmatrix} \Delta A_{af}(t) & \Delta A_{\tau f}(t) & \Delta A_{hf}(t) \end{bmatrix} = D_f F(t) \begin{bmatrix} E_{af} & E_{bf} & E_{cf} \end{bmatrix}, \quad (15)$$

where

$$\begin{aligned} D_f &= \begin{bmatrix} D_a \\ D_a - K_f D_b \end{bmatrix}, E_{af} = \begin{bmatrix} E_a & 0 \end{bmatrix}, \\ E_{bf} &= \begin{bmatrix} E_b & 0 \end{bmatrix}, E_{cf} = \begin{bmatrix} E_c & 0 \end{bmatrix}. \end{aligned} \quad (16)$$

The filtering design problem to be addressed is stated as follows.

Robust H_∞ filtering problem: For a given $\gamma > 0$, design a full order linear filter of form (6)-(8) such that the augmented system (11)-(13) is regular, impulse-free and internally exponentially stable, namely, there exist $\alpha > 0$ and $\beta > 0$ such that the solution x_f of (11),(13) with $w(t) = 0$ satisfies $\|x_f(t)\| \leq \alpha \sup_{-\tau' \leq s \leq 0} \|\varphi(s)\| e^{-\beta t}$ and under zero initial condition and for any nonzero $w(t) \in L_2[0, \infty)$, $z_f(t)$ satisfies $\|z_f(t)\|_2 \leq \gamma \|w(t)\|_2$.

III. ROBUST H_∞ PERFORMANCE ANALYSIS

In this section, we will concentrate our attention on the robust performance analysis for system (11)-(13). The following lemmas are useful in the proof of Theorem 1.

Lemma 1: Suppose that piecewise continuous real square matrices $A(t)$, X , and $Q > 0$ satisfy

$$A^T(t)X + X^T A(t) + Q < 0 \quad (17)$$

for all t . Then, the following hold

- (1). $A(t)$ and X are invertible,
- (2). $\|A^{-1}(t)\| \leq \delta$ for some $\delta > 0$.

Lemma 2: Suppose that a positive continuous function $f(t)$ satisfies

$$f(t) \leq \zeta_1 \sup_{t-\tau \leq s \leq t} f(s) + \zeta_2 e^{-\varepsilon t}, \quad (18)$$

where $\varepsilon > 0$, $\zeta_1 < 1$, $\zeta_2 > 0$ and $\tau > 0$. Then, $f(t)$ satisfies

$$f(t) \leq \sup_{-\tau \leq s \leq 0} f(s) e^{-\xi_0 t} + \frac{\zeta_2 e^{-\xi_0 t}}{1 - \zeta_1 e^{\xi_0 \tau}}, \quad t \geq 0, \quad (19)$$

where $\xi_0 = \min\{\varepsilon, \xi\}$ and $0 < \xi < -\frac{1}{\tau} \ln \zeta_1$.

Based on Lemmas 1 and 2, we are now in a position to state and establish the following theorem that gives sufficient conditions assuring a guaranteed γ level of noise attenuation to the filtering error systems of (11)-(13).

Theorem 1: Given scalars $0 < a < 1$ and $\gamma > 0$. Suppose that matrices $P, Q > 0$ and $T > 0$ are such that

$$PE_f = E_f^T P^T \geq 0, \quad (20)$$

$$\begin{bmatrix} (A_{af} + \Delta A_{af}(t))^T P^T + P(A_{af} + \Delta A_{af}(t)) \\ + L_f^T L_f + Q + T \\ (A_{\tau f} + \Delta A_{\tau f}(t))^T P^T \\ h(A_{hf} + \Delta A_{hf}(s))^T P^T \\ B_f^T P^T \\ P(A_{\tau f} + \Delta A_{\tau f}(t)) \quad hP(A_{hf} + \Delta A_{hf}(s)) \quad PB_f \\ -aQ \quad 0 \quad 0 \\ 0 \quad -(1-a)Q \quad 0 \\ 0 \quad 0 \quad -\gamma^2 I \end{bmatrix} < 0, \quad (21)$$

where $s \in [t-h, t]$. Then, the augmented system (11)-(13) is regular, impulse-free and internally exponentially stable and satisfies a prescribed H_∞ norm upper bound constraint, that is, $\|z_f(t)\|_2 \leq \gamma \|w(t)\|_2$ for any nonzero $w(t) \in L_2[0, \infty)$.

The proof of Theorem 1 is omitted due to the page limitation.

Remark 1: Theorem 1 can be viewed as an extension of the existing results to the descriptor systems with time-varying uncertainties and discrete and distributed delays. However, our analysis procedure is different from that in [16] and the derived stability in our paper is exponential stability.

Remark 2: For a special descriptor system with distributed delay terms, which is an equivalent system of the state-space system (1) in [15], stability analysis was given based on a generalized Lyapunov functional. From the view point of descriptor system theory, it can be seen from the proof of Theorem 2.1 in [15] that only stability of state of the slow subsystem was studied although it is enough for the paper [15]. For a general class of descriptor systems with delays, the stability of the two subsystems, namely, the “slow” subsystem and the “fast” subsystem, must be addressed in order to show the stability of the whole system. Instead of using decomposition technique, based on both a generalized Lyapunov functional and an algebraic function, a unified form of LMIs was proposed in our paper to show the exponential stability of the augmented system (11)-(13). The condition for assuring the stability

of the “fast” subsystem was implied from the unified form of LMIs, which has been shown in Remark 2 to be less conservative than the characteristic equation based conditions or matrix norm based conditions. It should be noted that for the system (5) in [15] in the case of $A_d(s) = 0$ and $C_1 = 0$ or the case of $d = h$, Corollary 2 in our paper has the equivalent condition as the one (8) in Theorem 2.1 in [15], while Corollary 2 can determine not only the stability of $x(t)$ but also the stability of $y(t)$ directly from the information of parameter matrices of (5) in [15].

IV. ROBUST H_∞ FILTER DESIGN

After finishing some necessary preparations in the last section, now we can devote ourselves to the design of filter parameters A_f and K_f . The expected filter parameters will be expressed in terms of the solutions of a set of LMIs, which can be realized in the following theorem.

Theorem 2: Given scalars $0 < a < 1$ and $\gamma > 0$. If there exist matrices P_o, P_1, P_2, P_3, Y and $R > 0$ and scalars $\varepsilon_i > 0$ ($i = 1, 2$), $\sigma > 0$ such that

$$P_1 E = E^T P_1^T \geq 0, \quad (22)$$

$$P_2 E = E^T P_2^T \geq 0, \quad (23)$$

$$P_3 E = 0, \quad (24)$$

$$\begin{bmatrix} A^T P_1^T + P_1 A + \varepsilon_1 E_a^T E_a + R & A^T P_3^T + A^T P_2^T - P_o^T - C^T Y^T & P_1 A_\tau + \varepsilon_1 E_a^T E_b \\ * & P_o^T + P_o + L^T L + \sigma I & P_3 A_\tau + P_2 A_\tau - Y C_\tau \\ * & * & -aR + \varepsilon_1 E_b^T E_b \\ * & * & * \\ * & * & * \\ * & * & * \\ * & * & * \\ hP_1 A_h & P_1 B_1 & \\ hP_3 A_h + hP_2 A_h - hY C_h & P_3 B_1 + P_2 B_1 - Y B_2 & \\ 0 & 0 & \\ -(1-a)R + \varepsilon_2 h^2 E_c^T E_c & 0 & \\ * & -\gamma^2 I & \\ * & * & \\ * & * & \\ P_1 D_a & P_1 D_a & \\ P_3 D_a + P_2 D_a - Y D_b & P_3 D_a + P_2 D_a - Y D_b & \\ 0 & 0 & \\ 0 & 0 & \\ 0 & 0 & \\ -\varepsilon_1 I & 0 & \\ * & -\varepsilon_2 I & \end{bmatrix} < 0, \quad (25)$$

then the robust H_∞ filtering problem for system (1)-(4) is solvable. Moreover, the parameters of the designed filter are given by

$$A_f = P_2^{-1}P_o, K_f = P_2^{-1}Y. \quad (26)$$

Remark 3: In Theorem 2, equality constraints are included, which will lead to numerical problems when checking such non-strict LMI conditions since equality constraints are often fragile and usually not met perfectly[16]. For the case that $\text{rank}(E) = q < n$, there exists a matrix $\Phi \in R^{n \times (n-q)}$ with $\text{rank}(\Phi) = n-q$ such that $\Phi^T E = 0$. Define $P_i = E^T \Theta_i + Z_i \Phi^T$ ($i = 1, 2$) and $P_3 = Z_3 \Phi^T$, where $\Theta_i \in R^{n \times n}$ ($i = 1, 2$) is positive definite, $Z_i \in R^{n \times (n-q)}$ ($i = 1, 2, 3$). Obviously, $P_i E = E^T P_i^T \geq 0$ ($i = 1, 2$) and $P_3 E = 0$ hold. Denote (25)' as the inequality for Theorem 2 after substituting $P_i = E^T \Theta_i + Z_i \Phi^T$ ($i = 1, 2$) and $P_3 = Z_3 \Phi^T$ into (25). Then the solution of P_i ($i = 1, 2, 3$) satisfying (22), (23) and (25) can be transformed into the solution of Θ_i ($i = 1, 2$) and Z_i ($i = 1, 2, 3$) satisfying the strict LMI (25)'. From (26), we can finally obtain the parameters of the designed filter as $A_f = (E^T \Theta_2 + Z_2 \Phi^T)^{-1} P_o$ and $K_f = (E^T \Theta_2 + Z_2 \Phi^T)^{-1} Y$.

Remark 4: If a and γ are fixed, the upper bound of h that guarantees the solution of the problem (25)' is feasible can be solved. Generally, a can be chosen as 0.5. In addition, when a and h are fixed, the smallest γ describing the disturbance attenuation level can be solved from the following optimization problem:

$$\text{Minimize } \vartheta,$$

$$\text{Subject to: } \Theta_i > 0, R > 0, \varepsilon_i > 0 (i = 1, 2) \text{ and (25)',} \quad (27)$$

and $\gamma = \sqrt{\vartheta^*}$, ϑ^* is the optimal value of problem (27). Furthermore, it is shown by the following example that appropriately adjusting the parameter a may lead to less conservative results.

V. EXAMPLE

Consider system (1)-(4) with parameters

$$E = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, A = \begin{bmatrix} -2 & 0 & 0.5 \\ 0.1 & -0.9 & 0.2 \\ 0 & 0.5 & 0.3 \end{bmatrix},$$

$$A_\tau = \begin{bmatrix} 0.2 & 0.1 & 0 \\ 0.2 & 0 & 0.15 \\ 0.1 & -0.23 & 0.1 \end{bmatrix}, A_h = \begin{bmatrix} 0.1 & 0 & 0.1 \\ 0.03 & 0.1 & 0 \\ 0.1 & 0.02 & 0.2 \end{bmatrix}$$

$$B_1 = \begin{bmatrix} 0.2 & 0 & 0 \\ 0 & 0.2 & 0 \\ 0 & 0 & 0.2 \end{bmatrix}, C = [1 \ 0 \ 0],$$

$$C_\tau = [0 \ 0.5 \ 0.4], C_h = [0.5 \ 0.3 \ 0.6],$$

$$B_2 = [0.3 \ 0.3 \ 0.3],$$

$$L = [1 \ 0.7 \ 0.8], D_a = 0.1I, E_a = E_b = E_c = I,$$

$$D_b = [0.1 \ 0.1 \ 0.1], h = 2.$$

Choose $\Phi = [0 \ 0 \ 1]^T$ and $a = 0.5$. For $\gamma = 1$, applying Theorem 2 and Remark 4 and combining (26), we can compute A_f and K_f as

$$A_f = \begin{bmatrix} -1.1727 & -1.7381 & -3.3248 \\ 0.0392 & -2.3829 & -3.8683 \\ -0.0517 & 0.4859 & 1.0229 \end{bmatrix},$$

$$K_f = \begin{bmatrix} -1.2634 \\ -1.2191 \\ 0.3661 \end{bmatrix}.$$

In fact, when a is chosen to be 0.5, one can find an upper bound of h that guarantees the feasibility of problem (25)' is 2.72. However, if one chooses a as 0.4, the upper bound of h can be 2.76. By optimization algorithm (27), we can find that the smallest γ is 0.6397 for $h = 2$ and $a = 0.5$ and the corresponding A_f and K_f are

$$A_f = 10^3 \times \begin{bmatrix} -0.4428 & -0.4829 & 0.4388 \\ -6.8267 & -7.4751 & -6.8018 \\ -0.0016 & 0.0007 & 0.0027 \end{bmatrix},$$

$$K_f = \begin{bmatrix} 0.5731 \\ 6.7398 \\ 0.9808 \end{bmatrix}.$$

VI. CONCLUSION

The robust H_∞ filtering problem has been addressed for continuous-time uncertain descriptor systems with discrete and distributed delays. The designed filter can guarantee the filtering error system is regular, impulse-free and exponentially stable and satisfies a prescribed H_∞ norm bound constraint. Decomposition-free method has been used to derive the LMI-based sufficient conditions, which can be efficiently solved by using interior-point optimization algorithm.

Acknowledgments: The research work of D.Yue was partially supported by the Teaching and Research Award Program for Outstanding Young Teachers at

Nanjing Normal University and the Foundation for University Key Teachers by the Ministry of Education. The research work of Q.-L.Han was partially supported by Central Queensland University for the project entitled "Delay Effects: Analysis, Synthesis and Applications" (2003-2006).

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