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A Modified PSO Algorithm for Constrained Multi-Objective Optimization

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Abstract -- This paper presents a modified PSO algorithm for solving constrained multi-objective optimization problems. Based on the constraint dominance concept, the proposed approach defines two sets of selection rules for determining the cognitive and social components of the PSO algorithm. The simulation results to the four constrained multi-objective optimization problems demonstrate the proposed approach is able to find Pareto-optimal solutions effectively.

Keywords- Constraint; Multi-objective; Optimization; PSO algorithm

I. INTRODUCTION

Most real-world search and optimization problems involve multiple objectives (MO) that need to be achieved simultaneously. The presence of constraints brings difficulties in optimization since the search space has to be restricted to a feasible region. In solving MO problems, three goals need to be achieved [1]: find a set of solutions as close as possible to the true Pareto-optimal front; find a set of solutions as diverse as possible; and find a set of solutions as many as possible.

Population-based optimization techniques such as Genetic Algorithms (GAs), Particle Swarm Optimization (PSO), Differential Evolution (DE) and Ant Colony Optimization (ACO) have been popular choices for MO problems. The main reason is that these algorithms are capable of finding a set of Pareto-optimal solutions in a single run. Such evolutionary optimization techniques have been extensively studied in last decade or two. Most of them are based on GAs [1, 2]. Research shows that the PSO algorithm offers some advantages over GAs, such as simplicity and computational inexpensive [3, 4]. With the success of

the PSO in single objective optimization, researchers are motivated to extend the use of PSO in MO problems. Up to now, more than twenty five [5] strategies that adopt the PSO algorithm in MO problems have been proposed. Some examples are: dynamic neighbourhood [6], grid method [7], weighted aggregation [8], multi-swarm [9], dominated tree [10], fitness sharing [11], maximin strategy [12] and Huo et al [13]. A detailed review can be found in [5]. However, most of these approaches are constraint free. How to integrate constraint-handling methods with the multi-objective PSO has motivated this research.

Two papers have addressed constrained multi-objective optimization using the PSO algorithm.

Ji [14] presented a divided range multi-objective PSO for distributed computing. For constraint-handling, the author adopted the symbiosis mechanism where the feasible particles evolve towards the Pareto-front; and infeasible particles evolve toward feasibility guided by an unfeasibility function. A gradually decreased threshold is used for the proportion of infeasible particles. The approach is tested by three constrained MO functions. The author claimed that the Pareto fronts are achieved but no detailed results are given. The unfeasibility function used to guide the infeasible particles is unclear.

Reddy and Kumar [15] presented EM-MOPSO which combines the PSO technique with Pareto dominance criteria to evolve non-dominated solutions. The constraint-handling is based on the constraint dominance concept [1]. The global best particle (gBest or lBest) is randomly selected from the ERP (an external repository) where all non-dominated solutions are stored. The personal best particle is determined by the Pareto-dominance concept. The approach is tested by four constrained MO functions with the promising results. However, the approaches that use ERP will

increase computing cost [5]. The implementation for this approach looks complicated.

In this paper, we also adopt the constraint dominance concept, and propose an easy-to-implement modified PSO algorithm for tackling constrained multi-objective optimization problems. The proposed approach defines two sets of rules for determining the cognitive and social components of the PSO algorithm. The approach has been tested by the four constrained multi-objective optimization problems and the results are presented in this paper.

The rest of the paper is organized as follows. Section II presents the proposed approach for constrained multi-objective optimization problems including the selection rules and the modified PSO algorithm. Section III presents the experiment results for the four test functions. Section IV is a performance study. Section V concludes the paper and indicates future work.

II. THE PROPOSED APPROACH

A. Problem description

A general multi-objective constrained optimization problem consists of a decision vector $x = (x_1, x_2, \dots, x_n)$, an objective function vector $f(x) = (f_1(x), f_2(x), \dots, f_k(x))$ and a constraint function vector $g(x) = (g_1(x), g_2(x), \dots, g_m(x))$.

The problem can be stated as to find x^* which

$$\begin{aligned} & \text{minimize} && f_j(x), && j = 1, 2, \dots, k \\ & \text{subject to} && g_i(x) \leq 0, && i = 1, 2, \dots, m \end{aligned} \quad (1)$$

where k is the total number of objective functions and m is the total number of constraints. All solutions to a multi-objective optimization problem are called Pareto-optimal solutions, the curve by joining these solutions is known as a Pareto-optimal front.

B. PSO Algorithm and Constraint Dominance Concept

A generic PSO algorithm can be described by as follows.

$$v^{(t+1)} = \chi [w v^{(t)} + c_1 r_1^{(t)} (pBest^{(t)} - x^{(t)}) + c_2 r_2^{(t)} (lBest^{(t)} - x^{(t)})] \quad (2)$$

$$x^{(t+1)} = x^{(t)} + v^{(t+1)} \quad (3)$$

where x is the decision vector in a D-dimensional search space; $pBest$ is the best previous position of a particle in the flight history; $lBest$ is the best particle of

the neighbourhood; v denotes the velocity; c_1 and c_2 are cognitive and social parameters respectively; w is the inertia weight and χ is a constriction coefficient; r_1 and r_2 are two random numbers uniformly distributed in the range $[0, 1]$; t denotes the iteration.

A solution's feasibility can be assessed by its total amount of constraint violations [16], described by

$$\Phi(x) = \sum_{i=1}^m \max(0, g_i(x)) \quad (4)$$

If a solution satisfies all constraints, $\Phi(x)$ returns a zero; the solution is feasible. Otherwise, $\Phi(x)$ returns a positive number; the solution is infeasible. Since an absolute equality is difficult to achieve in implementation, we use a feasibility criterion ε (where ε is small positive number) to evaluate a solution's feasibility, that is, a solution x is considered feasible if its $\Phi(x) \leq \varepsilon$.

The constraint dominance concept is adopted in this research. It can be described as following:

"A solution $x^{(1)}$ is said to 'constraint-dominate' a solution $x^{(2)}$, if any of the following conditions are true:

- Solution $x^{(1)}$ is feasible and solution $x^{(2)}$ is not.
- Solution $x^{(1)}$ and solution $x^{(2)}$ are both infeasible, but solution $x^{(1)}$ has a smaller constraint violation.
- Solution $x^{(1)}$ and solution $x^{(2)}$ are feasible and solution $x^{(1)}$ dominates solution $x^{(2)}$ in the usual sense" [1].

C. Selection Rules

Based on the constraint dominance concept, we propose the selection rules for determining $pBest$ and $lBest$ of the PSO algorithm, as follows.

Rule Set 1: Personal best particle updating rules

Suppose a particle's new location is $pNew$ and its personal best location in the history is $pBest$:

- If both $pNew$ and $pBest$ are feasible, and if $pNew$ dominates $pBest$, update $pBest$ with $pNew$;
- If $pNew$ is feasible and $pBest$ is not feasible, update $pBest$ with $pNew$;
- If $pNew$ is not feasible and $pBest$ is feasible, $pBest$ is not updated;
- If both $pNew$ and $pBest$ are infeasible, and if $pNew$ has a lower constraint violations (Φ) than $pBest$ has, then update $pBest$ with $pNew$.

Rule Set 1 is summarized in Table I.

TABLE I. PERSONAL BEST PARTICLE UPDATING RULES

$pNew$	$pBest$	$pNew$ dominates $pBest$?	$pNew$ has lower Φ than $pBest$ has?	Next $pBest$
feasible	feasible	yes		$pNew$
feasible	infeasible			$pNew$
infeasible	feasible			$pBest$
infeasible	infeasible		yes	$pNew$

Rule Set 2: Local (or global) best particle selection rules:

Among a neighbourhood, select the best performed k (k is number of objective functions) particles $lBest_i$ ($i=1$ to k) in each objective function:

- If all $lBest_i$ ($i=1$ to k) are feasible, randomly select one as $lBest$. In this way, a particle may follow $lBest_1$ at a time, and follow $lBest_2$ at another time. All $lBest_i$ ($i=1$ to k) will get the same probability to be selected as $lBest$;
- If there exist feasible particles and infeasible particles in all $lBest_i$ ($i=1$ to k), the $lBest$ is randomly selected from all the feasible particles. The infeasible particles are disregarded;
- If there are no feasible particles in all $lBest_i$ ($i=1$ to k), the particle with the lowest constraint violations is selected as $lBest$.

For two objective optimization problems, for example, if $lBest_1$ is the best particle in objective f_1 and particle $lBest_2$ is the best particle in objective f_2 . An arbitrary particle p in the same neighbourhood will have its $lBest$ determined by the rules listed in Table II.

TABLE II. LOCAL BEST PARTICLE SELECTION RULES

$lBest_1$	$lBest_2$	$lBest$
feasible	feasible	$rand(lBest_1, lBest_2)$
feasible	infeasible	$lBest_1$
infeasible	feasible	$lBest_2$
infeasible	infeasible	one with the lower Φ

The proposed selection rules for $pBest$ and $lBest$ have the following features:

- The feasibility is the top priority;
- The $pBest$ evolves towards to the bottom-left direction in objective space (assume all objectives are being minimized) since the Pareto dominance concept is adopted;
- The $lBest$ makes effort to extend the spread along the Pareto-optimal front since the best particle in one objective function is followed at a time.
- The rules can be applied in problems that have any number of objective functions.

D. Algorithm

Table III is the structure of the modified PSO algorithm for constrained multi-objective optimization problems.

TABLE III. STRUCTURE OF THE MODIFIED PSO ALGORITHM

Initialize particles
Calculate fitness values of particles under each objective
Calculate constraint violations of each particle
Set current locations as personal best locations
Set local best location for each particle according to Rule Set 2
Do
For each particle
Calculate new velocity by PSO formula
Calculate new location by PSO formula
Update personal best location according to Rule Set 1
End For
Set local best location for each particle according to Rule Set 2
End Do

III. EXPERIMENTS

A. Test Functions

Four functions (taken from [1]), named BNH, TNK, SRN and OSY have been selected for testing the proposed approach. The BNH function has a continuous convex Pareto-optimal set. The TNK function has a discontinuous, convex and nonconvex Pareto-optimal set. Both the SRN function and the OSY function have continuous linear (can be considered convex or nonconvex) Pareto-optimal sets. The TNK and the OSY functions are considered difficult problems since the TNK function has a discontinuous Pareto-front and the OSY function is a high dimensional and a highly constrained problem. The functions are described in equations (5) to (8).

BNH:

$$\begin{cases} \text{Minimize} & f_1(\mathbf{x}) = 4x_1^2 + 4x_2^2, \\ & f_2(\mathbf{x}) = (x_1 - 5)^2 + (x_2 - 5)^2, \\ \text{subject to} & g_1(\mathbf{x}) = (x_1 - 5)^2 + x_2^2 - 25 \leq 0, \\ & g_2(\mathbf{x}) = 7.7 - (x_1 - 8)^2 - (x_2 + 3)^2 \leq 0, \\ & 0 \leq x_1 \leq 5, \quad 0 \leq x_2 \leq 3. \end{cases} \quad (5)$$

TNK:

$$\begin{cases} \text{Minimize} & f_1(\mathbf{x}) = x_1, \\ & f_2(\mathbf{x}) = x_2, \\ \text{subject to} & g_1(\mathbf{x}) = 1 - x_1^2 - x_2^2 + 0.1 \cos(16 \arctan \frac{x_1}{x_2}) \leq 0, \\ & g_2(\mathbf{x}) = (x_1 - 0.5)^2 + (x_2 - 0.5)^2 - 0.5 \leq 0, \\ & 0 \leq x_1, x_2 \leq \pi. \end{cases} \quad (6)$$

SRN:

$$\left\{ \begin{array}{ll} \text{Minimize} & f_1(\mathbf{x}) = 2 + (x_1 - 2)^2 + (x_2 - 1)^2, \\ & f_2(\mathbf{x}) = 9x_1 - (x_2 - 1)^2, \\ \text{subject to} & g_1(\mathbf{x}) = x_1^2 + x_2^2 - 225 \leq 0, \\ & g_2(\mathbf{x}) = x_1 - 3x_2 + 10 \leq 0, \\ & -20 \leq x_1, x_2 \leq 20. \end{array} \right. \quad (7)$$

OSY:

$$\left\{ \begin{array}{ll} \text{Minimize} & f_1(\mathbf{x}) = -[25(x_1 - 2)^2 + (x_2 - 2)^2 + (x_3 - 1)^2 \\ & \quad + (x_4 - 4)^2 + (x_5 - 1)^2], \\ & f_2(\mathbf{x}) = x_1^2 + x_2^2 + x_3^2 + x_4^2 + x_5^2 + x_6^2, \\ \text{subject to} & g_1(\mathbf{x}) = 2 - x_1 - x_2 \leq 0, \\ & g_2(\mathbf{x}) = -6 + x_1 + x_2 \leq 0, \\ & g_3(\mathbf{x}) = -2 - x_1 + x_2 \leq 0, \\ & g_4(\mathbf{x}) = -2 + x_1 - 3x_2 \leq 0, \\ & g_5(\mathbf{x}) = -4 + (x_3 - 3)^2 + x_4 \leq 0, \\ & g_6(\mathbf{x}) = 4 - (x_5 - 3)^2 - x_6 \leq 0, \\ & 0 \leq x_1, x_2, x_6 \leq 10, \quad 1 \leq x_3, x_5 \leq 5, \quad 0 \leq x_4 \leq 6. \end{array} \right. \quad (8)$$

B. Results

For multi-objective optimization, the number of non-dominated solutions is directly linked to the population size. Therefore, a large size of population is required.

Twenty independent runs have been performed for each case. The PSO parameters are: $c_1 = 1.0$; $c_2 = 2.0$; $V_{\max} = 0.5 \cdot (U - L)$, where U and L are the upper and lower boundary for the decision variables; the population size is 200 for the BNH and the SRN functions; the population size is 500 for the TNK and the OSY functions; the maximum iteration $i_{\max}$ for all four cases is 1000; the inertia weight $w = 0.1$; $\chi = 0.63$; a potential solution is considered feasible when its $\Phi < \varepsilon = 1.0E - 05$; Neighbourhood topology is set to the circular ring local model with the neighbourhood size two.

Figures 1 to 4 illustrate the simulated Pareto-optimal fronts from the best runs for the four test cases. The best run means that the Pareto-optimal front has a small Spacing/Spread (S/D) value. 156 out of 200 non-dominated solutions are found for the BHN function. 111 out of 500 non-dominated solutions are found for the TNK function. 161 out of 200 non-dominated solutions are found for the SRN function and 56 out of 500 non-dominated solutions are found for the OSY function. The algorithm achieved reasonably good number of non-dominated solutions for the first three functions. However, the number of non-dominated solutions found for the last function (OSY) is small due to the complexity of the problem.

By observing the simulated Pareto-optimal fronts and the theoretical Pareto-optimal fronts [1], the proposed approach is able to converge to the Pareto-optimal solutions effectively. The final solution curves are reasonably dispersed.

Figure 1. Simulated Pareto-optimal front for test problem BNH

Figure 2. Simulated Pareto-optimal front for test problem TNK

Figure 3. Simulated Pareto-optimal front for test problem SRN

Figure 4. Simulated Pareto-optimal front for test problem OSY

IV. PERFORMANCE STUDY

Experiments are conducted based on the two metrics, “Spacing” and “Maximum Spread” (details in Appendix A). Table IV presents the results for the four functions. Since a smaller “Spacing” (S) value and a larger “Spread” (D) value are desirable, the minimum S and the maximum D are listed in the table. The average values and the standard deviations are also included.

The “Generational Distance” is not evaluated at this stage because of the lack of the real Pareto front data.

Due to the complexity and different user demands, there is not much data available for performance comparison. We include the “Spacing” data (EM-MOPSO) from [15] in Table V for a brief comparison. No “Max Spread” data available from [15]. The NSGA-II data in Table V is also taken from [15].

TABLE IV. SPACING AND MAXIMUM SPREAD FOR TESTING CASES BASED ON 20 RUNS

	Item	BNH	TNK	SRN	OSY
S	Min.	0.599672	0.004032	1.089612	1.040084
	Avg.	0.979688	0.012609	1.587708	2.943298
	Std.	0.258917	0.003361	0.423624	1.057702
D	Max.	143.5117	1.403277	302.6255	225.3124
	Avg.	136.9071	1.362672	276.0655	155.4652
	Std.	3.775605	0.031040	16.26418	21.64976

TABLE V. STATISTIC RESULTS BY DIFFERENT APPROACHES

		This Paper	NSGA-II	EM-MOPSO
BNH	Min.	0.599672	0.6408	0.6357
	Avg.	0.979688	0.7756	0.6941
	Std.	0.258917	0.0727	0.0385
SRN	Min.	1.089612	1.3402	1.0768
	Avg.	1.587708	1.5860	1.2439
	Std.	0.423624	0.1337	0.1055

According to Table V, our approach obtained the best “Spacing” value in function BNH. For function

SRN, our approach outperformed NSGA-II in the best “Spacing” value but slightly worse than EM-MOPSO. Regarding to algorithm consistency, our approach did not outperform the other two approaches in both functions but was in the acceptable range. More investigation needs to be done about this.

We realize that the “Spacing” value can only be applied in consecutive solutions which may not be suitable for those problems that have discontinuous solutions such as function TNK. In such cases, other performance metrics may be taken into consideration.

V. CONCLUSIONS AND FUTURE WORK

This paper proposed a simple and easy-to-implement modified PSO algorithm for solving constrained multi-objective optimization problems. By adopting the constraint dominance concept, the proposed approach defines two sets of rules for determining the personal best location $pBest$ and the best neighbourhood location $lBest$ of the PSO algorithm. The simulation results to the four constrained multi-objective optimization problems demonstrated the proposed approach is able to find the Pareto-optimal solutions effectively.

The performance of the proposed approach has been evaluated by two metrics. The results demonstrated the proposed approach is able to achieve similar good results in best “spacing” to the compared approaches. However, the proposed approach did not outperform the compared approaches in terms of consistency. Our future work includes more investigation in diversity control and testing the proposed approach in problems that have more than two objective functions.

APPENDIX A

Three metrics can be used for performance evaluation, as follows.

Spacing (S) [17] measures how well distributed (spaced) the solutions in the non-dominated set found. The formula is presented in (9),

$$S = \sqrt{\frac{1}{n} \sum_{i=1}^n (d_i - \bar{d})^2} \quad (9)$$

where n is the number of solutions in the obtained non-dominated set, $d_i = \min_{k \in n \wedge k \neq i} \sum_{m=1}^M |f_m^i - f_m^k|$ and $\bar{d}$ is the mean value of the above distance measure $\bar{d} = \sum_{i=1}^n d_i / n$; f_m is the m -th objective function value; M is the number of objective functions. When the solutions are near uniformly spaced, the corresponding

distance measure will be small. Thus, an algorithm finding a set of non-dominated solutions having a smaller spacing S is better.

Maximum Spread [18] gives a value which represents the maximum extension between the farthest solutions in the non-dominated set found. The formula is presented in (10). A bigger value indicates better performance.

$$D = \sqrt{\sum_{m=1}^M (\max_{i=1}^n f_m^i - \min_{i=1}^n f_m^i)^2} \quad (10)$$

Generational Distance (as cited in [1]) is a metric to find the average distance of the non-dominated set of solutions from the real Pareto optimal set. The formula is given in (11).

$$GD = \frac{\sqrt{\sum_{i=1}^n d_i^2}}{n} \quad (11)$$

where d_i is the Euclidean distance between solution i from the set of n non-dominated solutions found and the closest element from the real Pareto optimal set. A smaller value indicates the solutions found are closer to the real Pareto front.

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