Support Document (Years 1-7)

for

Queensland Primary Schools

Written by
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# Years 1 - 10 Mathematics Syllabus Support Document Years 1 - 7 for Queensland Primary Schools

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<td>Fig 4</td>
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<td>Division</td>
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<td>Introduce an awareness of the magnitude of the answer - whether it would be in the hundreds, tens or ones.</td>
<td>Fig 5</td>
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<td>Division</td>
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<td>Continue work on the front-end strategy. Introduce closer estimates. Extension work can involve exploration work on estimation strategies for division other than the front-end strategy such as: • rounding • compatible numbers, dividends to be no more than two digits • changing the divisor of 9 to 10.</td>
<td>3c(iv)</td>
<td>Fig 10</td>
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Estimation - Chapter 10 : 2
<table>
<thead>
<tr>
<th>Year</th>
<th>Operation</th>
<th>Yearly Expectations</th>
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<th>Ref</th>
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<td></td>
<td>- rounding</td>
<td></td>
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<tr>
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<td><strong>Multiplication</strong></td>
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<td></td>
<td>Continue work with rounding and front-end strategies with final adjustments.</td>
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<td></td>
<td></td>
<td>Consolidate rounding to 10 and 100.</td>
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<td></td>
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<td>Introduce rounding to 1000.</td>
<td>Fig 8</td>
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<td><strong>Division</strong></td>
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<td></td>
<td>Continue work with front-end estimation strategies with adjustments.</td>
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<td></td>
<td></td>
<td>Introduce compatible numbers for more than two digit dividends.</td>
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<td>Fig 13</td>
<td></td>
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<tr>
<td>Year 7</td>
<td>Addition &amp;</td>
<td>Continue work with:</td>
<td></td>
<td></td>
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<tr>
<td></td>
<td>Subtraction</td>
<td>- front-end with adjustments</td>
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<td></td>
<td></td>
<td>- rounding</td>
<td></td>
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<td></td>
<td>- grouping nice and large numbers</td>
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<td>Introduce approximating the mean.</td>
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<td><strong>Multiplication</strong></td>
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<td>Continue work with rounding with final adjustments</td>
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<td>Introduce rounding to:</td>
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<td>- 10,000</td>
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<td>- 25</td>
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<td>3d(iii)</td>
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<td></td>
<td><strong>Division</strong></td>
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<td></td>
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<td>Continue work with:</td>
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<td>- front-end estimation</td>
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<td>- strategies with adjustments</td>
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<tr>
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<td>- estimation with compatible numbers</td>
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<td>Introduce:</td>
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<td></td>
<td></td>
<td>- estimating by finding the first digit of the quotient</td>
<td></td>
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</tr>
<tr>
<td></td>
<td></td>
<td>- estimate by rounding the divisor</td>
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</table>

_Estimation - Chapter 10: 3_
The front-end strategy for Year 3 involves working with the left-most digit, for example:

\[ 56 + 73 \]  
Estimate 12 tens: 120.

The main technique for estimating answers to subtractions in Year 3 is the 'front-end' approach. It is important that the children have many opportunities to use this technique.

Front-end 64

\[ - 32 \]  
An estimate is made calculating with the left-hand digits only.

Think 6 tens - 3 tens is 3 tens. The estimate is 30.

624

- 212

Think 6 hundreds - 2 hundreds is 4 hundreds. The estimate is 400.

76

- 9

Think 7 tens - 0 tens is 7 tens. The estimate is 70.

The estimate is refined by looking quickly at the digits in the next column (6 tens and 6 tens) and deciding that there are enough tens to add on another hundred to the estimate (700).

\textbf{Front-end}

(a) 875

\[ - 358 \]  
An estimate is made by calculating with the left-hand digits only:

Think 8 hundred - 3 hundred \ldots \textit{estimate is 500}

Once students are confident with making initial estimates, they can try to get closer by considering the next column of numbers. 7 tens are more than the 5 tens that have been taken away, so the final estimate would be 'more than 500'.

(b) 825

\[ - 358 \]  
For this example, a 'front-end' estimate would again be 500, but this time there are not enough tens to take 5 tens away, so a closer estimate would be 'less than 500'.

Students may also estimate answers to calculations involving thousands, and verify their estimates either by checking the strategy that was used, or by using calculators. Examples with different numbers of digits in the numbers should be included.
Rounding for addition

17
32
+ 48
Think
Numbers are rounded to the nearest ten
and added mentally.

Rounding for subtraction

875
- 358
Numbers would be rounded to 900 and 400, giving an approximate answer of 500.

Looking for numbers that make 10'

This is a front-end strategy that makes convenient use of numbers
adding to ten. 6 tens and 4 tens are grouped to make 10 tens (100) and
the other 7 tens (70) are added (170).

The next column of numbers is quickly examined and the initial estimate is increased by another 10 (180).

Rounding and front-end estimation strategies for multiplication

Provide a word problem involving multiplication, e.g. Three children saved $38 each. How much money did they save altogether?

Ask students to:
- identify the operation involved (the teacher can record the calculation on the blackboard)
- quickly write an estimate of the answer
- explain the strategy used to make the estimates
- use calculators to find the exact answer
- compare the estimates, and identify the strategy that produced the closed estimate.

In the above case, students would have a closer estimate if they used 'rounding' as compared to 'front-end':

Rounding

\[ 38 \times 3 \]
Think round 38 to 40
\[ \rightarrow \frac{38}{4} = 9.5 \]
\[ \rightarrow 40 \]

Front-end

\[ 38 \times 3 \]
Think leave off the 8
\[ \rightarrow \frac{38}{9} = 4.22 \]
\[ \rightarrow 30 \]

Repeat this procedure with different numbers, and compare the accuracy of the strategies in each case. Students should come to realise that, in general, 'rounding' produces a closer estimate. Encourage students to practise estimating answers to multiplication calculations using this strategy.

Students can infer whether their estimates were more or less than the real answers by considering how they rounded the two-digit numbers. If the number was rounded to the next ten, then the real answer would be less than the estimate. If the number was rounded down to the lower ten, then the real answer would be more than the estimate.
The following procedure can be used as a model for questioning students to help them arrive at broad estimates. Determining the approximate size of an answer is a very important step, and students should be given plenty of practice and assistance where necessary. The model should enable students to decide whether an answer is in the tens, hundred, or less than ten, which is all that is expected at this year level.

(i) Pose problem.
- 5 children were given a 250 g chocolate bar to share. About how much should they have each if the chocolate is shared equally?

(ii) Understand the situation.
Ask the students:
- Which operation is needed? (divided by)
- How much is being shared? (250 g)
- How many are sharing? (5)
Teacher records on the blackboard: \( 5 \div 250 \mathrm{g} \)

(iii) Determine the approximate size of the answer.
Ask the students:
- Is there enough to give each person 100 g each? Why not? (No, that would be 500 g and there isn't enough)
- Will there be any hundreds in the answer? (No)
- Is there enough to give each person 10 g each? Why? (Yes, that's only 50 g and there's a lot left over)

(iv) Conclusion
- The answer must be in the tens (i.e. less than 100 and more than 10)
- Check the answer using your calculator.

Teachers could use a sheet or overhead transparency similar to the following to organise student's thinking about division problems and estimation. The questioning procedure outlined above should be employed, and calculators used to check answers. Warn the students that not all of the problems involve division!

<table>
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<th>Problem</th>
<th>Calculation</th>
<th>You would estimate the answer to be</th>
</tr>
</thead>
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<tr>
<td>5 children hired a trampoline for 90 mins. To be fair, how long should each child's turn take?</td>
<td>( 5 \div 90 )</td>
<td>in the hundreds</td>
</tr>
<tr>
<td>The stereo shop sold 3 identical radio/cassette players for a total of $372. How much does 1 cost?</td>
<td>( 372 \div 9 )</td>
<td></td>
</tr>
<tr>
<td>The Flying Doctor treated 36 patients in one month and 9 patients in the next month. How many patients were treated altogether?</td>
<td></td>
<td></td>
</tr>
<tr>
<td>A shop bought 28 m of material to cover 2 lounge suites exactly the same. How much material is needed for each one?</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Rounding

- Numbers are rounded mentally to 1 000 and 7 000, giving an approximate answer of 8 000.
- Numbers are rounded mentally to 7 000 and 2 000, giving an approximate answer of 5 000.

Front-end

- Make an initial estimate by calculating with the left-hand digits - 1 000 and 6 000 gives 7 000.

To get closer, glance at the next column of digits to see if there are enough hundreds to increase the initial estimate. In this case there are. The estimate would be 8 000.

In subtraction examples, calculate with the left-hand digits and compare the remaining parts of the numbers.

- Initial estimate (6 000)
- Compare the rest (748 is more than 435 or 700 is more than 400)
- Final estimate (over 6 000)

- Initial estimate (4 000)
- Compare the rest (261 is less than 942 or 200 is less than 900)
- Final estimate (under 4 000)

Both rounding and front-end strategies can be used for estimating answers to addition and subtraction. When used with the adjusting process, the front-end strategy usually gives a closer estimate.

Grouping to nice numbers

In addition, 'nice' numbers can be grouped to make 10, 20, 100 and so on, or may just add together easily. A front-end approach is still used.

- Initial estimate is $13. ($4 and $6 are $10 and another $3 is $13)
- Glance at the next column. (The cents make more than $1)
- Final estimate is over $14.

Teachers are encouraged to help students develop the following skills and understanding which are necessary for estimating effectively.

- multiplying numbers by 10 and 100 (mentally)
- renaming numbers (e.g. 28 hundred is 2 800)
- patterns
  - tens multiplied by ones gives tens
  - hundreds multiplied by ones gives hundreds
  - tens multiplied by tens gives hundreds
- multiplication facts
- rounding numbers to the nearest 10 or 100.
■ Rounding

Once students are proficient with forming an initial estimate, encourage them to identify whether the answer would be more or less than their estimate.

- Example:
  634 \times 4

  think

  6 hundred \times 4 is 24 hundred.
  Initial estimate is 2,400. We had more than 600 so the real answer is more than 2,400.

- Example:
  40 \times 4

  think

  40 \times 70 = 2,800
  or
  4 tens \times 7 tens = 28 hundred \rightarrow 2,800

■ Front-end

When a closer estimate is required, especially when dealing with money, the following strategy can be used by students who have the necessary skills:

- form an initial estimate by front-end
- consider the rest of the calculation and adjust the estimate if necessary - for example,
  - $2.19 \times 4$: initial estimate is $8
  - adjust up (about another $1)
  - final estimate is $9

- Example:

  $2.19 \times 4

  initial estimate is $8
  adjust up (about another $1)
  final estimate is $9

- Example:

  41 \times 68

  think

  40 \times 60 = 2,400
  or
  4 tens \times 6 tens = 24 hundred \rightarrow 2,400

- Involve students in comparing both rounding and front-end strategies to see which produces a more reliable estimate.

Figure 9 ....... Rounding to 10 or 100. An estimation strategy for multiplication (Introduce Year 5)

■ Rounding to 10

When a number is to be multiplied by 9, it can be multiplied by 10 instead to provide an estimate. For example, 16 \times 9 can be estimated by calculating 16 \times 10 (160).

- Example:

  41 \times 68

  think

  40 \times 60 = 2,400
  or
  4 tens \times 6 tens = 24 hundred \rightarrow 2,400

- Example:

  $2.19 \times 4

  initial estimate is $8
  adjust up (about another $1)
  final estimate is $9

- Example:

  41 \times 68

  think

  40 \times 60 = 2,400
  or
  4 tens \times 6 tens = 24 hundred \rightarrow 2,400

■ Rounding to 100

Similarly, numbers close to 100 can be changed to 100. The example 95 \times 5 would change to 100 \times 5.

Figure 10 ...... Rounding, working with nice numbers and changing the divisor. Estimation strategies for division. (Extension Year 5. Introduce & consolidate Year 6)

- Compare and discuss the following ways of estimating the answer to $4.254$.

  about 4 \underline{240}

  \underline{60}

  about 5 \underline{25} tens

  \underline{50}

  about 4 \underline{28} tens

  \underline{70}

  about 5 \underline{25}*

  \underline{50}

  about 4 \underline{24}*

  \underline{60}

Estimation - Chapter 10 : 8
Compatible Numbers

\[
\begin{array}{c|c|c}
7 & 36 & \text{think} \\
7 & 58 & \text{think} \\
7 & 26 & \text{think} \\
\end{array}
\]

Note: A number that is a multiple of another number is referred to as a 'compatible' number.

Changing a divisor of '9' to '10'

Students can rely on their understanding of numeration to divide numbers by 10. When the divisor is 9, students should recognise that 10 is a close substitute that is easy to work with.

\[
\begin{array}{c|c|c|c}
9 & 261 & \text{think} & 10 \ 261 & \text{about} 26 \\
9 & 407 & \text{about} \\
9 & 578 & \text{about} \\
9 & 778 & \text{about} \\
\end{array}
\]

Figure 11 .... Front-end, compatible numbers and grouping large numbers. Estimation strategies for addition & subtraction. (Consolidate Year 6)

Front-end addition

\[
\begin{array}{c|c|c|c}
$1.25 & 4.50 & \text{think} & 1 + 4 + 3 \\
+ $3.10 & & & \text{Estimate: over$8} \\
\end{array}
\]

Front-end addition with adjusting

\[
\begin{array}{c|c|c|c}
$6.24 & 2.90 & \text{think} & 6 + 2 + 5 = $13 \\
+ $5.75 & & & 24 + 75 \text{ is about$1} \\
\end{array}
\]

Front-end addition estimates with adjusting

When an estimate closer to the real total amount is required, especially when dealing with money, an initial front-end estimate can be adjusted to give a final estimate.

Front-end subtraction estimates with adjusting

\[
\begin{array}{c|c|c|c}
8 456 & -5 692 & \text{think} & 8 000 - 5 000 = 3 000 \\
& & & 8 000 - 5 000 = 3 000 \\
\end{array}
\]

Front-end subtraction with closer estimating

\[
\begin{array}{c|c|c|c}
832 & -357 & \text{think} & \text{Initial estimate: less than}$500 \text{ as } 32 \text{ is less than 57} \\
& & & \text{Consider the rest: looking} \\
& & & \text{at the tens place, 13 tens -} \\
& & & 5 \text{ tens gives 8 tens} \\
& & & \text{Closer estimate: 480} \\
\end{array}
\]

Estimation - Chapter 10: 9
Rounding to 10,100 or 1 000 for multiplication

When multiplying numbers with at least two digits, it is usual to round each number to the front digit and find the product of the rounded numbers. If a number, however, is close to 10, 100 or 1 000, another method is to round this number to 10, 100 or 1 000 (as appropriate) and multiply by the other number, e.g.

\[
394 \quad \text{think} \\
\times 96
\]

• Round: 96 is close to 100
• Estimate: 39 400

Have students complete the following estimates by rounding to 10, 100 or 1 000 as appropriate before multiplication.

\[
\begin{array}{cccc}
35 & 325 & 546 & 9 995 \\
\times 99 & \times 102 & \times 97 & \times 6
\end{array}
\]

The first digit found in the quotient during division provides a guide to the estimate.

\[
6 \quad \text{think} \\
7\ 4333
\]

• Find first digit of the quotient: 43 divided by 7 is 6
• Estimate: 6 indicates the quotient is in the 600s

This method does not always give the closest number to the actual number, but it provides a reasonable estimate.

Adjusting front-end estimates

Because some situations (e.g., shopping) demand more precise estimates, it is often appropriate to ‘adjust’ front-end estimates. For example:

\[
\begin{align*}
$7.24 & \\
$5.72 & \\
+ $2.99 &
\end{align*}
\]

Front end: \( 7 + 5 + 2 = 14 \)
Adjust-up: 24 and 72 are about $1 and 99¢ is nearly $1
Estimate: So... about $16.00

Estimation - Chapter 10 : 10
Front-end subtraction

\[
\begin{array}{c}
9374 \\
\underline{- 4269}
\end{array}
\]

9 thousand take away 4 thousand

Estimate: 5 thousand

Students may use their numeration skills to find a closer estimate by considering the first two digits; i.e. 93 (hundreds) take away 42 (hundreds) gives 51 (hundreds) or 5 100.

Similarly, to find a closer estimate, use a 'front-end' approach, and then adjust by considering the rest of the number. For example:

\[
\begin{array}{c}
9374 \\
\underline{- 4269}
\end{array}
\]

The difference between 374 and 269 is about 100.

Estimate: 5 100

Grouping nice numbers

\[
\begin{array}{c}
\$5.45 \\
\$1.82 \\
\$2.60 \\
\$3.21
\end{array}
\]

about $8.00

about $5.00

Estimate: $13.00

The group of numbers clusters around 80 000. The estimate of 560 000 is arrived at by multiplying the approximate mean by the number in the group.

Note: When estimating with large numbers, the other strategies are also appropriate. It is important that students' thinking be flexible enough to choose the best strategy. In the previous activity, for example, if Wednesday's total was 144 000, a combination of strategies would be required.

World Expo attendance numbers

<table>
<thead>
<tr>
<th>Day</th>
<th>Attendance</th>
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</thead>
<tbody>
<tr>
<td>Monday</td>
<td>78 385</td>
</tr>
<tr>
<td>Tuesday</td>
<td>81 824</td>
</tr>
<tr>
<td>Wednesday</td>
<td>79 108</td>
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<td>Thursday</td>
<td>76 821</td>
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<td>Friday</td>
<td>83 822</td>
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<td>Saturday</td>
<td>83 469</td>
</tr>
<tr>
<td>Sunday</td>
<td>80 875</td>
</tr>
<tr>
<td>Estimate</td>
<td>560 000</td>
</tr>
</tbody>
</table>

2 894

\[
\begin{array}{c}
2 894 \times 5 \\
3 000 by 5 is 15 000
\end{array}
\]

Estimate: About 15 000 or, more precisely, a little less than 15 000.

Variations to make rounding more precise

Sometimes it is possible to obtain a more precise estimate by rounding one factor and then mentally computing the answer. In the case of, for example, 64 x 22, 22 may be rounded to 20, and 64 x 20 may be calculated mentally to give 1 280.

When both factors are close to the middle of the range, a more precise estimate may be obtained by rounding one factor down and the other up. For example, in 85 x 66, the 85 may be rounded down to 80 and 66 may be rounded up to 70 to give an estimate of 5 600.

Here is a question for students to think about: when you round one number up and the other down, can it make a large difference if you round the other way? For example:

\[
\begin{array}{c}
85 \\
\times 66
\end{array}
\]

\[
\begin{array}{c}
80 \\
\times 70
\end{array}
\]

\[
\begin{array}{c}
90 \\
\times 60
\end{array}
\]

5 600

5 400

Can you generalise about these results?
Here students should realise that, by finding the first digit of the quotient, they have an acceptable estimate.
For example:
\[ 6 \overline{\sqrt{13\,825}} \]
The 2 in the quotient indicates that the quotient is in the 2 000s.

Sometimes the use of compatible numbers may not be appropriate. In these cases it may be better to round the divisor and find the first digit of the quotient.
For example:
\[ 43 \overline{\sqrt{2\,585}} \]

Due to the detailed Scope and Sequence and the figures which explain each increment in content, a year level expectation section is not required.
Teachers are encouraged to include estimation throughout the entire mathematics program and not present estimation only in separate lessons as good estimation and approximation skills enhance our ability to deal with many everyday situations. When calculating using the written algorithm or a calculator, students should practice estimating to predict answers and to check the reasonableness of answers.

Opportunities should be taken to practise estimation whenever possible in calculations involving concepts such as length, mass, money, time, volume, angles or area.

Ask students to give their estimates as quickly as they can. After estimates have been made, the emphasis should be on discussion of strategies. Encourage students to verify their estimates by checking their answers before calculating or using a calculator.

Remind students of the following features of estimation which make it useful:
- Estimates can be done quickly
- Estimates can be done mentally
- Estimates can provide adequate answers when precise answers are not needed.

If these points cannot be met, perhaps the original data should be used rather than making meaningless approximations.

The time taken for students to make each estimate should be controlled, to prevent them calculating the actual answer and then rounding the answer to make it look like an estimate. Controlling the time also helps the teacher identify students who are using inefficient strategies, but students would need to explain the strategy used before the teacher could diagnose the difficulty.

Some students who are efficient mental calculators find the uncertainty of estimation difficult to cope with, and try to calculate exactly. These students may be shown the inefficiency of their strategies by providing them with a longer list of amounts and comparing the time taken for them to arrive at an answer with the time taken for other estimators.

Students should also benefit from frequent, short practice lessons where estimation is used in meaningful contexts. They should realise that for estimation to be effective, it should be performed mentally, quickly, and give reasonably close answers.

The distinction between estimation and approximation depends upon the degree of prediction involved. When we estimate we are using judgement based on hunch or experience in previous similar situations to predict a number or measure (e.g. It looks like there are about 350 people in a crowd). When we approximate we have a count or measure, but have chosen either to collect it to a chosen unit (e.g. information collected to the nearest dollar) or to round it (e.g. rounding the number of people who came through the turnstile to 340). The approximate number or measures may then be used in computation (e.g. The gate takings should be about 340 x $8). The distinction is subtle and many people choose to use the term 'estimation' for both. Where an estimate is arrived at from approximate values, students should assess the consequences for the accuracy of the estimate.

Reference: A National Statement on Mathematics for Australian Schools.
Front-end - An estimation strategy

Front-end is the main technique for estimating answers to addition, subtraction, multiplication and division calculations. An estimate is made by calculating with the left-most digits only and refined by looking at the next column to see if there is sufficient value to necessitate an adjustment to the original estimate. When used with the adjusting process, the front-end strategy usually gives a closer estimate than the rounding strategy.

Front-end - An estimation strategy for addition

<table>
<thead>
<tr>
<th>Front-end</th>
<th>Front-end addition with adjusting</th>
</tr>
</thead>
<tbody>
<tr>
<td>$2.15</td>
<td>$6.24</td>
</tr>
<tr>
<td>$3.39</td>
<td>$2.90</td>
</tr>
<tr>
<td>+ $4.26</td>
<td>+ $5.75</td>
</tr>
</tbody>
</table>

- Initial estimate: 6 + 2 + 5 = $13
- Consider the rest: 24 + 75 is about $1
- Adjusted estimate: About $14.90

Front-end for subtraction

<table>
<thead>
<tr>
<th>Front-end</th>
<th>Front-end with adjustment</th>
</tr>
</thead>
<tbody>
<tr>
<td>9 374</td>
<td>8 456</td>
</tr>
<tr>
<td>- 4 269</td>
<td>- 5 692</td>
</tr>
</tbody>
</table>

- Initial estimate: subtract the front-end digit to get an initial estimate, 8 000 - 5 000 = 3 000
- Initial estimate: 3 000
- Consider the rest: 456 is less than 692
- Adjusted estimated: under 3 000

Front-end for multiplication

<table>
<thead>
<tr>
<th>Front-end</th>
<th>Front-end with adjustment</th>
</tr>
</thead>
<tbody>
<tr>
<td>38 x 3</td>
<td></td>
</tr>
</tbody>
</table>

- Leave off the 8
- 3 so 30
- x 3 x 3
- 9 90

- Initial estimate is $8
- Adjust up (about another $1)
- Final estimate is $9

Front-end for division

Present a word problem to students.
Five small tents cost $352 altogether. About how much did each tent cost?

Check students' understanding of the situation.
Ask the students:
- Which operation is needed? (division)
- How much is being shared? ($352)
- How many are sharing? (5)

Record $352 on the blackboard/OHT.
Involve students in determining the magnitude of the answer.

Ask the students:
- What do we share out first? (the hundreds)
- How many hundreds are there to share? (3)
- Are there enough hundreds to give each person one (hundred) each? If there are enough hundreds to give each person one (hundred), then students can conclude that the answer will be in the hundreds. Will there be any hundreds in the answer? (no)
- Are there enough tens to give each person one (ten) each? (yes, there are 35 tens)

Discuss the estimate.
The answer must be 'in the tens' (less than 100 and more than 10).
Have students check the answer with calculators.

Front-end with adjustment

Make a closer estimate.
Students must be able to determine confidently the place value of the answer, before they are expected to identify the approximate number of tens or hundreds in the answer.
To help students make a closer estimate, use the same questioning procedure as explained earlier, adding the extra question as shown.
- Are there enough tens to give each person one (ten) each?
- About how many tens will each person receive?
  Students think '5 sevens are 35?'
  5 sevens are 35.
  The estimate is '7 tens'.
- Conclusion - the answer will be about $70.

Include practice examples where:
- the lead digit/s of the dividend are multiples of the divisor
  e.g. $3 \overline{652}$ $4 \overline{207}$
- the lead digit/s of the dividend are not multiples of the divisor
  e.g. $3 \overline{782}$ $4 \overline{223}$

**Rounding – an estimation strategy**

Rounding is an estimation strategy where numbers involved in the calculation are rounded mentally to give an approximate answer. The front-end estimation strategy usually gives a more accurate answer than the rounding strategy when it is used with the adjustment process.

Through practising with different examples, students should understand the following:
- **When both numbers are rounded down**, the answer will be greater than the estimate. For example:
  
  $\begin{align*}
  \text{72 round to } & 70 \\
  \times \text{23 round to } & \times 20 \\
  \text{1400} & \text{1400 is an underestimate so the answer is greater than 1400.}
  \end{align*}$

- **When both numbers are rounded up**, the answer will be less than the estimate. For example:
  
  $\begin{align*}
  \text{56 round to } & 60 \\
  \times \text{28 round to } & \times 30 \\
  \text{1800} & \text{1800 is an overestimate, so the answer is less than 1800.}
  \end{align*}$

- **When one number is rounded up and the other is rounded down**, the estimated cannot be classified as being an overestimate or an underestimate without further calculation. For example:
  
  $\begin{align*}
  \text{47 round to } & 50 \\
  \times \text{33 round to } & \times 30 \\
  \text{1500} & \text{1500}
  \end{align*}$
### Rounding for addition

\[
\begin{align*}
1372 + 6885 & = 8257 \\
\text{Numbers are rounded mentally to 1000 and 7000, giving an approximate answer of 8000.}
\end{align*}
\]

### Rounding for subtraction

\[
\begin{align*}
6540 - 2478 & = 4062 \\
\text{Numbers are rounded mentally to 7000 and 2000, giving an approximate answer of 5000.}
\end{align*}
\]

### Rounding for multiplication

#### Rounding

<table>
<thead>
<tr>
<th>Rounding</th>
<th>38</th>
<th>3</th>
<th>4</th>
<th>12</th>
<th>40</th>
<th>120</th>
</tr>
</thead>
<tbody>
<tr>
<td>x3</td>
<td>38</td>
<td>3</td>
<td>4</td>
<td>12</td>
<td>40</td>
<td>120</td>
</tr>
</tbody>
</table>

#### Variations to make rounding more precise.

Sometimes it is possible to obtain a more precise estimate by rounding one factor and then mentally computing the answer. In the case of, for example, \(64 \times 72\), 22 may be rounded to 20, and \(64 \times 20\) may be calculated mentally to give 1280.

When both factors are close to the middle of the range, a more precise estimate can be obtained by rounding one factor down and the other up. For example, in \(85 \times 66\), the 85 may be rounded down to 80 and 66 may be rounded up to 70 to give an estimate of 5600.

Here is a question for students to think about: when you round one number up and the other down, can it make a large difference if you round the other way? For example:

\[
\begin{align*}
85 & \times 66 \\
80 & \text{ or } 90 \\
66 & \times 70 \\
5600 & \text{ or } 5400
\end{align*}
\]

Can you generalise about these results?

#### Rounding to 10, 100, 1000 or 10000 for multiplication

When multiplying numbers with two or more digits, it is usually effective to round each number to the lead digit and to find the product of the rounded numbers.

However, if one of the numbers is close to 10, 100, 1000 or 10000, another strategy is to round this number to 10, 100, 1000 or 10000 as appropriate and multiply by the other number. For example:

\[
\begin{align*}
4327 & \times 97 \\
4327 \text{ multiplied by 100} & \text{ Estimate: 432700}
\end{align*}
\]

#### Rounding to 25 or 50

When one of the factors is close to 25 or 50, it may be rounded to 25 or 50 and the product may be calculated mentally by using fractions of one hundred. For example, to estimate \(64 \times 27\), one could round 27 to 25 and think \(64 \times 25\) is equal to \(6400\), because 25 is one quarter of 100, \(64 \times 25\) will be one quarter of \(6400\), or 1600.

**Estimation - Chapter 10 : 16**
An objective of estimating an answer to a division is perhaps better met by the front-end or compatible number strategy than the rounding strategy. In most cases, the rounding of a dividends may not be as advantageous as using the left most digits or selecting the nearest compatible number. (A number that is a multiple of another number is referred to as a "compatible" number).

Looking to make ten, grouping to nice numbers, compatible numbers and grouping large numbers are names of estimation strategies for addition which have cause some confusion because they are used interchangeable in the sourcebooks. To avoid further confusion the author suggests the following applications.

Looking to make ten, grouping nice numbers and grouping large numbers should refer to the strategy where addends are grouped if they add together nicely, possible to sums such as 10, 20, 50, 100, 1 000, 50 000 etc. (Warning: some examples in the sourcebooks are referred to as compatible numbers. This brief is designed by the author to clear some confusion which exists about these terms).

Looking to make ten is the name given to the lowest level of complexity for introduction in Year 4. The terms ‘nice numbers’ and ‘grouping large numbers’ should refer to the exact same strategy only with bigger addends.

6 tens and 4 tens are grouped to make 10 tens (100) and the other 7 tens (70) are added (170).

The next column of numbers is quickly examined and the initial estimate is increased by another 10 (180).

Initial estimate: 25 cents + 76 cents is about $1
Final estimate: about $1.40

Initial estimate: 75 thousand and 30 thousand are about 100 thousand
Second initial estimate from the rest: 9 thousand and 190 thousand are about 200 thousand
Final estimate: about 300 000

Estimation - Chapter 10: 17
The term compatible numbers should be reserved for reference to a number that is a multiple of another. In the following example 4 and 20 are compatible numbers. Therefore the compatible number strategy refers to an estimation strategy only for division.

\[
\begin{array}{c}
4 \overline{\div} 275 \\
\text{think}
\end{array}
\]

- Select multiple of divisor closest to dividend: 27 is close to 28, which is a multiple of 4.
- Estimate: 280 divided by 4 is 70

\[
\begin{array}{c}
4 \overline{\div} 275 \\
\text{think}
\end{array}
\]

- 28 tens
- Estimate: 70

To use this strategy, students need to be able to recognise which multiple of the divisor is closer to the dividend.

This strategy is also valuable for 2-digit divisors. For example:

\[
32 \overline{\div} 957 \\
3 \times 32 = 96
\]

Therefore, a reasonable estimate is 30.

Point out to students that both the dividend and divisor can be changed, as in the following example:

\[
48 \overline{\div} 958 \quad 48 \overline{\div} 960 \quad \text{or} \quad 50 \overline{\div} 1000
\]

Obviously, students must be proficient with multiples if they are to apply this strategy successfully.

---

**Reading and Recording Estimation**

Due to the detailed Scope and Sequence and the figures which explain each increment in content, a Reading and Recording section is not required.
<table>
<thead>
<tr>
<th>Year</th>
<th>Language</th>
<th>School Use</th>
</tr>
</thead>
<tbody>
<tr>
<td>Year 1</td>
<td>guess</td>
<td></td>
</tr>
<tr>
<td>Year 2</td>
<td>as for Year 1</td>
<td></td>
</tr>
</tbody>
</table>
| Year 3 | front-end
digit
first
left
about
estimate
strategy
Optional
• round up to
• round down to
• looking to make ten |            |
| Year 4 | as for previous years
less than
more than
nearest
closest estimate
adjust up
adjust down
answer would be in the ones, tens, etc. |            |
| Year 5 | as for previous years
grouping to nice numbers
initial estimate
final estimate
compatible numbers
divisor
dividend
quotient
multiples |            |
| Year 6 | as for previous years
grouping large numbers |            |
| Year 7 | as for previous years
approximating the mean
precise     |            |
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  Year 5
  Year 6
  Year 7

Year Level Expectations .....................................2

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  Numerator, denominator & vinculum ..............3b
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  Equivalence ..............................................3e
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  Decimal fractions .......................................4c

Language ......................................................5
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<th>Ref</th>
<th>Sourcebook</th>
<th>School Use</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st</td>
<td></td>
<td>p. 167</td>
<td></td>
</tr>
<tr>
<td>2nd</td>
<td></td>
<td>pp. 167-168</td>
<td></td>
</tr>
<tr>
<td>3rd</td>
<td></td>
<td>pp. 168-169</td>
<td></td>
</tr>
<tr>
<td>4th</td>
<td></td>
<td>p. 170</td>
<td></td>
</tr>
<tr>
<td>5th</td>
<td></td>
<td>pp. 170-171</td>
<td></td>
</tr>
<tr>
<td>6th</td>
<td></td>
<td>p. 172</td>
<td></td>
</tr>
</tbody>
</table>

**Decimal Fractions**

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| 10th      |     | pp. 178-181|            |
| 11th      |     | pp. 178-181|            |
| 12th      |     |            |            |

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| 14th      |     | p. 190     |            |
| 15th      |     | Fig 2      | pp. 190-192|
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<th></th>
<th>Example</th>
<th>Ref</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st</td>
<td>Tenths (no regrouping)</td>
<td>Fig 3</td>
</tr>
<tr>
<td>2nd</td>
<td>Ones and tenths</td>
<td></td>
</tr>
<tr>
<td></td>
<td>- no regrouping</td>
<td>0.2</td>
</tr>
<tr>
<td></td>
<td>- with empty houses</td>
<td>+ 0.5</td>
</tr>
<tr>
<td></td>
<td>- regrouping in the ones</td>
<td>1.4</td>
</tr>
<tr>
<td></td>
<td>- regrouping in the tenths</td>
<td>3.4</td>
</tr>
<tr>
<td></td>
<td>- regrouping throughout</td>
<td>8.4</td>
</tr>
<tr>
<td>3rd</td>
<td>Tens, ones and tenths</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(regrouping throughout)</td>
<td></td>
</tr>
</tbody>
</table>

NOTE: HARMLESS ZEROS are those which occur usually in the subtrahend and do not necessitate regrouping.

### Decimal Subtraction Algorithms

<table>
<thead>
<tr>
<th></th>
<th>Example</th>
<th>Ref</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st</td>
<td>Tenths (no regrouping)</td>
<td>Fig 4</td>
</tr>
<tr>
<td>2nd</td>
<td>Ones and tenths</td>
<td></td>
</tr>
<tr>
<td></td>
<td>- no regrouping</td>
<td>0.9</td>
</tr>
<tr>
<td></td>
<td>- regrouping in tenths</td>
<td>- 0.5</td>
</tr>
<tr>
<td>3rd</td>
<td>Tens, ones and tenths</td>
<td></td>
</tr>
<tr>
<td></td>
<td>- regrouping in the ones</td>
<td>2.6</td>
</tr>
<tr>
<td></td>
<td>- regrouping throughout</td>
<td>- 1.3</td>
</tr>
<tr>
<td></td>
<td>- zeros and regrouping throughout</td>
<td>7.3</td>
</tr>
</tbody>
</table>

NOTE:

Every attempt should be made to ensure that the situations which involve these written calculations are practical and meaningful to students.

Students should use estimation and approximation before written computations with whole numbers and decimals. The same habit should apply with calculator usage because students may make keying errors. Therefore approximation strategies must be discussed and reinforced regularly; this has the added bonus of providing opportunities for mental calculation practice.
<table>
<thead>
<tr>
<th>Fractions</th>
<th>Ref.</th>
<th>Sourcebook</th>
<th>School Use</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Year 5 Scope and Sequence</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>1st</strong></td>
<td>Revise Year 4 work involving modelling, reading, recording and calculating with tenths.</td>
<td>Part 1</td>
<td>Year 5 pp. 95-97, 101</td>
</tr>
<tr>
<td><strong>2nd</strong></td>
<td>Multiplying with tenths.</td>
<td>Fig 5</td>
<td>p. 96</td>
</tr>
<tr>
<td><strong>3rd</strong></td>
<td>Introduce hundredths through:</td>
<td>3f</td>
<td>p. 97</td>
</tr>
<tr>
<td></td>
<td>• modelling</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>• writing as 13.26</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>• renaming</td>
<td>pp. 97-98</td>
<td></td>
</tr>
<tr>
<td></td>
<td>• count</td>
<td>p. 98</td>
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<tr>
<td><strong>4th</strong></td>
<td>Explore equivalence relationships between:</td>
<td>3e</td>
<td>pp. 98-100</td>
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<tr>
<td></td>
<td>• common fractions</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>• decimal fractions</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>• common and decimal fractions</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>5th</strong></td>
<td>Commence work on the Scope and Sequences for the addition and subtraction algorithms involving hundredths. Also commence work on the Scope and Sequence for multiplication algorithms. Limit work to ones and tenths by single digit whole number multipliers.</td>
<td>Scope &amp; Sequence</td>
<td></td>
</tr>
<tr>
<td><strong>6th</strong></td>
<td>Relate common fractions and decimal fractions to measures:</td>
<td>3e</td>
<td>p. 99</td>
</tr>
<tr>
<td></td>
<td>• quarters to parts of a metre</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>• halves to parts of a metre</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>• three quarters to parts of a metre</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>• quarters to parts of a kilogram and litre</td>
<td>3e</td>
<td>p. 99</td>
</tr>
<tr>
<td></td>
<td>• halves to parts of a kilogram and litre</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>• three-quarters to parts of a kilogram and litre.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
### Year 5 - Scope and Sequence

**Year 5 Sourcebook pp. 100-109**

<table>
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<th>Decimal Fractions Addition Algorithms</th>
<th>Example</th>
<th>Ref</th>
<th>School Use</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st Revise addition involving whole numbers &amp; tenths</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2nd Ones, tenths and hundredths with no regrouping</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>• then with some empty houses</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3rd Ones, tenths and hundredths with regrouping in the hundredths</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>• then with some empty houses</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4th Ones, tenths and hundredths with regrouping in the tenths and hundredths</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>• then with some empty houses</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5th Addition algorithms with larger numbers involving tenths and hundredths with and without regrouping as they arise in practical situations.</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Decimal Fractions Subtraction Algorithms</th>
<th>Example</th>
<th>Ref</th>
<th>School Use</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st Revise subtraction involving whole numbers &amp; tenths</td>
<td></td>
<td></td>
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<tr>
<td><strong>Ones, Tenths and Hundredths</strong></td>
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<tr>
<td>2nd Ones, tenths and hundredths with no regrouping</td>
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<tr>
<td>• then with harmless zeros</td>
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<tr>
<td>• then with three digit take one or two digit</td>
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<tr>
<td>3rd Ones, tenths and hundredths with regrouping in the hundredths</td>
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<tr>
<td>• then with harmless zeros</td>
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<tr>
<td>• then with three digit take one or two digit</td>
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<tr>
<td>4th Ones, tenths and hundredths with regrouping in the tenths and hundredths</td>
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<tr>
<td>• then with harmless zeros</td>
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<tr>
<td>• then three digit take two digit</td>
<td></td>
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<tr>
<td>5th Ones, tenths and hundredths with regrouping caused by zeros</td>
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<tr>
<td>• then with three digit take one or two digit</td>
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</tr>
<tr>
<td>6th Continue work on subtraction algorithms with larger numbers involving tenths, hundredths, zeros with or without regrouping as they arise practical situations</td>
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</tbody>
</table>

**Note:** The language pattern used to develop the algorithm with whole numbers is maintained with algorithms involving decimal fractions. Knowledge of the inverse relationship between addition and subtraction should contribute to students' understanding of addition and subtraction algorithms. The use of estimation to check the reasonableness of answers must be encouraged at all times. The use of the inverse operation and calculators should be encouraged to check the accuracy of answers.

*Fractons - Chapter 11:5*
Year 5 - Scope and Sequence

Only single digit whole numbers should be used as multipliers and the numbers being multiplied should focus on ones and tenths. Many values outside these limits can be changed into whole numbers for calculating. For example, $5.65 may be renamed as 565 cents. Calculators are useful for exploring multiplication.

<table>
<thead>
<tr>
<th>Year 5 Sourcebook pp. 104-106</th>
<th>Ref</th>
<th>School Use</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Fraction Multiplication Algorithms</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1st Consolidate numeration well enough so that students understand that 0.4 x 6 is 24 tenths or 2.4</td>
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</tr>
<tr>
<td>2nd Tenths by a single digit whole number multiplier, no regrouping</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3rd Ones and tenths by a single digit multiplier, no regrouping</td>
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<td></td>
</tr>
<tr>
<td>4th Tenths by a single digit whole number multiplier, with regrouping.</td>
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<td></td>
</tr>
<tr>
<td><strong>Ones and tenths with regrouping</strong></td>
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<td></td>
</tr>
<tr>
<td>5th Ones and tenths by a whole single digit multiplier with regrouping in the tenths</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6th Ones and tenths by a single digit multiplier with regrouping in the ones</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7th Ones and tenths by a single digit multiplier with regrouping in the ones.</td>
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</tbody>
</table>

Year 5 Scope and Sequence - Division with Decimal Fractions

Division with decimals is introduced in Year 6. When division is involved in Year 5, have students rename the values so they are operating with whole numbers - for example, 3.25 m may be renamed as 325 cm - or use calculators.

"$5.65 may be renamed as 565 cents."

Fractions - Chapter 11 : 6
### Fractions

<table>
<thead>
<tr>
<th>Week</th>
<th>Task</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st</td>
<td>Revise Year 4 work</td>
<td>Part 1</td>
</tr>
<tr>
<td>2nd</td>
<td>Introduce the concept of percentage and continue work throughout to link with Step 9</td>
<td></td>
</tr>
<tr>
<td>3rd</td>
<td>Models to represent fractions:</td>
<td></td>
</tr>
<tr>
<td></td>
<td>• consolidate fraction models based on area</td>
<td></td>
</tr>
<tr>
<td></td>
<td>• introduce: set models; length models</td>
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</tr>
<tr>
<td>4th</td>
<td>Consolidate equivalence relationships with common fractions (omit those which are uncommon such as 11ths and 13ths)</td>
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</tr>
<tr>
<td></td>
<td>Introduce equivalence relationships with percentages</td>
<td>Chap</td>
</tr>
<tr>
<td>5th</td>
<td>Revise tenths and hundredths</td>
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<tr>
<td></td>
<td>Link decimals to percentage</td>
<td>Chap 14</td>
</tr>
<tr>
<td>6th</td>
<td>Introduce thousandths using:</td>
<td>Fig 10</td>
</tr>
<tr>
<td></td>
<td>• symmetry of names</td>
<td></td>
</tr>
<tr>
<td></td>
<td>• models</td>
<td>3f</td>
</tr>
<tr>
<td></td>
<td>• place value charts and slides</td>
<td></td>
</tr>
<tr>
<td></td>
<td>• calculators</td>
<td></td>
</tr>
<tr>
<td></td>
<td>• number expanders</td>
<td></td>
</tr>
<tr>
<td></td>
<td>• games</td>
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</tr>
<tr>
<td></td>
<td>Apply thousandths to units of measure such as:</td>
<td>3e</td>
</tr>
<tr>
<td></td>
<td>• length (millimetres and metre)</td>
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<tr>
<td></td>
<td>• mass (grams and kilograms)</td>
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</tr>
<tr>
<td></td>
<td>• volume (millilitres and litres)</td>
<td></td>
</tr>
<tr>
<td>7th</td>
<td>Ensure students are fluent with addition and subtraction algorithms involving tenths and hundredths with regrouping</td>
<td>Year 5 Scope &amp; Sequence</td>
</tr>
<tr>
<td></td>
<td>Revise Year 5 multiplication algorithms which were restricted to ones and tens with a single digit whole number as multiplier</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Commence work on the Year 6 Scope &amp; Sequence for addition, subtraction and multiplication algorithms with decimal fractions to thousandths</td>
<td>Scope &amp; Sequence</td>
</tr>
<tr>
<td></td>
<td>Introduce division with decimals and</td>
<td>Scope &amp;</td>
</tr>
<tr>
<td></td>
<td>Link common fractions and decimal fractions using</td>
<td>Sequence</td>
</tr>
<tr>
<td></td>
<td>the quotient aspect of common fractions</td>
<td>Fig 11</td>
</tr>
<tr>
<td>8th</td>
<td>Continue work with percentages using:</td>
<td></td>
</tr>
<tr>
<td></td>
<td>• estimation</td>
<td></td>
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<tr>
<td></td>
<td>• rectangles (including a square) divided into a hundred equal parts</td>
<td></td>
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<tr>
<td></td>
<td>• a percentage finder</td>
<td></td>
</tr>
<tr>
<td></td>
<td>• discussion throughout</td>
<td></td>
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<tr>
<td>Year 6 Scope and Sequence</td>
<td>Example</td>
<td>Ref</td>
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<tr>
<td>---------------------------</td>
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<tr>
<td>Year 6 Sourcebook pp. 64-66</td>
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</tr>
</tbody>
</table>

**Decimal Fraction Subtraction Algorithms**

1st  Revise subtraction involving whole numbers, tenths and hundredths

2nd  Ones, tenths, hundredths and thousandths with no regrouping
      • then with harmless zeros
      • then with less digits

3rd  Ones, tenths, hundredths and thousandths with regrouping in the thousandths
      • then with harmless zeros
      • then with less digits

4th  Ones, tenths, hundredths and thousandths with regrouping in the hundredths
      • then with harmless zeros
      • then with less digits

5th  Ones, tenths, hundredths and thousandths with regrouping throughout
      • then with harmless zeros
      • then with less digits

6th  Ones, tenths, hundredths and thousands with regrouping throughout
      • then with harmless zeros
      • then with less digits

7th  Ones, tenths and hundredths with regrouping caused from zeros
      • then with less digits

8th  Continue work on subtraction algorithms with larger numbers involving tenths, hundredths, thousandths, zeros with or without regrouping as they arise in practical situations.
<table>
<thead>
<tr>
<th>Year 6 Scope and Sequence</th>
<th>Example</th>
<th>Ref</th>
<th>School Use</th>
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</thead>
<tbody>
<tr>
<td>Year 6 Sourcebook pp. 64-66</td>
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</table>

**Decimal Fraction Addition Algorithms**

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<tbody>
<tr>
<td>1st</td>
<td>Revise addition involving whole numbers, tenths and hundredths with regrouping throughout</td>
<td></td>
<td>Part 1-4</td>
</tr>
<tr>
<td>2nd</td>
<td>Ones, tenths, hundredths and thousandths with no regrouping</td>
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<tr>
<td></td>
<td>• then with some empty houses</td>
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<tr>
<td>3rd</td>
<td>Ones, tenths, hundredths and thousandths with regrouping in the thousandths</td>
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<td></td>
<td>• then with some empty houses</td>
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<tr>
<td>4th</td>
<td>Ones, tenths, hundredths and thousandths with regrouping in the hundredths</td>
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<td></td>
<td>• then with some empty houses</td>
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<tr>
<td>5th</td>
<td>Ones, tenths, hundredths and thousandths with regrouping in the tenths</td>
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<td></td>
<td>• then with some empty houses</td>
<td></td>
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<tr>
<td>6th</td>
<td>Ones, tenths, hundredths and thousands with regrouping throughout</td>
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<td></td>
<td>• then with some empty houses</td>
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<tr>
<td>7th</td>
<td>Addition algorithms with larger numbers involving tenths, hundredths, thousandths, with or without regrouping as they arise in practical situations.</td>
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<tr>
<td>Year 6 - Scope and Sequence</td>
<td>Example</td>
<td>Ref</td>
<td>School Use</td>
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<tr>
<td>Decimal Fractions Multiplication Algorithms</td>
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<tr>
<td>1st</td>
<td>Revise multiplication of ones and hundredths by single digit whole numbers with regrouping</td>
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<tr>
<td>Ones, Tens and Hundredths</td>
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<tr>
<td>2nd</td>
<td>Ones, tenths and hundredths by single digit whole numbers, no regrouping</td>
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<td></td>
<td>• then with zeros</td>
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<tr>
<td>3rd</td>
<td>Ones, tenths and hundredths by single digit whole numbers with regrouping in the hundredths</td>
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<tr>
<td></td>
<td>• then with zeros</td>
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<tr>
<td>4th</td>
<td>Increase the size of the whole number in the multiplicand.</td>
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<tr>
<td>Ones, Tens, Hundreds and Thousandths</td>
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<tr>
<td>5th</td>
<td>Ones, tenths, hundredths and thousandths with no regrouping</td>
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<td>• then with zeros</td>
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<td>6th</td>
<td>Ones, tenths, hundredths and thousandths with regrouping in the thousandths</td>
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<td></td>
<td>• then with zeros</td>
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<td>7th</td>
<td>Ones, tenths, hundredths and thousandths with regrouping in the hundredths</td>
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<td></td>
<td>• then with zeros</td>
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<td>8th</td>
<td>Ones, tenths, hundredths and thousandths with regrouping in the tenths</td>
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<td></td>
<td>• then with zeros</td>
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<tr>
<td>9th</td>
<td>Ones, tenths, hundredths and thousandths with regrouping throughout</td>
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<td></td>
<td>• then with zeros</td>
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<tr>
<td>Multiplying by Decimal Fractions</td>
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<tr>
<td>10th</td>
<td>Multiply:</td>
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<tr>
<td></td>
<td>• single digit whole number by tenths</td>
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<td>• two digit whole number by tenths</td>
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<tr>
<td>11th</td>
<td>Multiply:</td>
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<td></td>
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<tr>
<td></td>
<td>• single digit whole number by hundredths</td>
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<td></td>
<td>• two digit whole number by hundredths</td>
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<tr>
<td></td>
<td>• three digit whole number by hundredths</td>
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<tr>
<td>Multiply Decimal Numbers by Decimal Fractions</td>
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<tr>
<td>12th</td>
<td>Multiply:</td>
<td></td>
<td></td>
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<tr>
<td></td>
<td>• tenths by tenths</td>
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<td></td>
</tr>
<tr>
<td></td>
<td>• ones and tenths by tenths</td>
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<td></td>
<td>• hundredths by tenths</td>
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<tr>
<td></td>
<td>• tenths and hundredths by tenths</td>
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<td></td>
</tr>
<tr>
<td></td>
<td>• ones, tenths and hundredths by tenths</td>
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</tbody>
</table>
### Decimal Fractions Division Algorithms

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</thead>
<tbody>
<tr>
<td><strong>1st</strong></td>
<td>Relate division to multiplication using small numbers</td>
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<tr>
<td><strong>2nd</strong></td>
<td>Division of ones and tenths by a single digit whole number with:</td>
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<td></td>
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<tr>
<td></td>
<td>• no regrouping</td>
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<td></td>
<td>• regrouping in the ones</td>
<td></td>
<td></td>
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<tr>
<td></td>
<td>• zeros in the dividend</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>3rd</strong></td>
<td>Division of ones, tenths and hundredths by a single digit divisor with:</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>• no regrouping</td>
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</tr>
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<td></td>
<td>• regrouping in the ones</td>
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<td>• regrouping in the tenths</td>
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<td></td>
<td>• regrouping in the ones and tenths</td>
<td></td>
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<tr>
<td></td>
<td>• zeros in the dividend</td>
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<tr>
<td><strong>4th</strong></td>
<td>Division of ones, tenths, hundredths and thousandths by a single digit divisor with:</td>
<td></td>
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<tr>
<td></td>
<td>• no regrouping</td>
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<td>• regrouping in the ones</td>
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<td>• regrouping in the tenths</td>
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<td>• regrouping in the ones and tenths</td>
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<td>• regrouping in the hundredths</td>
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<td></td>
<td>• regrouping throughout</td>
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<td></td>
<td>• zeros in the dividend</td>
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</table>

*Fig 11*
### Year 7 Scope and Sequence

Much of Year 7 fraction work involves consolidation of previous work with areas of extension and with wider applications.

<table>
<thead>
<tr>
<th>Year 7 Sourcebook pp. 37-62</th>
<th>Ref</th>
<th>Sourcebook</th>
<th>School Use</th>
</tr>
</thead>
</table>
| 1st | Consolidate decimal fraction place value work to thousandths using:  
- symmetry of names  
- models  
- place value charts and slides  
- calculators  
- number expanders  
- games  
- M.A.B.  
- metric measures | 3f |            |            |
| 2nd | Consolidate Year 6 Scope and Sequences of algorithms for addition, subtraction, multiplication and division involving decimal fractions to thousandths | Yr 6 Scope & Seq |            |            |
| 3rd | Consolidate common fraction work using the:  
- area model  
- array model  
- length model  
Introduce the terms denominator, numerator and vinculum (optional) | 3f |            |            |
| 4th | Consolidate work on equivalence of:  
- common fractions to thousandths  
- decimal fractions to thousandths  
- common fractions and decimal fractions with a denominator of 2 or 5 or factors of 2 and or 5 i.e. those which have a finite number of decimal places | 3f |            |            |
| 5th | Consolidate work on equivalence between common fractions, decimal fractions and percentage  
Introduce percentages over 100% | Chap |            |            |
| 6th | Introduce algorithms of decimal fractions to include:  
- explorations of patterns, i.e.  
tenths x tenths = hundredths  
hundredths x tenths = thousandths  
tenhs x hundredths = thousandths  
- division as per Year 6 with remainders  
- division with decimal fraction divisors by modification involving powers of ten | Yr 6 S & S Fig11 |            |            |
<table>
<thead>
<tr>
<th>Year</th>
<th>Sourcebook pp. 37-62</th>
</tr>
</thead>
</table>
| 7th  | Introduce calculations of:  
  * simple interest  
  * discounts  
  * credit cards  
  * borrowing money |
| 8th  | Introduce addition and subtraction of common fractions with:  
  * like denominators  
  * with unlike denominators |
| 9th  | Work with fractions beyond thousandths as they arise in practical situations |
| 10th | Introduce work on equivalence relationships of common fractions and decimal fractions with:  
  * a denominator of 2 or 5 or factors of 2 and or 5 i.e. those which have a finite number of decimal places  
  * a denominator which is a combination of the first two i.e. the decimal equivalent of this type has a non-repeating and repeating part |

---

**Introduce calculations of:**  
* simple interest  
* discounts  
* credit cards  
* borrowing money

---

**Fractions - Chapter 11 : 13**
Figure 1 Introducing the common fraction symbol
(Introduce Year 4)

Common fraction representation is new for most students entering Year 4. The other number representations (whole numbers and decimal fractions) are based on a base ten place value system. Even though the numerical representations are different for common and decimal fractions, the models and language are identical for both.

An approach which reinforces the concept of equal partitioning and the selection of a number of those equal parts will be used to introduce the symbols for common fraction format.

The terms numerator and denominator need not be learned or used by the students during these activities. The respective numbers can be referred to (if necessary) as the top number and the bottom number. The vinculum, though it has a different mathematical meaning, can be interpreted for convenience as meaning ‘out of’.

It should be noted that this ‘out of’ interpretation will only be used in the introduction of the common fraction format. It cannot be continued indefinitely because it becomes confusing when used with improper fractions such as: \( \frac{5}{3} \).

- How many equal parts is the whole cut into? (6 equal parts)
- What can we call these parts? (Sixths)
- How many parts are shaded? (2 equal parts)
- How much is shaded? (2 or the 6 equal parts or two-sixths is shaded).

The special form for common fractions can be introduced: $\frac{2}{6}$

\[ \frac{2}{6} \text{ or } \frac{2}{6} \]

Figure 2 Introducing the symbol for mixed numbers
(Introduce Year 4)

When introducing the symbol for mixed numbers the types of questions that should be asked include:

- How many halves are there altogether?
- How many halves are shaded/unshaded?
- How many halves in one, two, then three whole pizzas?

Repeat this procedure using thirds, quarters and other simple fractions. The following types of diagrams could be used.

\[ \frac{7}{3} \text{ is the same as } 2\frac{1}{3} \]

\[ \frac{5}{4} \text{ is the same as } 1\frac{1}{4} \]
Figure 3......Addition of ones and tenths with regrouping in the tenths (Introduce Year 4)

The following example shows the type of language which should be used with the addition algorithm when adding numbers involving tenths. Teachers should keep in mind the sequence of difficulty suggested above when working through examples using the algorithm.

<table>
<thead>
<tr>
<th>Language</th>
<th>Symbolic recording</th>
</tr>
</thead>
<tbody>
<tr>
<td>How many tenths are there? (7 tenths and 5 tenths = 12 tenths). Are there enough tenths to make a one? (Yes, one and two tenths). Record the one in the ones place.</td>
<td>Ones</td>
</tr>
<tr>
<td>2</td>
<td>7</td>
</tr>
<tr>
<td>+3</td>
<td>5</td>
</tr>
<tr>
<td>Record the tenths in the tenths place. The decimal point shows us where tenths are.</td>
<td>Ones</td>
</tr>
<tr>
<td>1</td>
<td>7</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
</tr>
<tr>
<td>+3</td>
<td>2</td>
</tr>
<tr>
<td>Add the ones. Are there enough to make a ten? (No)</td>
<td>Ones</td>
</tr>
<tr>
<td>1</td>
<td>7</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
</tr>
<tr>
<td>+3</td>
<td>2</td>
</tr>
<tr>
<td>Record the ones in the ones place.</td>
<td>Ones</td>
</tr>
<tr>
<td>1</td>
<td>7</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
</tr>
<tr>
<td>+3</td>
<td>2</td>
</tr>
</tbody>
</table>

Note: Most students should be familiar with regrouping as a result of their whole number experiences. However, those experiencing difficulty understanding this algorithm should practise the operation using cardboard models as shown below.

```
10 cm

1 cm

10 cm

representing 'one'

representing 'one-tenth'
```
Figure 4 .......Introducing the common fraction symbol

(Introduce Year 4)

To be consistent with the development of the algorithm for whole numbers, the decomposition method should be used. The following example shows the type of language which should be used throughout the development of the subtraction algorithm involving tenths. Once again teachers should keep in mind the suggested algorithm involving tenths. Once again teachers should keep in mind the suggested sequence of difficulty when setting examples to be worked through using the algorithm.

<table>
<thead>
<tr>
<th>Language</th>
<th>Symbolic recording</th>
</tr>
</thead>
<tbody>
<tr>
<td>What is subtracted first? (Tenths). How many tenths do we have? (3). Can we take away 6? (No). What must we do? (Trade one for 10 tenths).</td>
<td>Ones</td>
</tr>
<tr>
<td>7</td>
<td>3</td>
</tr>
<tr>
<td>-4</td>
<td>6</td>
</tr>
<tr>
<td>Record the trade. Subtract the tenths.</td>
<td>Ones</td>
</tr>
<tr>
<td>6</td>
<td>13</td>
</tr>
<tr>
<td>7</td>
<td>9</td>
</tr>
<tr>
<td>-4</td>
<td>6</td>
</tr>
<tr>
<td>Record the tenths. (7) The decimal point shows us where tenths are. Subtract the ones.</td>
<td>Ones</td>
</tr>
<tr>
<td>6</td>
<td>13</td>
</tr>
<tr>
<td>7</td>
<td>9</td>
</tr>
<tr>
<td>-4</td>
<td>6</td>
</tr>
<tr>
<td>2</td>
<td>7</td>
</tr>
</tbody>
</table>

The algorithm should be practised using the cardboard model suggested in 3(a) if any students need assistance.

Figure 5 .......Multiplying with tenths

(Introduce Year 5)

Models of fractions can be used initially to help students develop the pattern with multiplying tenths. The language can reinforce tenths as a ‘name’:

3 groups of 4 is 12
3 groups of four-tenths is 12 tenths.

Have students use models to show 3 groups of four-tenths. Ask students if there are enough tenths to make one whole. Use the models to demonstrate.

12 tenths is the same as one whole and two-tenths (\(\frac{12}{10}\) or 1.2)

Reinforce the commutative principle by having students work different examples on the calculator, keying in the numbers in different orders. See 3(f) for the explanation of how to use the multiplication constant function on the calculator to find multiples of fractions.

Fractions - Chapter 11 : 16
**Figure 6** Addition of ones, tenths and hundredths with regrouping in the hundredths *(Introduce Year 5)*

Example: Two pieces of rope measured 3.74 m and 4.18 m. What is the total length?

<table>
<thead>
<tr>
<th>Verbal (Student thinks or says)</th>
<th>Symbolic (Teacher of student writes)</th>
</tr>
</thead>
<tbody>
<tr>
<td>What operation is involved? (Addition)</td>
<td>Ones . Tenths Hths</td>
</tr>
<tr>
<td>What numbers are we adding? (3.74 and 4.18)</td>
<td>1</td>
</tr>
<tr>
<td>What do we add first? (Hundredths)</td>
<td>3 . 7 4</td>
</tr>
<tr>
<td>How many hundredths are there? (4 hundredths and 8 hundredths: 12 hundredths)</td>
<td>+4 . 1 8</td>
</tr>
<tr>
<td>Are there enough hundredths to make a tenth? (Yes, 1 tenth and 2 hundredths)</td>
<td>2</td>
</tr>
<tr>
<td>(Students record the 2 hundredths and the 1 tenth in the appropriate places).</td>
<td></td>
</tr>
<tr>
<td>What is added next? (Tenths)</td>
<td>Ones . Tenths Hths</td>
</tr>
<tr>
<td>How many tenths are there altogether? (1 tenth and 7 tenths and 1 tenth: 9 tenths)</td>
<td>1</td>
</tr>
<tr>
<td>Are there enough tenths to make one? (No)</td>
<td>3 . 7 4</td>
</tr>
<tr>
<td>(Students record the tenths).</td>
<td>+4 . 1 8</td>
</tr>
<tr>
<td>What is added next? (Ones)</td>
<td>9</td>
</tr>
<tr>
<td>How many ones are there? (3 ones and 4 ones: 7 ones)</td>
<td>Ones . Tenths Hths</td>
</tr>
<tr>
<td>(Student record the ones).</td>
<td>1</td>
</tr>
<tr>
<td>What is 3.74 m plus 4.18 m? (7.92 m)</td>
<td>3 . 7 4</td>
</tr>
<tr>
<td></td>
<td>+4 . 1 8</td>
</tr>
<tr>
<td></td>
<td>7 . 9 2</td>
</tr>
</tbody>
</table>

**Figure 7** Subtraction of ones, tenths and hundredths with regrouping in the tenths and hundredths *(Introduce Year 5)*

Example: A 1.27 m length of timber was cut off the end of a board measuring 6.52 m. What length of timber remained?

<table>
<thead>
<tr>
<th>Verbal (Student thinks or says)</th>
<th>Symbolic (Teacher of student writes)</th>
</tr>
</thead>
<tbody>
<tr>
<td>What operation is it? (Subtraction: 6.52 take 1.27)</td>
<td>Ones . Tenths Hths</td>
</tr>
<tr>
<td>What do we take first? (Hundredths)</td>
<td>4 12</td>
</tr>
<tr>
<td>Are there enough hundredths to take 7? (No)</td>
<td>6 . 8 2</td>
</tr>
<tr>
<td>What do we do? (Change 1 tenth into hundredths: 4 tenths and 12 hundredths)</td>
<td>-1 . 2 7</td>
</tr>
<tr>
<td>(Record the trade).</td>
<td>5</td>
</tr>
<tr>
<td>What is 12 hundredths take 7 hundredths? (5 hundredths)</td>
<td>(Record the 5 hundredths).</td>
</tr>
<tr>
<td>What do we take next? (Tenths)</td>
<td>Ones . Tenths Hths</td>
</tr>
<tr>
<td>Are there enough tenths to take 2 tenths? (Yes: 4 tenths take 2 tenths is 2 tenths)</td>
<td>4 12</td>
</tr>
<tr>
<td>(Record the 2 tenths).</td>
<td>6 . 8 2</td>
</tr>
<tr>
<td>What is taken next? (Ones)</td>
<td>-1 . 2 7</td>
</tr>
<tr>
<td>Are there enough ones to take 1 one? (Yes: 6 ones take 1 one is 5 ones)</td>
<td>5</td>
</tr>
<tr>
<td>(Record the 5 ones)</td>
<td>6 . 8 2</td>
</tr>
<tr>
<td>What is 6.52 take 1.27? (5.25)</td>
<td>-1 . 2 7</td>
</tr>
</tbody>
</table>
Figure 8: Patterns (Introduce Year 5)

Before students are introduced to the written algorithm for multiplication in Year 5 they should understand numeration well enough to be able to say that 0.4 x 6 is 24 tenths of 2.4. Refer to Activity 1(b)(iii) for suggestions of how to develop this idea.

Involve students in investigating and verifying patterns involving decimals until they are confident in predicting answers to different examples such as those shown below. Use a calculator to investigate the following:

\[
\begin{array}{cccc}
6 & 0.6 & 6 & 0.6 \\
x \times 8 & x \times 8 & x \times 0.8 & x \times 0.8 \\
48 & 4.8 & 4.8 & 0.48
\end{array}
\]

Use place-value names to reinforce numeration.

Figure 9: Multiplication of ones and tenths by a whole number with regrouping in the tenths (Introduce Year 5)

Example: The caterer bought 3 bags of potatoes with a mass of 2.5 kg each. What is the total mass?

<table>
<thead>
<tr>
<th>Verbal (Student thinks or says)</th>
<th>Symbolic (Teacher or student writes)</th>
</tr>
</thead>
<tbody>
<tr>
<td>What operation is involved? (Multiplication)</td>
<td>Ones . Tenths</td>
</tr>
<tr>
<td>What is to be multiplied? (2 and 5 tenths)</td>
<td>2 . 5</td>
</tr>
<tr>
<td>What are we multiplying by? (3)</td>
<td>x . 3</td>
</tr>
<tr>
<td>What do we multiply first? (Tenths)</td>
<td></td>
</tr>
<tr>
<td>How many tenths are there? (3 lots of 5 tenths or 5 tenths by 3: 15 tenths)</td>
<td>Ones . Tenths</td>
</tr>
<tr>
<td>Are there enough tenths to make one? (Yes: 1 and 5 tenths)</td>
<td>1 . 5</td>
</tr>
<tr>
<td>(Students record the 5 tenths in the tenths place, and the 1 in the ones place).</td>
<td>x . 3</td>
</tr>
<tr>
<td>What do we multiply next? (Ones)</td>
<td>Ones . Tenths</td>
</tr>
<tr>
<td>How many ones are there? (6 ones and an extra one: 7)</td>
<td>1 . 5</td>
</tr>
<tr>
<td>Record the 7 ones)</td>
<td>x . 3</td>
</tr>
<tr>
<td>What is 2.5 x 3? (7 and 5 tenths)</td>
<td>7 . 5</td>
</tr>
</tbody>
</table>

Figure 10: Introducing Thousandths (Introduce Year 6)

If students are confident with and understand the place-value system with tenths and hundredths, thousandths can be introduced to build on that knowledge. Some students will find it difficult to understand decimal fractions to thousandths (and smaller). Models should be used to provide concrete examples to allow students to relate to thousandths.

The metric measures allow thousandths to be modelled in comparison with a whole unit. Blackboards marked with the square-metre hundred board can also be used to show relative sizes. A calculator can be used to count in thousandths and to illustrate place-value patterns. The number expander can be constructed to represent number and to illustrate the various equivalences. The abacus might also be used to represent numbers though students should have a sound understanding before using it.

The symmetrical nature of the place-value names around the 'ones' place is useful to illustrate differences in spelling and pronunciation as well as to locate particular places.
Thousandths are best introduced through the place-value method - for example, 0.763 is the same as:

7 tenths 6 hundredths 3 thousandths
763 thousandths

Using these forms helps with the understanding of place-values associated with written algorithms as they are introduced. Common fractions should not be used to introduce thousandths as very few students will have sufficient understanding of common fractions. Representing the number in previous example as

\[
\frac{763}{1000}
\]

can follow later as understanding develops.

**Figure 11... Division – decimal-fraction divisors**

*Introduce single digit divisors Year 6, two digit divisors Year 7*

Examples that involve decimal-fraction divisors have to be modified before students can use a written algorithm to compute the answers. This change involves:

- multiplying the divisor by a power of ten (10, 100, 1000) to make it a whole number;
- multiplying the dividend by the same power of ten.

This result, which is now in a form that students can handle, is thus equivalent to the original situation. Present some examples with decimal-fraction divisors to students.

- Discuss the difficulty of dividing by a decimal fraction; to do so use the sharing language and try to relate the situations to model.

- Compare previous division examples with the present ones, asking the students to notice the difference. (The divisors are decimal fractions in the present cases).

- Discuss whether the examples can be changed into a form that has whole number divisors which allow the sharing language to be used once again. (Multiply the divisor by an appropriate power of ten).

- Ask students whether the result will be the same now that they are dividing by a larger number.

- Discuss what else should be done to compensate. (The dividend is multiplied by the same power of ten as was the divisor).


Year Level Expectations

Fractions 2

**Year 4**
- Conceptual development of common fractions and decimal fractions to tenths.
- Algorithms for addition and subtraction with decimal fractions developed.

*Year 4 Sourcebook pp. 163-194*

Children entering Year 4 will have little to no background knowledge of fractions. Year 4 is a significant year for fraction work. The first part of year 4 will be concerned with concept development using concrete materials and oral language only. Symbolic representation should only be introduced after students have developed understanding of the fraction as a concept and familiarity with associated language.

**Common fractions**

Work begins with common fractions to tenths. Care must be taken when introducing the fraction names half, third and quarter because these names are not obviously related to the number of equal parts in the whole as fraction names such as sixths, sevenths, etc.

**Decimal fractions**

An understanding of the concept of a fraction is a necessary prerequisite for work with decimal fractions. Once the initial concept is understood, numeration ideas for tenths can be developed in a manner which parallels, and is based on, whole number concept development. These numeration ideas for decimals must be viewed as an extension of the base ten system of numeration. Year 4 decimal fraction work is limited to tenths. Algorithms for addition and subtraction with decimal fractions are developed to include whole number and tenths (refer to Part 1).

**Year 5**

Introduction of hundredths, multiplication algorithms, equivalence and relating fractions to units of measure.

*Year 5 Sourcebook pp. 93-109*

**Introducing hundredths**

Year 4 activities focused on fractions to tenths. Ensure students have a good understanding of tenths before introducing activities which involve hundredths. Initially, encourage students to read numbers so that place value is reinforced. For example, 1.6 can be read as ‘one and six-tenths’ and ‘3.78’ can be read as ‘three and 78 hundredths’. Use the abbreviated language (e.g. ‘three point seven eight’ for 3.78) after understanding of fractions has been developed.

**Equivalence**

Equivalence relationships are explored and include:
- common fractions
- decimal fractions
- common and decimal fractions

**Relating to measures**

Year 5 students are involved in activities which relate to tenths and hundredths to metre, litre and kilogram.

**Operations with decimal fractions**

Students will already be using the addition and subtraction algorithms with decimal fractions. Previous work is built on with the introduction of hundredths.
The multiplication algorithm is introduced in this year's Sourcebook. When division is involved, have students rename the values so they are operating with whole numbers - for example, 3.25 m can be renamed as 325 cm - or use calculators. Calculators are also useful for working with more complicated numbers.

**Years 6**

Introduce thousandths, percentages and division with decimal fractions

*Year 6 Sourcebook pp. 47-74*

In Years 4 and 5, both decimal fractions and common fractions are introduced to students using models and specific language patterns. The emphasis during Year 6 is to consolidate understanding of these concepts.

- **Decimal Fractions**

Decimal fractions are emphasised with thousandths being introduced. Calculators, place-value charts and number expanders are used to explore and reinforce decimal-fraction concepts. In terms of operations, decimal fractions are added and subtracted to thousandths, and multiplied and divided to hundredths. Calculators are used to explore the algorithms and to cope with any computations which the students cannot do mentally or in writing.

- **Operations with decimal fractions**

Practical contexts are provided for operations with decimal fractions to hundredths and thousandths. These contexts include the metric measures, perimeters and areas of plane shapes, and the use of money in budgeting and shopping activities.

- **Percentages**

Percentages and the way they relate to decimal fractions and to monetary transactions are also introduced. At this stage, no written algorithms are used with percentages. The emphasis is on thinking about the concepts and on mental computations. Students are asked to estimate percentages of numbers and quantities using benchmarks such as a half (50 per cent) and a tenth (10 per cent). *Refer to Chapter 14 on Percentage.*

- **Common fractions**

With common fractions, students continue to discriminate between equal and unequal subdivisions of models and the naming of the equal parts. Estimation related to common fractions is continued, with students being required to make decisions about the relative size of parts of shapes. Students are asked, for example, to shade a fraction of a shape or to decide what fraction of another shape has been shaded.

- **Equivalence**

Some activities with equivalence of common fractions are included in the preparation for the addition and subtraction activities to begin in Year 7.

Games are also used to allow application of concepts and processes in atypical situations. These games involve such simple ideas as matching equivalent representations, finding the 'odd' one out or estimating. Individual or team formats are suitable for the activities.

Games offer a valuable opportunity for students to practise or revise fraction concepts. In such a different context, students' misconceptions can often become apparent and be remediated immediately.
Adding and subtraction with common fraction.
Division algorithm with two digit divisors.

Year 7 Sourcebook pp. 35-62

During Years 4, 5 and 6 the teaching of common fractions emphasise that students should recognise and construct appropriate models to represent simple fractions. Associated with these activities have been comparison of common fractions with whole numbers, estimation of the size of common fractions and recognition of the necessity for equal partitioning. Equivalence was introduced as a prerequisite for beginning operations with common fractions. It is important that students understand all these aspects and can demonstrate their understanding. The denominators of fractions have been kept to reasonable limits, omitting those which are uncommon, such as 11ths and 13ths.

- **Decimal fractions beyond thousandths**

Decimal fractions to thousandths were studied in year 6 and now, if practical situations arise, fractions beyond thousandths can be used by students. It would be wise to explore the place names so that students can read calculator displays and round where appropriate. Operations with decimal fractions should include real data whenever it is possible, incorporating money and measurement. Calculation of the area and perimeter of plane shapes also provides suitable contexts.

- **Operations with decimal fractions**

The four operations are used with decimal fractions and percentages are linked with equivalent operations using decimal and common fractions. Many of the decimal operations can be linked with money and measurement. On occasions the computations can be carried out mentally; on others use calculators for speed and accuracy; but sometimes the aim is to check the student's proficiency with written algorithms.

Students introduced to the division algorithm with whole numbers in Year 5 and to examples involving decimal fractions in the dividend in Year 6. In all cases, the divisors were single-digit whole numbers. These procedures should be revised and reinforced along with the estimation activities outlined earlier.

When students demonstrate that they understand and are competent with examples using single digit divisors, they can be introduced to the algorithm for two digit divisors.

- **Adding and subtracting with common fraction**

In Year 7, students are first introduced to addition and subtraction with common fractions. Examples with like denominators will be examined first, so that students develop some of the prerequisite skills for calculating with unlike denominators.

The formal setting out of operations will begin in a vertical form to improve student's understanding and to explain the abstract process of calculating with common fractions. The vertical form has its own kind of place value, remind students of similar procedures with whole-number calculations. With common fractions, there may still be some trading and renaming. Students will not be asked to operate with mixed numbers which can be introduced, if necessary, in later years.
Glossary of Terms

Fractions

The Concept of Decimal Fractions

(3a) Introduce Tenths Yr 4, Hundredths Yr 5, Thousandths Yr 6

An understanding of the concept of a fraction is a necessary prerequisite for work with decimal fractions. Once the initial concept is understood, numeration ideas for tenths can be developed in a manner which parallels, and is based on, whole numbers concept development.

These numeration ideas for decimals must be viewed as an extension of the base ten system of numeration.

The unit or one is the focal reference point for both decimal fractions and for whole numbers. For whole numbers, the ones were grouped to form tens, and the tens were then grouped to form hundreds, and so on. With decimal fractions, the one is partitioned into ten equal parts called tenths. These tenths can be further subdivided to form hundredths.

The place value chart and the calculator will be used to explore the relationships between the places. Other materials such as area models will be used to introduce the decimal concepts and language.

The decimal point only identifies the positions of the ones and tenths places and does not represent a place itself. Once the position of the point is known the ones and tenths places are also immediately known. In other countries, particularly in Europe, a comma is used instead of a point.

Numerator, Denominator and Vinculum (optional)

(3b) Introduce Year 7

The terms numerator and denominator need to be learned or used by the students until Year 7. From Year 4 to Year 6 the respective numbers can be referred to (if necessary) as the 'top number' and the 'bottom number'. The vinculum, though it has a different mathematical meaning, can be interpreted for convenience as meaning 'out of'.

It should be noted that this 'out of' interpretation will only be used in the introduction of the common fraction format. It cannot be continued indefinitely because it becomes confusing when used with improper fractions such as: \( \frac{6}{5} \).

The numerator is the 'top number' of a fraction and tells how many parts of the whole are involved. The denominator is the 'bottom number' and tells how many equal parts in the whole.

Fractions, Improper Fractions and Mixed Numbers

(3c)

A fraction is part of a whole quantity or number. The fraction \( \frac{2}{3} \) means 2 parts out of a total of 3 equal parts.

It is important to stress to the students that a fraction refers to an equal part/s of a whole and not unequal parts. Year 4 activities should enable students to discriminate between equal and unequal partitioning and subsequent naming of equal parts of fractions.
An improper fraction has a numerator which is greater than its denominator and therefore represents more than one whole. Improper fractions can also be written as mixed numbers. Mixed numbers consist of a whole number and a fraction. Using mixed numbers is another way of representing improper fractions.

### Decimal Fractions

A decimal fraction is a fraction which is written using the decimal place value system i.e. 'base ten' system. A decimal point is placed on the line between the one and the tenths. Refer to part 4 of this chapter for the correct reading and recording of decimal fractions.

It is important students identify the symmetry which exists in the 'base ten' system.

#### The symmetry of names

Discuss with students the symmetrical pattern of place names around the ones place. Emphasise the spelling and pronunciation of the place names.

<table>
<thead>
<tr>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>7</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Ones</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Tens</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Hundreds</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Thousands</td>
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<td></td>
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<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Have students extend the pattern in either direction for a few more places to show that numbers can be infinite in relation to whole number places or decimal-fraction places.

### Equivalence

It is important for students to be able to interpret different ways of representing numbers and understand the place-value relationships. When using calculators, for example, zeros on the ends of fractional numbers are automatically deleted from the display.

Students need an understanding of these relationships in order to deal effectively with everyday situations which involve fractions. From Year 4 activities should be used to develop equivalence relationships involving both common fractions and decimal fractions.

Investigate relationships such as:

- common fractions (\(\frac{2}{4}\) and \(\frac{3}{6}\) are the same as \(\frac{1}{2}\))
- decimal fractions (0.6 is the same as 0.60)
- common and decimal fractions (0.2 is the same as \(\frac{2}{10}\), \(\frac{1}{5}\) is the same as 0.25).

Emphasise the language associated with the equivalent fractions and the symbols used. Students will need these understandings in Year 6 for analysing how the symbols relate to each other.

### Relating fractions to measures

'One-quarter', 'one-half' and 'three-quarters' are names commonly used in describing measures relating to length, mass and volume. Students need to be able to relate these terms to the written forms for common fractions and decimal fractions. In order to write these fractions in decimal form, equivalence relationships must be established.
Use mass to develop the following relationships:
- 250 g is one-quarter of a kilogram (1/4 kg, 0.25 kg)
- 500 g is one-half of a kilogram (1/2 kg, 0.5 kg)
- 750 g is three-quarters of a kilogram (3/4 kg, 0.75 kg)

Use length to develop the following relationships:
- 250 mm is one-quarter of a metre (1/4 m, 0.25 m)
- 500 mm is one-half of a metre (1/2 m, 0.5 m)
- 750 mm is three-quarters of a metre (3/4 m, 0.75 m)
- 10 cm is one tenth of a metre (1/10 m, 0.1 m)
- 50 cm is one half of a metre (1/2 m, 0.5 m)

Use volume to develop the following relationships:
- 250 ml is one-quarter of a litre (1/4 L, 0.25 L)
- 500 ml is one-half of a litre (1/2 L, 0.5 L)
- 750 ml is three-quarters of a litre (3/4 L, 0.75 L)

Modelling Fractions (Continue to model from Year 4 to 7)

It is essential that students be given ample and ongoing opportunity to model fractions and decimal fractions. Many of the modelling materials used in whole number work can be modified for fraction work.

Cuisenaire Rods

Use Cuisenaire rods to identify relationships involving halves, thirds, quarters, fifths and tenths (e.g. 1/3 is the same as 2/6)

Abacuses

From Year 5 use pairs of abacuses to make and compare numbers involving decimal fractions. The place values along the abacuses can be changed to suit the size of the numbers concerned as shown following. Using such models helps students identify where the whole numbers are in each model and where the first decimal place (tenths) begins.
Place Value Charts

Extend the place-value chart to include decimals and discuss the symmetry of the place-value headings around the ones place. Encourage students to use these headings when writing numbers.

The place-value chart can be used to display various equivalent forms as they are discussed and modelled.

The illustration shows the common aspects of the equivalent forms - each of the numbers has a '5' in the tenths place. The 'zeros' fill empty places and do not add any more value to the number, and so the three representations are equivalent.

Number Expanders

The number expander is used for decimal fractions in the same way as for whole numbers. The expander should have enough 'folds' to cater for the size of decimal fraction relevant i.e. tenths, hundredths, thousandths.

The number expander provides a very convenient way to view the relationships within the place value system. It concentrates on place names and therefore will not have a decimal point.

If the expanders are covered with clear 'contact' film, students can write their own numbers on them with felt pens, which can later be cleaned off when new numbers have to be written.

With whole numbers like 54, students could write a zero in the tenths place to allow comparison with the other numbers.

Note: Because no decimal point is used, a place number must always be visible so that students have a reference upon which to base their assessment of the value of the number. These place names will generally be ones and tenths together or just tenths (because this is the focal point of the activity). Alternatively, the decimal places might be coloured differently to the whole numbers to indicate to students where the 'ones' are. Whenever students use the expander, a symbolic representation, e.g. 3.8 should always be displayed for comparison and association.

Note: When using the place value chart to interpret numbers, students' attention must be directed to hints given by the number name. For example, 12 tenths indicates that the last digit (2) must be placed in the tenths place.

<table>
<thead>
<tr>
<th>Tens</th>
<th>Ones</th>
<th>Tenths</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>5</td>
<td>4</td>
</tr>
<tr>
<td>3.</td>
<td>ones</td>
<td>5</td>
</tr>
<tr>
<td>3.</td>
<td>ones</td>
<td>5</td>
</tr>
<tr>
<td>3.</td>
<td>4</td>
<td>hundredths</td>
</tr>
</tbody>
</table>

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Models

Use models to identify equivalent fractions:

Four-tenths (0.4) is the same as 40 hundredths (0.40)

Grid Paper

Grid paper can be used to create models resembling the following model comparing 3 tens and 3 tenths.

Have students observe partitioning of a model of a one into tenths, then hundredths. The model might resemble the following diagram.

Use grid paper for students to make their own models of ones, tenths and hundredths and to compare the relative sizes of each.

M.A.B.

The MAB blocks can be used to demonstrate decimal fractions, provided that students can adapt their thinking to assign a unit value (1 whole) to the large block. The smaller blocks then take on values of 1 tenth, 1 hundredth and 1 thousandth.

Once students understand these relationships, numbers can be represented using the blocks - students describe each part of the numbers and record them on a place-value chart.
For example:
- Count out 100 of the small blocks
- Name the number (100 thousandths)
- Record the number on the place-value chart (0.100)
- Trade ten of the small blocks for one of the long blocks
- Continue trading as long as possible
- Name the number (10 hundredths)
- Record the number on the chart (0.10)
- Continue trading for flat blocks, naming the number and recording it
- Compare the three forms (0.100; 0.10; 0.1)
- Repeat for other decimal fractions.

There are three ways to model fractions. These are:
- based on area (Year 4 and 5)
- the set model (Introduce Year 6)
- the length model (Introduce Year 6)

Understanding is enhanced if these models are used in practical activities. The set model and the length models involve division and sharing, so care needs to be taken to include both division and fraction language in activities.

**Area Models**

'Area' models can be created using pieces of paper, material, wood, tiles, plastic, bread or any other substance which can be cut up or subdivided easily and reassembled for observation and discussion of equivalence.

Use the above materials to involve students in the following type of process:
- cut a loaf of bread into halves and name each piece (half);
- reassemble the pieces and discuss what the 2 halves make:
- cut the bread again to make quarters;
- move the pieces slightly to show the quarters;
- discuss the relationship between the half and the 2 quarters;
- discuss the relationship between the whole, 2 halves and 4 quarters.

**Set Models**

The set model is introduced to students in Year 6 as an alternative way to represent common fractions.

Counters, bottle-tops, stickers, tiles, etc can be used in array form to represent fractions. Students need to be comfortable with the fact that if there are 12 counters, each counter represents one-twelfth.

Each counter represents 1 twelfth

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Because the model is difficult for some students to interpret, have students work with practical activities - for example, have the students study sets of counters, and name the fraction of each set which is different from the rest.

\[ \bullet \quad \bullet \quad \bullet \quad \bullet \quad \bullet \quad 3 \text{ counters out of 12 counters are black} \]
\[ \bullet \quad \bullet \quad \bullet \quad \bullet \quad \bullet \quad \frac{3}{12} \text{ 3 twelfths} \]

Have the students undertake the practical activities following:

- A box of 12 stickers is to be shared equally among 3 children.
  - How many will each receive? (4)
  - What fraction will each receive? (one-third, 1/3)
  - Draw a model of the sharing.

\[ \quad \frac{4}{12} \quad \frac{4}{12} \quad \frac{4}{12} \quad \text{one third (1/3)} \]

**Length Models**

- Using pieces of string or wool, have students cut off fractions of the whole length (, , ). Initially, these fractions should be worked out without measuring, so students will 'problem solve' to find ways to determine the required fraction of the length. To find thirds, for example, students may need to work in pairs of small groups and wind the string around two pivots (pins) until three identical lengths are found.

Adjust the distance between the pivots. —

\[ \text{Mark where each third ends.} \]

Measure to check.

Students should be encouraged to demonstrate and explain other methods.

**Modelling Equivalence - using the area model**

To prepare for operations with common fractions, students need to experience and understand equivalence relationships. As this understanding does not come readily, models of fractions will need to be used. Introduction to symbolic representations too early will not help students to understand equivalence relationships. Observe and talk with students as they manipulate the models to gain an indication as to when they no longer need to use concrete examples. The models used to develop and reinforce the concept of a fraction should also be used to illustrate the equivalence of fractions and percentages.

**Using the area model for example.**

Have students investigate equivalence involving division by reversing the procedure used above with a set of tiles as follows:

- arrange 12 tiles into a 4 by 3 array;
- discuss the fact that each tile represents \( \frac{1}{12} \) of the area;

- arrange the tiles so that \( \frac{8}{12} \) can be observed;
- observe that \( \frac{4}{12} \) makes up the remainder of the tiles;

- keeping the separate groups (\( \frac{8}{12}, \frac{4}{12} \)), arrange the tiles into pairs to make sixths;

- discuss the fractions which now appear (\( \frac{4}{6}, \frac{2}{6} \));
- equate the fractions \( \frac{8}{12} \) and \( \frac{4}{6} \) because they involve the same tiles;
• lead the students to observe that both numerator and denominator have been halved;
• reform the original fractions (\(\frac{8}{12}, \frac{4}{12}\));
• arrange the tiles into groups of 4 to form thirds;
• discuss and equate the fractions (\(\frac{8}{12}, \frac{3}{3}\));
• after the division by 4 has been established, relate the 3 equivalent fractions using the models, showing that each of the fractions uses the same number of tiles (\(\frac{8}{12}, \frac{8}{12}, \frac{6}{12}\));
• repeat the process with other fractions, such as \(\frac{2}{12}, \frac{8}{10}, \frac{6}{9}\), etc. Paper-folding models may be used easily with fractions having denominators such as 2, 4 and 8.

**Calculating with common fractions (Introduce Year 7)**

For activities and explanations on the addition and subtraction of common fractions with like and unlike denominators, refer to pages 57 - 59 of the Year 7 Sourcebook.

**Approximating and the algorithms involving decimal fractions**

It is important that students gain enough confidence with approximating to be able to make quick and reasonably accurate estimates of answers to addition and subtraction problems. When estimations are being practised, the calculator should be used to confirm the quality of the estimate. Using the calculator means that feedback is immediate and that many estimates can be attempted in a short time.

To approximate numbers which have whole numbers and tenths, the students most focus on the tenths digit (if they are rounding to ones). The two most important rules at this stage are:

(i) If the tenths digit is 5 or more, students round to the next whole number, e.g. 4.6 is round to 5. Revise the 'point of no return'.

(ii) If the tenths digit is 4 or less, then the number is rounded to the whole number already there, e.g. 3.4 is rounded to 3.

These rounding 'rules' often need to be reinforced visually, e.g. teacher-prepared number lines where the 'distances' to the whole numbers can be compared.

Students could use the rounding strategies above or they could use a 'front-end' or other strategies.

For example, in

\[2.3 + 8.8 + 3.7\]

it is possible to obtain a rough estimate of 13 by focussing on the 'front-end', in this case the ones. This estimate can be refined by considering the tenths to obtain a more accurate estimate of 15.

Similarly, for

\[1.4 + 12.6\]

an efficient strategy would draw on the 'compatibility' of the numbers under consideration, i.e. the tenths add up to make one whole.

Some of the best learning takes place when students have to explain and justify their methods. Ask students to explain the strategy or strategies they used and how they arrived at their estimate.
Addition and Subtraction

Students need a quick and easy method of estimating in order to obtain some idea of how large an answer should be. The procedure used will depend on the nature of the example. Both rounding and front-end strategies are useful for addition and subtraction.

**Rounding**
- Round 2.74 to 3, and 1.08 to 1
- 3 and 1 are 4

**Front-end**
- (Add up the numbers in the left column)
- 2 and 1 are 3
- (A quick look at the next column shows that the estimate would be more than 3, about 4).

**Rounding**
- Round 9.47 to 9 and 6.73 to 7.
- 9 take 7 is 2.

**Front-end**
- 9 take 6 is 3
- (A glance at the next column shows that the answer would be less than 3 because there are not enough tenths to take 7.)

In an example such as 0.14 + 0.9 + 0.52, where there are no ones, students might use a front-end approach by calculating the tenths (1 tenth and 5 tenths: 6 tenths), then glancing at the next column of numbers to identify that the answer would be more than 6 tenths. Encourage students to use estimation to predict answers and to check the reasonableness of answers that have been calculated by the written algorithm or calculators.

Multiplication

Rounding as opposed to front-end usually produces close estimates when multiplication is involved. Have students focus on the whole number part of the calculation when estimating, unless the number to be multiplied is less than one.

**About 5**
- 4.8
- x 6
- 30
- (Real answer is less than 30, because 4.8 is less than 5.)

**About $6**
- $6.23
- x 8
- 48
- (Real answer is more than $48 because $6.23 is more than $6.)

Estimation should be practised as students calculate with both the written algorithm and calculators.

Division

Students need to consider the results of situations that involve division by a decimal fraction. The estimates in situations like this should be restricted to judging whether a quotient will be larger than or smaller than the original dividend.

For example: 0.6 \( \overline{11.42} \) Will the result be larger than 11.42? smaller than 11.42?

Approximating with money

Because students are so familiar with dollars and cents, some exercises can be given involving approximating amounts of money to the nearest dollar.

The fact that there are two ‘decimal places’ should not be a problem because students basically interpret those places as whole cents. ‘Half way’ or ‘point of no return’ is therefore established as 50 cents.
Fraction work should emphasise the fact that the whole has been divided into equal parts. Teachers should stress the total number of equal parts and the number of those parts that are being referred to, e.g.

There are 5 equal parts.
4 parts are shaded.
4 out 5 equal parts are shaded.
Four-fifths is shaded.

Emphasis must also be placed on the idea that the whole, or one, is ‘five out of five equal parts of five-fifths’, ‘eight out of eight equal parts or eight-eights’, and soon.
Care must be taken when introducing the fraction names ‘half’, ‘third’, and ‘quarter’, because these names are not obviously related to the number of equal parts in the whole.
Some recommended questions are:

How many equal parts make up the whole? (Seven)
What could you call these parts? (Sevenths)
How many sevenths make up the whole? (Seven)
How much is shaded? (3 sevenths - encourage students to use the appropriate name; at this stage they could also identify the number of parts which are not shaded).

Since symbolic representation should only be introduced after students have developed some understanding of the fraction as a concept and familiarity with associated language, the following format could be used to help show where the number names are placed.

3 parts out of 7 equal parts
- three-sevenths are shaded

4 parts out of 4 equal parts
- four-quarters or one whole is shaded

The terms numerator and denominator need not be learned or used by the students during these activities. The respective numbers can be referred to (if necessary) as the top number and the bottom number. The vinculum, though it has a different mathematical meaning, can be interpreted for convenience as meaning 'out of'.
It should be noted that this ‘out of’ interpretation will only be used in the introduction of the common fraction format. It cannot be continued indefinitely because it becomes confusing when used with improper fractions such as: $\frac{5}{3}$
Mixed Numbers

Two and one-fifth might be depicted and read as follows:

\[\begin{array}{c}
\text{1 whole} \\
\text{1 whole} \\
\text{1 fifth}
\end{array}\]

2 wholes shaded and one-fifth shaded.
Two and one-fifth shaded or 11 fifths shaded.

The following table can be started for fractions and wholes, and the students asked to continue the pattern:

<table>
<thead>
<tr>
<th>Whole</th>
<th>Parts</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 whole</td>
<td>5 fifths</td>
</tr>
<tr>
<td>2 wholes</td>
<td>10 fifths</td>
</tr>
<tr>
<td>_ wholes</td>
<td>_ fifths</td>
</tr>
</tbody>
</table>

Provide students with the opportunity to present in the following way:

\[\begin{array}{c}
\text{thirds} \\
\text{or} \quad \text{and} \quad \text{thirds} \\
\text{or} \quad \text{thirds}
\end{array}\]

Decimal Fractions

The decimal point only identifies the positions of the ones and tenths places and does not represent a place itself. Once the position of the point is known the ones and tenths places are also immediately known. In other countries, particularly in Europe, a comma is used instead of a point.

It is becoming quite common to place the point on the line (i.e. a full stop) because of the increased use of typewriters and computers printers. This is the recommended format for students to use. When a decimal fraction is less than one, a ‘zero’ must be placed in the units place, e.g. 0.03, 0.5.

Initially encourage students to read numbers so that place value is reinforced such as 0.7 should be read as seven-tenths, but in later years may be referred to as point seven.

1.6 may be read as ‘one and six-tenths’ and ‘3.78’ may be read as ‘three and 78 hundredths’. Use the abbreviated language (e.g. ‘three point seven eight’ for 3.78) after understanding of fractions has been developed.

The following format will help develop consistency within school.

<table>
<thead>
<tr>
<th>Year 4</th>
<th>Year 5</th>
<th>Year 6 &amp; 7</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.7 – seven tenths</td>
<td>0.7 – seven tenths</td>
<td>3.789 – three and seven hundred and eighty-nine thousandths</td>
</tr>
<tr>
<td>1.6 – one and six tenths</td>
<td>1.6 – one and six tenths</td>
<td>.07 – point seven</td>
</tr>
<tr>
<td>3.78 – three and seventy-eight hundredths</td>
<td>3.78 – three and seven hundred eight</td>
<td>1.7 – one point seven</td>
</tr>
<tr>
<td>3.08 – three and eight hundredths</td>
<td>3.789 – three point seven eight</td>
<td>3.78 – three point seven eight nine</td>
</tr>
</tbody>
</table>

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<table>
<thead>
<tr>
<th>Year</th>
<th>Language</th>
<th>School use</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>whole</td>
<td></td>
</tr>
<tr>
<td></td>
<td>part</td>
<td></td>
</tr>
<tr>
<td></td>
<td>equal parts</td>
<td></td>
</tr>
<tr>
<td></td>
<td>unequal parts</td>
<td></td>
</tr>
<tr>
<td></td>
<td>fraction</td>
<td></td>
</tr>
<tr>
<td></td>
<td>halves</td>
<td></td>
</tr>
<tr>
<td></td>
<td>thirds</td>
<td></td>
</tr>
<tr>
<td></td>
<td>quarters</td>
<td></td>
</tr>
<tr>
<td></td>
<td>fifths</td>
<td></td>
</tr>
<tr>
<td></td>
<td>sixths</td>
<td></td>
</tr>
<tr>
<td></td>
<td>sevenths</td>
<td></td>
</tr>
<tr>
<td></td>
<td>eights</td>
<td></td>
</tr>
<tr>
<td></td>
<td>ninths</td>
<td></td>
</tr>
<tr>
<td></td>
<td>tenths</td>
<td></td>
</tr>
<tr>
<td></td>
<td>mixed fraction</td>
<td></td>
</tr>
<tr>
<td></td>
<td>decimal fraction</td>
<td></td>
</tr>
<tr>
<td></td>
<td>mixed decimal fraction</td>
<td></td>
</tr>
<tr>
<td></td>
<td>top number</td>
<td></td>
</tr>
<tr>
<td></td>
<td>bottom number</td>
<td></td>
</tr>
<tr>
<td></td>
<td>'out of'</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>hundredths</td>
<td></td>
</tr>
<tr>
<td></td>
<td>fractions beyond tenths</td>
<td></td>
</tr>
<tr>
<td></td>
<td>equivalent fraction</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>thousandths</td>
<td></td>
</tr>
<tr>
<td></td>
<td>percentage</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>numerator</td>
<td></td>
</tr>
<tr>
<td></td>
<td>denominator</td>
<td></td>
</tr>
<tr>
<td></td>
<td>vinculum (optional)</td>
<td></td>
</tr>
</tbody>
</table>
Ratio & Proportion

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<tr>
<th>Year 7</th>
<th>Content</th>
<th>School Use</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st</td>
<td>Concept development.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>• Use concrete materials to compare two whole number quantities.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>• Compare measures</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Use the word 'to' and not the symbol in both oral and written forms.</td>
<td></td>
</tr>
<tr>
<td>2nd</td>
<td>Introduce the symbol for ratio.</td>
<td></td>
</tr>
<tr>
<td>3rd</td>
<td>Introduce the idea of equivalence and proportion.</td>
<td></td>
</tr>
<tr>
<td>4th</td>
<td>Model and use equivalent ratios.</td>
<td></td>
</tr>
<tr>
<td>5th</td>
<td>Generate equivalent ratios using multiplication.</td>
<td></td>
</tr>
<tr>
<td>6th</td>
<td>Express ratios in simpler forms using division.</td>
<td></td>
</tr>
</tbody>
</table>
Ratio and proportion work does not begin until Year 7.

In Year 7 students are introduced to the concept of ratio using models and appropriate language. Numbers and quantities are first compared using the work 'to' in both oral and written forms before the ':' symbol is introduced. This step eases students into acceptable interpretation of the ratio symbol in later exercises.

After the concept of equivalence and proportion have been introduced, equivalent ratios are examined and generated: firstly by multiplication followed by division methods to generate simpler forms.
Ratios involve the comparison of numbers or quantities. Although ratios are used extensively throughout mathematics, many students do not understand the concept and, therefore, have difficulty applying ratios in practical situations. Research results have illustrated that inappropriate strategies, particularly addition, are used by students in ratio situations. (See K.M. Hart [Ed.], Children’s Understanding of Mathematics: 11-16, John Murray Ltd, London: 1981.)

While ratio and fraction concepts overlap in some respects, the following model illustrates why they should not be interchanged:

Usually illustrates the ratio 4 : 5
Usually illustrates the common fraction $\frac{4}{9}$

The confusion arises basically because ratios most often compare a part of the model to another part (4 shaded parts to 5 unshaded parts), and fractions compare a part to a whole (4 shaded parts out of 9 parts altogether).

As with fractions, the models used to illustrate ratios are very important in helping students understand the situation. The set model (shown above) seems to be the best one for introducing the concept, with the length model being used in follow-up activities.
Equivalent ratios and proportion are based on multiplication and division.

Proportions involve relationships between ratios where an increase in one also results in an increase in the other. For example, if two numbers are in the ratio 3 : 2 and the first number doubles, then the second one also doubles to maintain the same proportion.

Therefore, 3 : 2 is in the same proportion as 6 : 4. Let students see this by using groups of 3 boys and 2 girls. If the number of boys doubles, the number of girls must also double to keep the same proportion.

Counters or other materials can be used to demonstrate this idea with different ratios. Emphasize the use of multiplication to generate these ratios. The original ratios can be retrieved by using division.

Addition and subtraction strategies should not be applied with equivalent ratios. Students should experiment with materials to illustrate these facts.

For example:

- ask students to model the ratio 3 : 1 using red and blue counters;
- there are 3 red counters for every 1 blue counter;
- add 3 more red counters and 3 more blue counters to the piles;
- determine the new ratio of the colours (it is now 6 : 4);
- discuss why this new ratio is not in the same proportion as the first;
- decide that addition cannot be used to generate equivalent ratios.

The new ratio cannot be in the same proportion as the first ratio for a number of reasons, but it can easily be seen that the ‘3 reds for every 1 blue’ relationship has not been maintained. Therefore the ratios are not equivalent.

Allow students to experiment with subtraction as well. They will soon notice that this operation is not reliable for generating equivalent ratios. Again, emphasize the use of multiplication and division for determining equivalent ratios.
Ratios involve the comparison of numbers or quantities. Although they are often written in common fraction form, it is strongly recommended that only the ‘:’ form be used in Year 7.

While ratio and fraction concepts overlap in some respects, the following model illustrates why they should not be interchanged:

Usually illustrates the ratio 4 : 5

Usually illustrations the common fraction \[
\frac{4}{9^*}
\]

The confusion arises basically because ratios most often compare a part of the model to another part (4 shaded parts to 5 unshaded parts), and fractions compare a part to a whole (4 shaded parts out of 9 parts altogether).

As with fractions, the models used to illustrate ratios are very important in helping students understand the situation. The set model (shown above) seems to be the best one for introducing the concept, with the length model being used in follow-up activities.

Before introducing students to the ‘:’ symbol, numbers and quantities can be compared using the word ‘to’ in both oral and written forms. This step will ease students into an acceptable interpretation of the ratio symbol in later exercises.

After the preliminary investigations, guiding students to use the ratio symbol should be relatively straightforward. Instead of recording the word ‘to’, students substitute the ratio symbol. It is read and interpreted as before. For example, 5 : 3 is still read as 5 to 3.

Ratios do not always have to be seen or written in the standard form using the ‘:’ symbol—it may be more convenient to construct a table of equivalent values. The information from a table can be processed and written elsewhere, in ratio form if necessary.

<table>
<thead>
<tr>
<th>Shadow 1</th>
<th>30 cm</th>
<th>50 cm</th>
<th>1 m</th>
<th>1.2 m</th>
<th>1.5 m</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shadow 2</td>
<td>60 cm</td>
<td>1 m</td>
<td>2 m</td>
<td>2.4 m</td>
<td>3 m</td>
</tr>
<tr>
<td>Year</td>
<td>Language</td>
<td>School Use</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>------</td>
<td>----------------------------------</td>
<td>------------</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Year 7</td>
<td>5 is to 15 equivalence proportion ratio equivalent ratios simpler form</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The red one is longer than the blue one...

The red one is twice as long as the blue one...

The ratio of the lengths is 16 to 8
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## Scope and Sequence

### Length

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</table>
The purpose of this scope and sequence is two-fold.

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Year Level Expectations
Length

Year 1
Arbitrary Units
- Year 1 Sourcebook pp. 92-97
In Year 1, children compare the general size of people and objects. Activities involve estimating and measuring by sight and using whole arbitrary units. Conservation of length is also explored.

Year 2
Introduce Metre
- Year 2 Sourcebook pp. 196-204
In Year 2, consolidation of conservation of length and the use of arbitrary units is continued. The metre as a standard unit of length is introduced. (Kilogram and litre as the standard units for mass and volume respectively and are also introduced in Year 2). Although centimetres are not formally introduced in Year 2, teachers are encouraged to make use of incidental moments when these units are mentioned to reinforce comparative size.

Perimeter/Circumference
Activities investigating the concepts of perimeter and circumference as boundaries are begun.

Year 3
Introduce Centimetres and Concept of Perimeter
- Year 3 Sourcebook pp. 289-308
Measurement using arbitrary units and metres continues in Year 3 where centimetres are introduced. Techniques for using a ruler to measure and for using a ruler and pencil to draw lines should also be taught. Mixed units of measure, such as 3 metres 30 centimetres, may be used as an interim measure.

Perimeter/Circumference
The concept of perimeter is introduced in Year 3 and it is intended that it be treated informally. Explore perimeter as a measure of length from a starting point around the boundary of a shape and back to the starting point is used. The use of trundle wheels, rulers, geoboards and string are recommended.

Year 4
Consolidate Metre, Centimetre and the concept of Perimeter
- Year 4 Sourcebook pp. 317-329
Through Year 4, it is necessary to continue the estimating and measuring activities with metres and centimetres to help students begin to visualise the length of a metre and multiples of centimetres. For example, it is desirable that students are able to judge, with some accuracy, lengths of approximately 10 cm, 30 cm and 1 m, as these are useful 'yardsticks' when estimating. A knowledge of body measurements may also be advantageous, e.g. hand span, foot lengths, arm span and stride length. The procedure of estimation followed by validation should be basic to the measurement process.

Perimeter/Circumference
Work on the concept of perimeter is continued in Year 4. To avoid later confusion between area and perimeter, students should be afforded ample opportunity to thoroughly understand what perimeter is. No formulae work is suggested in this Sourcebook, though some examples may lead students to discover some formulae for themselves.
Year 5
Introduce kilometre and the formulae for perimeter of rectangles (squares)
Year 5 Sourcebook pp. 191-210
Throughout Year 5 it is necessary to continue the estimating and measuring activities begun in the earlier year levels to further consolidate students' understanding of centimetres and metres. Depending on the level of understanding of fractions, decimal forms may also be used for recording length. For example, 1 m 35 cm may be written as 1.35 m. The difficult concept of kilometre is introduced formally in Year 5.

Perimeter/Circumference
The formulae for finding perimeters of rectangles (including squares) is developed for the first time in Year 5. It is very important students build up an understanding of the formulae for themselves and not simply be told a formula to use. The use of calculators for applying formulae is encouraged.

Year 6
Introduce millimetre and circumference
Year 6 Sourcebook pp. 138-154
By Year 6, students should be quite proficient at measuring with centimetres and metres. Work will need to continue with kilometres and with the relationships between units of length. It is during Year 6 that the concept of millimetre is introduced.

Perimeter/Circumference
Work with the formulae for finding perimeters of rectangles (including squares) continues. In Year 6, perimeters of other polygons are investigated and calculated. Students are introduced to the idea of circumference of circles as a measure of the distance around a circular shape and the relationship between radius, diameter and circumference. Once students show proficiency with the concept of 'pi' the formula $C = \pi D$ may be developed and explored.

Formal application of formulae may be practised on occasion but it contributes very little to a real understanding of perimeter.

Year 7
Calculate fluently all measures, perimeters and circumferences
Year 7 Sourcebook pp. 101-117
By end of Year 7, children should be estimating and calculating, converting and measuring fluently with millimetres, centimetres, metres and kilometres. Children should also be competent at selecting the most appropriate unit of measure and at using various measuring devices.

Perimeter/Circumference
In Year 7, activities leading to the consolidation of measuring and calculating perimeters of polygons, circles and combined shapes are attempted.
Glossary of Terms

Length

Estimations

Since the ability to estimate is an essential skill, measurement activities should always be practical in nature and involve students in estimations and measurements to validate their estimations. The ability to estimate is not an easily acquired skill and students need to be encouraged in their efforts. Estimating and measuring are mutually supportive skills and should be carried out together. Because students learn from each other, much may be gained from allowing work to be done in pairs or small groups where discussion estimates is encouraged before measuring and verifying.

Children should always be encouraged to validate their estimates. To help children gain confidence in estimating, use phrases such as 'make a close guess' rather than 'right' or 'wrong', when validating their estimates.

When the length is not exactly a multiple of 1 m, make approximations using words such as 'almost', 'about', or 'a little bit less than'.

Teachers should frequently ask students to discuss and to compare the strategies they are using to estimate length, and identify which strategies should be used. One good strategy is to compare against a known length or choosing a known length as a unit, and visualising how many would be needed to cover the length.

Arbitrary Units

Arbitrary Units are the first forms of measurement encountered by children.

Estimating and measuring using arbitrary units begins with children using dissimilar units to measure the length of an object, a second object and for comparing the two. (Year 1 pp. 105-106). Estimating and measuring length using arbitrary units usually involves the use of the same unit repeated or body parts. Measurements should be in whole units only, so if there is a space at the end which is less than one unit, approximations such as 'about 11 armspans' or '10 armspans and a bit more' are.

Metre (Introduce Year 2)

The metre unit is the first unit introduced and is the basic standard unit of measure. When children begin measuring with metre sticks, rulers or tape measures, these should be placed end to end with no overlapping. Later children should measure and mark one metre intervals as they use a metre stick, ruler or tape. It is more difficult to use one measure, mark the end and then move the measure for the second metre, than it is to use a number of metre sticks placed end to end.

For this reason, begin measuring using the end to end method.
Centimetre  (Introduce Year 3)

Centimetres are taught in Year 3 and follow the introduction of metres in Year 2. The relationship between the standard units within each measurement involves a factor of 1,000 and this fact needs to be pointed out to students. The only exception involves centimetres and its relationships to millimetres and to metres. The learning of these relationships should be relatively straightforward with students only having to be careful if centimetres are involved.

\[ \text{mm} \rightarrow \text{m} \rightarrow \text{km} \]
\[ \text{g} \rightarrow \text{kg} \rightarrow \text{t} \]
\[ \text{mL} \rightarrow \text{l} \]

all factors of 1,000

\[ \text{mm} \rightarrow \text{cm} \text{ factor of 10} \]
\[ \text{cm} \rightarrow \text{m} \text{ factor of 100} \]

Mixed units of measure (such as 3 metres 50 centimetres) are not recommended by the Metric Conversion Board of Australia. However, it may be of assistance to children in Year 3 to use a mixed form in addition to the form '350 centimetres' as an interim step to the use of decimal forms, such as 3.5 m, which will be introduced in Year 4.

Kilometre  (Introduce Year 5)

The kilometre is introduced in Year 5. It is difficult for many adults and students to visualise a kilometre. The relationship of 1,000 m = 1 km should be constantly reinforced and simple fractions of a kilometre (halves, tenths) should be discussed, e.g., 1.5 km is the same as 1,500 m or 'one and a half kilometres'.

Estimating in kilometres poses many more difficulties than in any of the smaller and more manageable metric units. Establishing and maintaining student competence in dealing with references relating to kilometres, therefore, is also difficult. As for all units of measure, it is important to help students gain an idea of the magnitude of a kilometre through extensive activities.

*NOTE: The correct pronunciation for kilometre is kilo - metre where the word 'kilo' sounds like the word 'killer' i.e. killer metre.*

Millimetres  (Introduce Year 6)

During Year 6, millimetres (mm) are introduced formally to students. It would be unusual, however, if the measurement had not been noticed, discussed and even used by students previously. The main aims of activities involving millimetres are:

- to help students see the reason for having such a small unit;
- to nominate occasions when millimetres would be the most appropriate unit of measure;
- to understand the relationship between metres and centimetres;
- to provide opportunities for estimating and measuring in millimetres.

The meaning of the prefix 'milli', which means 1 thousandth, may help relate millimetres to the basic unit, metre.

When object graphs are recommended, a vinyl sheet may be marked with plastic tape to form a grid. This grid may then be used for all object graphs and also the people graphs.

Refer to the Scope and Sequence for length on page _ for Year level appropriateness of measuring devices.
Materials

For children to develop a sense of the size of each standard unit, it is important to use measuring devices on which the unit of measure is clearly visible. Children cannot develop a sense of the magnitude of one metre, for example, by using equipment such as trundle wheels. In such devices, the length of one metre is not obvious so children tend to listen just for the clicks. Trundle wheels are introduced in Year 3 Sourcebook activities, at a time when children are more familiar with the size of one metre.

Use improvisations of metre sticks so that all children have access to measuring devices. Timber or dowelling may be cut into one-metre lengths to produce a metre stick.

After measuring the length of objects, children compare and classify these measurements. For this purpose, children construct object and picture graphs. When object graphs are recommended, a vinyl sheet may be marked with plastic tape to form a grid. This grid may then be used for all object graphs and also for people graphs.

Refer to the Scope and Sequence for length on page ___ for Year level appropriateness of measuring devices.

How to use a Ruler (Introduce Year 3)

The following is a list of teaching points which may be useful to help children to learn correct use of a ruler and pencil to draw straight lines.

- Have the children hold the ruler firmly on the paper near the middle of the part being used, with fingers comfortably separated. Fingertips should not protrude over the ruling edge.
- Have the children to hold the point of the pencil against the ruler's edge, with the pencil almost vertical.
- The pencil should be moved along the ruling edge gently but firmly and steadily.
- Discuss with the children the position of the zero (0) mark on the ruler and the significance of this when measuring a line. The children should start measuring from the zero mark and not from the end of the ruler.

Techniques for using a ruler to measure are as follows:

- Place the ruler so that the numbers read from left to right.
- Align the left-hand end of the ruler or the zero mark (whichever applies) with the left-hand end of the line to be measured.
- Count along the ruler to the end of the line.
- Record the length.

Points to note when ruling lines of specific length:

- Place the ruler with the numbers reading from left to right.
- Hold the ruler firmly by spreading one's fingers along the ruler.
- Pull rather than push the pencil from the zero mark or the ruler's end to the required length.
- Check that the measurement is correct before removing the ruler.

References:
Year 3 Sourcebook p. 48 & p. 298
Conversions between units

The chief advantage of the metric system is that nearly all the standard units are interlinked by factors of 100. The only exceptions involve centimetres and hectares.

Conversions for length may prove a little difficult as factors of 10, 100 and 1 000 apply.

\[
\begin{align*}
1000 \text{ mm} & = 1 \text{ m} \\
100 \text{ cm} & = 1 \text{ m} \\
10 \text{ mm} & = 1 \text{ cm}
\end{align*}
\]

Initially, let students use calculators when converting, because:
- the important decision is whether to multiply or divide;
- some may still have difficulties with the numeration aspects;
- calculators handle the most complex examples easily;
- early success will help 'converting' become a quick mental activity.

For students who are not sure of themselves, break the activities down into a number of small steps. For example:
- Present students with a small number of examples:
  \[
  \begin{align*}
  \text{Change:} & \quad 2348 \text{ g} = \ldots \text{ kg} \\
  & \quad 5.5 \text{ L} = \ldots \text{ mL} \\
  & \quad 0.78 \text{ t} = \ldots \text{ kg} \\
  & \quad \ldots \text{ mm} = 34.5 \text{ cm} \\
  & \quad \ldots \text{ mm} = 52.45 \text{ m}
  \end{align*}
  \]
  - Have them examine the units involved in each conversion.
  - Ask students to indicate (underline) which unit is the larger one.
  - Discuss whether to divide or multiply.
  (Converting from a large unit to a smaller one, we multiply. Converting from a small unit to a larger one, we divide).
  - Decide which factor is involved.
  - Perform the necessary calculation.
  - Explain why the decisions were made.

Therefore, ask students to make only one decision—whether to multiply or divide—and to explain why they chose that operation in each case.

Students then have to decide what to multiply by. If only the common units other than centimetres are involved, the factor is 1000. Centimetres have a factor of 10 with millimetres and 100 with metres.

Benchmarks

Benchmarks are introduced formally in Year 5 and are arbitrary units which assist with approximations. It is important for children to develop the idea that different benchmarks are appropriate for different situations.

For example, the walking pace may be the best arbitrary unit to estimate the length of a room, whereas a person's height may be the best benchmark to use in estimating the height of a door.
Perimeter (Introduce Boundaries from Year 2, Concept of Perimeter from Year 3)

Perimeter is a term which may be applied to the distance around the boundary of polygons (which have straight sides), circles (though it is generally called the circumference), ellipses, and figures which have a combination of straight and curved boundaries.

\[ \text{perimeter} = \text{peri} : '\text{around}' + \text{metron} : '\text{measure}' \]

In Year 3, when the term perimeter is introduced, children should think of the perimeter as the distance from a starting point around the boundary of a shape and back to the starting point.

Perimeter of a Square (Introduce Year 5)

Please note the square is a rectangle because, like rectangles, it has 2 pairs of equal and parallel sides and four right angles.

Squares may be singled out for particular discussion to lead children to the rule that to find the perimeter of a square, the length of one side is multiplied by four.

This may be abbreviated to \( P = S \times 4 \) but if students prefer other forms e.g. \( P = \text{Side} \times 4 \) or \( S + S + S + S \), these are also correct and acceptable.

Perimeter of a Rectangle (Introduce Year 5)

Legitimate forms of the formulae for perimeter of rectangles may include:

- \( P = L + W + L + W \)
- \( P = 2L + 2W \)
- \( P = 2(L + W) \)

The students should be encouraged to use the rule that has greatest meaning for them and which reinforces their understanding of perimeter. The first one is the most general one, providing a method to find the perimeter of any polygon, i.e. adding the length of the sides. The other formulae might begin to make more sense to students as they start to search for ways to abbreviate their own formulae.

Circumferences of a Circle (Introduce Year 6)

In Year 6, students are to be introduced to the idea of circumference of circles as a measure of the distance around the circular shape and the relationship between radius, diameter and circumference.

Once students show proficiency with mental calculations and explanations about circumference, they may be told that a more accurate calculation of the circumference of a circle may be made if the diameter is multiplied by 3.14. It must be stressed, however, that this figure also gives an approximation, but closer to the actual answer.

The numeral 3.14 is called a 'constant' and is represented by the Greek letter \( \pi \), called 'pi'. The rule for finding circumference is written as:

\[ C = \pi D \]

The students should verbalise the rule to illustrate that they understand what it implies, e.g. "To find the circumference of a circle, I multiply the diameter of the circle by 3.14".

Length - Chapter 16 : 10
The terms “metre”, “centimetre” and “kilometre” may be written in words or in symbolic form. Note that neither a full stop nor an ‘s’ for plurals is used in symbolism. Unlike the symbols for units of volume, all symbols for length are in lower case.

<table>
<thead>
<tr>
<th>Unit</th>
<th>Symbol</th>
<th>Meaning</th>
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</thead>
<tbody>
<tr>
<td>metre</td>
<td>m</td>
<td>metre is the base unit for length</td>
</tr>
<tr>
<td>centimetre</td>
<td>cm</td>
<td>‘centi’ means ‘one hundredth of’</td>
</tr>
<tr>
<td>kilometre</td>
<td>km</td>
<td>‘kilo’ means ‘a thousand’</td>
</tr>
<tr>
<td>millimetres</td>
<td>mm</td>
<td>‘milli’ means ‘one thousandth of’</td>
</tr>
</tbody>
</table>

It should be noted that mixed units such as 1 metre 50 centimetres, is not a correct method of recording metric units. However, to assist students in recording length, this mixed format may be used as an intermediate step to the correct format of 1.5 metres. The use of the correct format needs to develop with Year 1 level expectations for decimal fractions.

The following format will help develop consistency within schools:

<table>
<thead>
<tr>
<th>Year 2</th>
<th>Year 3 and 4</th>
<th>Year 5</th>
<th>Year 6</th>
</tr>
</thead>
<tbody>
<tr>
<td>metre</td>
<td>1 metre 54 centimetres</td>
<td>1.54 metres</td>
<td>1.575 metres</td>
</tr>
<tr>
<td>‘more than a metre’</td>
<td>154 centimetres</td>
<td>1 kilometre 575 metres</td>
<td>Decimal notation to thousandths may be used because it is consistent with Year 6 decimal work.</td>
</tr>
<tr>
<td>‘less than a metre’</td>
<td></td>
<td>1575 metres</td>
<td></td>
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</table>

Decimal notation to hundredths may be used because it is consistent with Year 5 decimal work.
During length measuring activities, teachers should introduce and model the following terms and encourage the children to use them appropriately within their own language.

<table>
<thead>
<tr>
<th>Year 1</th>
<th>Year 2</th>
<th>Year 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>tall / short high / low taller / shorter higher / lower same / different as long as long / short shorter / longer longest / shortest widest / narrowest thickest / thinnest long, longer, longest short, shorter, shortest wide, wider, widest narrow, narrower, narrowest thick, thicker, thickest thin, thinner, thinnest tall, taller, tallest high, higher, highest low, lower, lowest</td>
<td>Include previous language</td>
<td>Include previous language</td>
</tr>
<tr>
<td>deep, deeper, deepest shallow, shallower, shallowest far, further, furthest close, closer, closest length width height depth distance boundary metre</td>
<td></td>
<td>centimetre between about almost nearly estimate guess between length around outside measure perimeter</td>
</tr>
</tbody>
</table>

If a space measures less than a whole unit, approximating terms such as 'about', 'almost', 'nearly', 'a bit more than', 'a little more than', or 'close to' should be used.

To help children gain confidence in estimating, use phrases such as 'guessing the same number' or 'making a close guess' rather than 'right' or 'wrong' when they verify their estimates.

Children should learn to use these words in context. They should come to understand that different people will attach different meanings to such words as 'almost', 'short', 'nearly', and 'about'.
Fathom, yard, cubit, foot, span, hand.

1 fathom
1 yard
1 cubit

Width of a finger, 1 inch

3 grains of barley = 1 finger
1 finger = 1 inch
4 fingers = 1 hand
4 hands = 1 foot
2 cubits = 1 arm
1 arm = 1 yard
2 yards = 1 fathom
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Year 1

Arbitrary Units and beam balances
*Year 1 Sourcebook pp. 98-101*

In Year 1 children compare masses by lifting and also by measuring using beam balances and arbitrary units. The notion that apparent size does not always determine whether one object is heavier than another is also developed. Children construct object and picture graphs to represent comparisons.

Year 2

Introduction of Kilogram
*Year 2 Sourcebook pp. 205-211*

The ideas presented in Year 1 are revised in Year 2 and the standard unit of measure for mass, the kilogram is introduced. Children should estimate and measure mass as "1 kg", "more than 1 kg" or "less than 1 kg". Even though grams are not formally introduced in Year 2, teachers are encouraged to make use of incidental moments when these units are mentioned to reinforce comparative size. After measuring the mass of objects, children compare and classify these measures by constructing object and picture graphs. Children should compare and classify measurements of mass through the construction of objects and picture graphs.

Year 3

Consolidate Kilogram, introduce bathroom scales
*Year 3 Sourcebook pp. 193-202*

Although arbitrary units should still be used, Year 3 activities involving mass centre on the standard unit of mass, the kilogram. Balance scales should be the main instrument for measuring mass although for greater accuracy it may be necessary to use kitchen scales. Use of bathroom scales may be introduced in Year 3 and recordings should be written as "a bit more than 28 kg" or "nearly 30 kg" in cases where the mass is not a complete kilogram. The term kilogram may be written in word form or recorded in the symbolic form "kg". Bar graphs may be constructed to record results of activities.

Year 4

Introduce Grams
*Year 4 Sourcebook pp. 235-248*

In Year 4, mass activities focus on refining the measurement of mass to include grams. The terms "kilogram" and "gram" may be written in words or recorded in the symbolic forms "kg" or "g". Grams are explored in multiples of 100 before refining to less than 100. The beam balance should continue to be the main instrument for measuring mass, although for increased accuracy, it may be necessary to use kitchen type scales. Units of measure, such as 1 kilogram, 500 grams and 1500 grams may be used in Year 4 as an intermediate step to 1.5 kilograms as used in Year 6.
Year 5
Consolidation of Grams
Year 5 Sourcebook pp. 135-144

Kilograms and grams continue to be the focus of Year 5 activities involving the measuring and comparing of mass. The recording of mass using non decimal form such as 3 kg 500 g will continue although decimal form to hundredths may be taught. Decimal work to hundredths is consistent with Year 5 fraction work, so it is appropriate to focus on tenths (1.2 kg) and hundredths (2.75 kg) but not thousandths (1.275 kg). Year 5 activities should reinforce the relationship between grams and kilograms by encouraging students to rename masses in different ways. Calculators may be used to convert between kilograms and grams.

Year 6
Introduce Tonne, mean and average
Year 6 Sourcebook pp. 137-154

Exploration of kilogram and gram and the relationship between them continues in Year 6. The concept of "tonne" is introduced and recording of mass using non decimal forms (3 kg 500 grams) is no longer necessary as thousandths become part of the children's fraction repertoire. The concept of benchmarks may also be explored e.g. children's own mass for kilograms and the mass of a car or truck for a "tonne". The "mean" or "average" mass may be investigated by arranging a series of masses such as body masses in order from lightest to heaviest and then see which mass is in the middle of the range.

Year 7
Introduce 'Mass and Weight' conversions
Year 7 Sourcebook p. 101

Year 7 activities focus broadening student's concepts of the units of mass through estimation, calculation and exploration of relationships. The concept of a tonne is a difficult one and will require considerable reinforcement. Students must also practice using "benchmarks" to assist with estimations. Year 7 children should be aware of "mass" as the amount of matter contained in an object or body of material while weight is a force relating to the pull of gravity. By Year 7 mass must be the only acceptable term used for quantity of matter. Conversions from one unit of measure to another is investigated using calculators initially and then through mental and written computation. Emphasis should be placed on whether to multiply or divide, why and by what. Milligrams are introduced in Year 8.
Estimations

Since the ability to estimate is an essential skill, measurement activities should always be practical in nature, involving students in estimations and measurements to validate their estimations. The ability to estimate is not an easily acquired skill and students need to be encouraged in their efforts. Estimating and measuring are supportive skills and should be carried out simultaneously. Because students learn from each other, much may be gained from letting them work in pairs or in small groups to discuss estimates before measuring.

When working with mass, students should be encouraged to base their estimates on "feel" rather than size, as size may be very misleading. Where possible, when attempting an estimate in mass, children should be given the opportunity to handle the materials.

Children should always be encouraged to validate their estimates. To help children gain confidence in estimating, use phrases such as "guessing the same number" or making a close guess rather than "1 right" or "1 wrong" when validating their estimates.

Arbitrary Units

Arbitrary units are introduced in Year 1 to measure mass. At first, children balance items on beam balances using a variety of different objects. Later they are encouraged to use groups of similar objects to measure and compare mass. Arbitrary units could include cotton reels, pencils, sticks of chalk, unifix cubes, marbles, bolts, copies of the same reading book etc.

As a lead up to the introduction of standard units of measure, the inadequacies of arbitrary units of measure may be discussed, namely:

- lack of meaning if a common knowledge of these units is not shared e.g. explaining that a parcel is the same mass as five full boxes of rods has no meaning to a person unfamiliar with rods;

- the lack of portability of arbitrary units of measure.
Kilograms (Introduce Year 2)

Kilograms are the first standard unit of mass introduced. The word kilogram and its symbolic form "kg" is used. From Year 2 to Year 4 when grams are introduced, measurements should be in whole kilograms or approximations using words such as "almost", "about", "a little bit less than" or "more than".

When children find their mass on bathroom scales in Year 3 the mass may be written as "a bit more than 28 kg" or "nearly 29 kg" in cases where the mass is not a complete kilogram.

Grams (Introduce Year 4)

Children should be introduced to grams in Year 4 when they are experienced in dealing with kilograms. They should also be aware of the need for a standard unit of measure which is less than a kilogram.

Allow students to identify everyday objects which have a mass of less than a 100 grams. After children have been introduced to the one gram mass involve them with heavier objects using multiples of 100 grams as these larger masses are more manageable.

Follow up by returning to masses of less than 100 g and link it with number work dealing with counting in 5's, 10's, 20's and with the topic of money.

\[ e.g. \quad 50 = 20 + 20 + 10 \]
\[ 50 \text{ g} = 20 \text{ g} + 20 \text{ g} + 10 \text{ g} \]
\[ 50 \text{ cents} = 20 \text{ cents} + 20 \text{ cents} + 10 \text{ cents} \]

Tonnes (Introduce Year 6)

Tonnes, like kilometre is a very large unit of measure and maybe difficult to visualise. Since it is not possible for children to "feel" a tonne, benchmarks such as trucks, cars, fork lifts will assist with the concept. Activities should be practical in nature and involve children with real life applications of tonne. Some schools may have access to weighbridges, refineries, quarries, mines and manufacturing industries which could provide examples of measuring materials in tonnes. The Queensland Year Book for Agricultural Crops is also a useful document to explore everyday application of tonne.

Trucks often display signs which relate to the mass of the vehicle and its maximum load
\[ e.g. \]
CROSS is the total mass of the vehicle and its load expressed in tonnes
TARE is the mass of the vehicle expressed in tonnes
NET is the actual mass of the load expressed in tonnes
The chief advantage of the metric system is that most common units are interlinked by factors of 100. The only exceptions involve centimetres and hectares.

Since all the units for mass do use a factor of 1,000, learning the relationships should be straightforward.

\[
\begin{align*}
1,000 \text{ g} & = 1 \text{ kg} \\
1,000 \text{ kg} & = 1 \text{ t}
\end{align*}
\]

Initially, let students use calculators when converting, because:
- the important decision is whether to multiply or divide;
- some may still have difficulties with the numeration aspects;
- calculators handle the most complex example as easily as the simplest;
- early success will help "converting" become a quick mental activity.

For students who are not sure of themselves, break the activities down into a number of small steps. For example:

- Present students with a small number of examples:

\[
\begin{align*}
\text{Change} & - \quad 2,348 \text{ g} = \ldots \text{kg} \\
& 5.5 \text{ L} = \ldots \text{mL} \\
& 0.78 \text{ t} = \ldots \text{kg} \\
& \ldots \text{mm} = 34.5 \text{ cm} \\
& \ldots \text{mm} = 52.45 \text{ m}
\end{align*}
\]

- Have them examine the units involved in each conversion;
- Ask students to indicate (underline) which unit is the larger one;
- Discuss whether to divide or multiply (Converting from a large unit to a smaller one, we multiply; Converting from a small unit to a larger one, we divide.);
- Decide which factor is involved;
- Perform the necessary calculation;
- Explain why the decisions were made.

Therefore, ask students to make only one decision — whether to multiply or divide — and to explain why those chose that operation in each case.

Students then have to decide what to multiply by. If only the common units other than centimetres are involved, the factor is 1,000. Centimetres have a factor of 10 with millimetres and 100 with metres.
Mass/Weight (Differentiate Year 7)

Teachers should be aware of the distinction between "mass" and "weight". Mass is the amount of matter contained in an object and is constant, whereas the weight of an object is dependent on the effect of gravity. Beyond earth's gravity, astronauts experience weightlessness. Their weight becomes zero but their mass remains the constant.

Although it is inevitable that students will hear the words "weigh", "weighing" and "weight" used in everyday language, teachers are encouraged to use the term "mass" in its correct context, in an effort to avoid confusion between the terms. For example — "Measure your mass using the bathroom scales" or "These two objects have the same mass".

It is not expected that students will comprehend the difference between these two terms until Year 7 when children are expected to use the terms correctly. Teachers should use the terms correctly in all grades.

Year 7 students must identify mass as the amount of matter contained in an object or body of material and weight as a force relating to the pull of gravity. Please note that the Metrichation Board recommends the term "mass" be the only one acceptable term for quantity of matter.

The verb "to weigh" will continue to be used in everyday language — however, schools should concentrate on the idea of measuring mass, and include instructions such as:

- Find the mass of the box
- This shoe has a mass of 345 g
- Which has the greater mass, the shoe or the box?
- What unit will you use to measure the mass of the truck?

Materials

Beam balances and improvisations of beam balances should be the first measuring devices used in activities. Sea-saws and buckets attached to a coat hanger make good improvisations of beam balances. Discuss how to use the various scales and balances. For example, masses and objects should never be dropped into pans — place masses carefully to prevent damage.

For children to develop a sense of the size of each standard unit, it is important to use measuring devices on which the unit of measure is clearly visible. Children cannot develop a sense of the magnitude of 1 kg by using equipment such as bathroom scales. In such devices, the mass of 1 kg is hidden so children merely read the number being displayed. Bathroom scales are introduced in Year 3 when children are more familiar with 1 kg.

After measuring mass children are involved in comparing and classifying these measurements by constructing object, picture, bar and line graphs.

In lower grades when object graphs are recommended in the following activities, a vinyl sheet may be marked with plastic tape to form a grid. This grid may then be used for all object graphs and people graphs.
The terms kilogram, gram and tonne may be written in words or in symbolic form. Note that neither a full stop nor an "s" for plurals is used in symbolism.

<table>
<thead>
<tr>
<th>kilogram</th>
<th>kg</th>
<th>&quot;kilo&quot; means &quot;a thousand&quot;</th>
</tr>
</thead>
<tbody>
<tr>
<td>gram</td>
<td>g</td>
<td>gram is from the Greek word meaning &quot;small&quot;</td>
</tr>
<tr>
<td>tonne</td>
<td>t</td>
<td></td>
</tr>
</tbody>
</table>

It should be noted that mixed units, e.g. 1 kilogram, 500 grams are not commonly used when recording metric measurements. However, to assist students in recording masses, this format along with "1500 grams" may be used from Year 4 as an intermediate step to the more correct format 1.5 kilograms. The use of the correct format needs to develop with year level expectations for decimal fractions.

The following format will help develop consistency within schools:

<table>
<thead>
<tr>
<th>Year 2 to Year 4</th>
<th>Year 5</th>
<th>Year 6</th>
</tr>
</thead>
<tbody>
<tr>
<td>kilogram</td>
<td>1 kg 500 g or 1500 g</td>
<td>1.575 kg</td>
</tr>
<tr>
<td>&quot;more than a kilogram&quot;</td>
<td>1.5 kg and 1.57 kg</td>
<td>Decimal notation to thousandths may be used because it is consistent with Year 5 decimal work.</td>
</tr>
<tr>
<td>&quot;less than a kilogram&quot;</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Students need plenty of discussion about the concept and language associated with mass. They should be aware of a number of important points. Mass is defined as the amount of matter contained in an object or body of material; weight is a force relating to the pull of gravity. The Metrication Board recommended that the term "mass" be the only one acceptable for quantity of matter.

On going changes take place in common use of language. The verb "to weigh" will continue to be used in everyday language; however, schools should concentrate on the idea of measuring, estimating and comparing mass, and include instructions such as:

- Find the mass of the box
- This shoe has a mass of 345 g
- Which has the greater mass, the shoe or the box?
- What unit will you use to measure the mass of the truck?

Language is used in different ways depending on the context. Terms such as "heavier", "lighter" and "the same weight" are used in conversation. These terms are related to the terms that will be met in the classroom — "greater mass", "less mass" and "the same mass". Students need to be able to relate everyday language to the language they hear in school.
<table>
<thead>
<tr>
<th>Year</th>
<th>Year 6</th>
<th>Year 4/5</th>
<th>Year 3</th>
<th>Year 2</th>
<th>Year 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>mass</td>
<td>less than 1 kilogram, equal, more than 1 kilogram, same/different, weighs less, weighs more, weighs less, weighs more, weighs less, more than</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>weight</td>
<td>less than 1 kilogram, equal, more than 1 kilogram, same/different, weighs less, weighs more, weighs less, more than</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>language</td>
<td>less than 1 kilogram, equal, more than 1 kilogram, same/different, weighs less, weighs more, weighs less, more than</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Encourage the children to use them appropriately within their own language patterns. Children should hear measurement terms used correctly in activities that interest them. Teachers should model the terms correctly and
Volume

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### Scope and Sequence

#### Volume

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<td>1(a)</td>
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<td>using parts of litres</td>
<td>pp. 315-319</td>
<td>1(b)(c)</td>
<td>pp. 347-348</td>
<td>1(a)(b)</td>
<td>p.213</td>
<td>1(a)</td>
<td></td>
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<tr>
<td>millilitres</td>
<td>319</td>
<td>1(d)</td>
<td></td>
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<td>combinations</td>
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<td>conversions</td>
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<td>Conservation of Volume</td>
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<td>p.320</td>
<td>1(e)</td>
<td>p.348</td>
<td>1(c)</td>
<td></td>
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<td>Activities for investigating and Creating Problems</td>
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<td>4</td>
<td>pp.149-15</td>
<td>p.115</td>
</tr>
</tbody>
</table>

The purpose of this Scope and Sequence is two-fold:

1. to indicate the Year of Introduction
2. to help you access quickly and easily appropriate information and activities in the sourcebooks
Year Level Expectations
Volume

Year 1
Arbitrary Units
Year 1 Sourcebook pp. 115-118

In Year 1 children will measure and compare the volume of containers by:
• filling one container and then transferring that filler to other containers
• using multiple arbitrary units
• using a single arbitrary unit repetitively
Conservation of volume is explored.

Year 2
Introduce Litre (L)
Year 2 Sourcebook pp. 212-222

The concepts in Year 1 should be revised in Year 2 and the standard unit of measure, litre, should be introduced. Even though millilitres are not formally introduced in Year 2, teachers are encouraged to make use of incidental moments when millilitres are mentioned to reinforce their comparative size. Familiarize children with the word litre and its abbreviation "L".

Year 3
Compare and order volume, and concept of half.
Year 3 Sourcebook pp. 309-326

In Year 3, the emphasis in teaching volume is on the continuing development of the concept of volume (both solid and liquid), including comparing and ordering volume, appropriate language, and conservation of volume. These objectives may best be served by involving the children in hands-on activities, employing both solid and liquid volume.

By using arbitrary units, the children should come to appreciate the need for common units, and then standard units, to facilitate reliable comparisons. The children should practise estimation constantly before validation.

In Year 3, volume is measured and recorded in litres or parts of a litre - that is, half a litre, more than half a litre, less than half a litre. The term "millilitre" is not introduced until Year 4. It is recommended that decimal recording not be used at this level.

Throughout the activities, the term "volume" may be used interchangeably with the term "capacity". Although "volume" often refers to the amount of space occupied by a container and "capacity" usually refers to the total space inside a container the distinction need not be made by children at this stage.

Teachers should use the available opportunities throughout these activities to talk about the concept of one-half, for example "Ben drank half the volume of water that Tom drank". In Year 3 the children should encounter only the written form, "one-half", not \( \frac{1}{2} \) or 0.5. Formal work on fractions is begun in Year 4.
Year 4

Introduce Millilitre (mL)
Year 4 Sourcebook pp. 235-247

Measuring comparing and ordering activities with liquids involving the standard one litre unit, as well as a range of informal units should be continued in Year 4. *Millilitres are to be introduced in Year 4* once children have had extensive practical experience with liquid measures using the standard one litre unit as well as a range of informal units. The background of the word "litre", "milli" and millilitre would be discussed. Strategies for estimating capacity should also be discussed and appropriate ones encouraged.

Year 4 students should be able to recognise the range of scale on graduated containers (e.g. 0 - 250 mL or 0 - 1000 mL) and be able to identify the sequence of numbers on the scale e.g. multiples of 10, 50, 100. Calculations involving volume and whole number concepts should be experienced in Year 4.

Year 5

Explore fractions of a litre and displacement
Year 5 Sourcebook pp. 211-221

Year 5 consolidates the understanding of the standard units of volume, litre and millilitre. The relationship between litres and millilitres should be revised i.e. 1000 mL = 1 L. Concepts explored in Year 5 include: fractions of a litre, displacement and soma cubes. Volumes of solids using arbitrary units are measured and compared.

Year 6

Formal decimal notation of units
Year 6 Sourcebook pp. 137-154

The activities of the Year 6, length, mass and volume chapter only deals with units of volume related to capacity. Millilitres (mL) and litres (L). The other volume units (cm³ and m³) are combined with the study of three dimensional shapes.

Explorations involving litres and millilitres and their inter relationship continues in Year 6. Recording of volume using non-decimal notation (1 litre 500 millilitres) is no longer necessary as thousandths become part of the childrens fraction repertoire. Benchmarks are explored to assist children with estimations.

Year 7

Conversions
Year 7 Sourcebook pp. 143-153

Year 7 activities focus on helping students increase their understanding of the units of capacity, on inter relationships, and on increasing students' ability to estimate and calculate with the unit. Children should also practice using benchmarks to assist with estimations. Conversions from one unit of measure to another are investigated using calculators, then through mental and written computation. Emphasis should be placed on whether to multiply or divide, why and by what.
Estimations (Introduce Year 1)

Since the ability to estimate is an essential skill, measurement activities should always be practical in nature involve students in estimations and measurements to validate their estimations. The ability to estimate is not an easily acquired skill and students need to be encouraged in their efforts. Estimating and measuring are supportive skills and should be carried out together. Because students learn well from each other, much may be gained from allowing work to be done in pairs or small groups where discussion of estimates is encouraged before measuring and then verifying.

Children should always be encouraged to validate their estimates. To help children gain confidence in estimating, use phrases such as 'make a close guess' rather than 'right' or 'wrong', when validating their estimates.

When the volume is not exactly a multiple of 1 L make approximations using words such as "almost", "about" or "a little bit less than".

Teachers should frequently ask students to discuss and compare the strategies they are using to estimate capacity, and identify which strategies are more appropriate for particular situations. Students who are not using appropriate strategies should be used. One good strategy is to compare against a known capacity, or choosing a known capacity as a unit, and visualising how many would be needed to fill the container.

Arbitrary Units (Introduce Year 1)

From Year 1 arbitrary units are used to measure the volume of containers which vary in height and diameter, such as egg cups, matchboxes, jugs, bottles, cups, tumblers, lids and bottle tops. As well as filling these containers with coloured water children should also use a variety of other fillers, such as sand, shell grit, seeds, dried beans, small stones, rice, salt, sawdust, foam beads, wheat, breakfast cereals and flour. Measurements should be in whole units only. If the volume cannot be measured exactly in whole units, use approximations such as "about 24 egg cups or 23 egg cups and a bit more!"

As a lead up to the introduction of standard units of capacity, the inadequacies of arbitrary units may be discussed namely:

- Lack of meaning if a common knowledge of these units is not shared e.g. explaining that a bottle holds four cupfuls has no meaning to a person unfamiliar with the size of the cups
- The lack of practicability of arbitrary units of measure.

The arbitrary units used to measure the volume of a 3-D shape should be three dimensional and identical in size and shape. They should begin to see that units need to fit together easily without gaps. i.e. they should tessellate. This fact may be deduced by students as they observe the many spaces which occur when units such as marbles and beads are used. Sand and sugar also exhibit this fact. Water may still be added to a bucket/cup apparently filled with either sand or sugar.

Encourage students to estimate volume before measuring. When measuring to check a volume estimate of an object which cannot easily be filled by the arbitrary units, students should try to reduce the spaces to a minimum and approximate the size of the unfilled sections. This situation should be followed by a discussion of the suitability of the arbitrary unit used to measure the given volume.
As a result of activities students should realise that:

(i) large arbitrary units are not easily used to measure small volumes and that small arbitrary units are not easily used to measure large volumes;
(ii) objects that do not fit together easily (i.e. marbles, cylinders) leave many gaps when stacked in boxes; therefore, it is difficult to estimate and measure volumes using these objects;
(iii) objects that fit together easily without gaps (i.e. MAB blocks, matchboxes) may be stacked tightly and are more easily counted then some other units.

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**Volume - Chapter 18 : 6**

**Litre (Introduce Year 2)**

In Year 2 litre is introduced as the standard unit of measure for capacity. "Litre" comes from the French word "litron", meaning a unit of capacity. The word litre and its symbolic form "L" is used. The capital letter L is used for litre to avoid confusion between the lower case "l" and the numeral 1. From Year 2 to Year 4 when millilitres are introduced measurements should be in whole kilograms only, so when the capacity is not exactly multiples of 1 litre, approximations should be made using words like "almost", "about", "a little bit less" or "more than".

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**Millilitre (Introduce Year 4)**

Children should be introduced to millilitres in Year 4 when they are experienced in dealing with litres and informal units. They should also be aware of the need for a standard unit of measure which is less than a litre.

In working with measures which are multiples of 100 mL, students should be able to discover that ten lots of 10 mL total 1 000 mL and that 1 000 mL fills exactly a one litre container. This work could be related back to the number study ideas involving multiples. The background of the word "litre", "milli-" and "millilitre" could be discussed in Year 4:

**Litre:** comes from the French word "litron", meaning a unit of capacity.

**Milli-:** means one thousandth and is related to the French word "mille", indicating a thousand.

**Millilitre:** one thousandth of a litre, i.e. 1 000 mL = 1 L

In Year 5, descriptions of the volume of containers should include 250 mL as "a quarter of a litre" and similarly 750 mL as "three-quarters of a litre". These relationships may help students when they begin to use decimal notation to record this information.
In Year 5 fractions of a litre may be investigated by providing the following graduated containers and by considering these types of questions:

(i) How many times may the 250 mL container be filled from the one litre container?
(ii) What fraction of one litre is 250 mL?
(iii) Which container holds \( \frac{1}{2} \) litre?
(iv) How many times must the 50 mL container be filled to make \( \frac{1}{4} \) litre?
(v) If the 200 mL container is filled twice from a \( \frac{1}{2} \) litre container, how many millilitres of water would be left in the larger container?

Results of these activities may be validated using the actual containers.

**Capacity/Volume (Correct usage Year 5)**

*Capacity* usually refers to the total amount of space available in a container.

*Volume* usually refers to the amount of space occupied by the material inside a container.

Until Year 5 the term volume may be used interchangeably with the term capacity. In Year 5 however, children may begin to discuss the differences.

**Conversions Between Units**

The chief advantage of the metric system is that nearly all the common units are interlinked by factors of 1000. The only exceptions involve centimetres and hectares.

Since the units for capacity do use a factor of 1000, learning the relationship should be straightforward.

\[ 1000 \text{ mL} = 1 \text{ L} \]
Initially, let students use calculators when converting, because:
• the important decision is whether to multiply or divide;
• some may still have difficulties with the numeration aspects;
• calculators handle the most complex example as easily as the simplest;
• early success will help “converting” become a quick mental activity.

For students who are not sure of themselves, break the activities down into a number of small steps. For example:
• Present students with a small number of examples:

  Change -
  2.348 g = ... kg
  5.5 L = ... mL
  0.78 t = ... kg
  ...mm = 34.5 cm
  ...mm = 52.45 m

• Have them examine the units involved in each conversion.
• Ask students to indicate (underline) which unit is the larger one.
• Discuss whether to divide or multiply. (Converting from a large unit to a smaller one, we multiply. Converting from a small unit to a larger one, we divide.) Why this is so must be clearly established.
• Decide what factor is involved.
• Perform the necessary calculation.
• Explain why the decisions were made.

Therefore, ask students to make a crucial decision - whether to multiply or divide - and to explain why they chose that operation in each case.

Students then have to determine what to multiply by. If common units other than centimetres are involved, the factor is 1000. Centimetres have a factor of 10 with millimetres and a factor of 100 with metres.

**Benchmarks (Introduce Year 6)**

Benchmarks for volume are introduced formally in Year 6 and are designed to assist with approximations. It is important for children to develop the idea that different benchmarks are appropriate for different situations. A drinking glass or tea cup, a teaspoon and a bucket can act as references upon which students may base their estimates.
Reading and Recording Volume

Recording Capacity (Abbreviations)

The terms litre and millilitre may be written in word form or symbolic form. Note that neither a full stop nor an "s" for plural are used in symbolism. A space should be left between the numeral and the symbol.

*The capital letter "L" is used for litre* to avoid confusion between the lower case "l" and the numeral "1".

\[
\begin{align*}
\text{litr}e & = L \\
\text{millil}itre & = mL \text{ (yes, a mixture of upper and lower case is correct)}
\end{align*}
\]

The background of the word "litre", "milli-" and millimetre may be discussed in Year 4.

*Litre:* comes from the French word "litron", meaning a unit of capacity.

*Milli:* comes from the French word "mille", meaning one thousand.

*Millilitre:* one thousandth of a litre i.e. 1 000 mL = 1 L

It should be noted that mixed units e.g. 1 litre 500 millilitres, are not recommended by the International System of Units. However to assist students in recording volumes, this format together with 1500 mL may be used as an intermediate step to the more correct format of 1.5 L.

The use of the correct format needs to develop in accordance with year level expectations for decimal fractions.

As for all decimal notation the decimal point should be placed on the line to be consistent with calculators and computers e.g. 3.5 L - a space should be left between the number and the unit.

The following format will help develop consistency within school:

<table>
<thead>
<tr>
<th>Year 2 to Year 4</th>
<th>Year 5</th>
<th>Year 6</th>
</tr>
</thead>
</table>
| litre
"more than a litre"
"less than a litre" | 1 litre 500 millilitres or 1500 millilitres | 1.575 L
Decimal notation to thousandths may be used because it is consistent with Year 6 decimal fractions |
| 1.5 L and 1.57 L
Decimal notation to tenths and hundredths may be used because it is consistent with Year 5 decimal fractions |
Children should hear measurement terms used correctly in activities that interest them. Teachers should model the terms correctly and encourage the children to use them appropriately within their own language patterns.

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<th>Year 4</th>
<th>Year 5</th>
<th>Year 6/7</th>
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</thead>
<tbody>
<tr>
<td>full</td>
<td>Revise previous inside language</td>
<td>Revise previous language</td>
<td>Revise previous language millilitre</td>
<td>Revise previous language capacity/volume</td>
<td>Revise previous language.</td>
</tr>
<tr>
<td>empty</td>
<td>about a litre volume litre</td>
<td>about a litre more than a litre less than a litre about a litre</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>overflow</td>
<td>holds the most holds the least fills holds as much as almost full mainly full holds the same as</td>
<td></td>
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# Scope and Sequence

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<td>p. 161 2</td>
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<td>p. 157 1(b)</td>
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</table>

The purpose of this Scope and Sequence is two fold:

1. To indicate the year of introduction
2. To help you access quickly and easily appropriate information and activities in the Sourcebooks.
Year 1

Explore the concept of Area

*Year 1 Sourcebook p. 36*

Year 1 area activities involve children covering areas using everyday objects. Children may compare how they covered their desks with how other children have done it.

Children should consider:
- the shape of the objects chosen to cover the surface;
- the amount of desk-top left uncovered;
- the number of objects covering the surface.

Activities such as this are repeated using:
- a variety of plane shapes - e.g. paper squares, rectangles, triangles and parallelograms,
- multiple copies of the same item - e.g. reading books, matchboxes or Cuisenaire rods, box-lids;
- multiple copies of the same plane shape - e.g. paper squares.

Encourage verbal descriptions of what has been done.

Year 2

Exploration of Area continued

*Year 2 Sourcebook pp. 185-189*

In Year 2, activities focus on area concepts by involving children in:
- spreading shapes over surfaces to give the appearance of covering them;
- fitting shapes together to completely cover areas (with no gaps or overlaps);
- measuring and comparing the area of surfaces with a variety of arbitrary units.

Year 3

Exploration of Area extending into best fit and coverage

*Year 3 Sourcebook pp. 246-252*

In Year 3, children are introduced to the idea of area as ‘covering a surface’, and a variety of arbitrary units are used, including shapes which tessellate. (Refer to section 3a for information about tessellation). Initially children may use a variety of measures and be content to leave parts of the surface uncovered before progressing to using the unit, but still leaving parts uncovered.

Experiences with many types of materials allow the children to decide on the shape that gives the best fit. Using these ideas, children in Year 4 will investigate choosing common units and part units to allow for comparisons. Activities investigating the concept of standard units for measuring area (that is, square centimetres and square metres) is Year 5 work.

It is important that the children begin to develop skills in estimating the area of regular and irregular shapes. Before covering an area and counting the number of units, the children should give an estimate of the number of shapes necessary to cover the space.

Note: Year 3 Introductory Area concepts are included in the plane shapes topic of the Year 3 Sourcebook because it provides a convenient context. *pp. 246-252*
Consolidation of the concept of Area

Year 4 Sourcebook pp. 331-362

These activities are aimed at helping students gain a 'feel' for area concepts. In the Year 3 Sourcebook, activities suggested involved students in covering areas with a variety of units including shapes which 'tessellate' (i.e. fit together neatly without any gaps or overlapping). Refer to Section 3a. These activities will be extended in Year 4 to include the use of common units to compare areas, and part units to help cover an entire area.

By investigating a variety of units, students should realise that it is much easier to cover surfaces with common units which tessellate. Many activities involving tessellations may integrate area and plane shape concepts, and may be approached through colouring and design activities in Art.

Although units the size of square centimetres and square metres might be used in some of the activities, these terms need not be used until Year 5 when students are introduced to standard units for measuring area.

Year 5

Introduce square metres and square centimetres

Year 5 Sourcebook pp. 199-210

To this stage, students have measured areas using arbitrary units only. Ensure that they may confidently use arbitrary units to find areas, before introducing them to the standard units - square metres and square centimetres. Either unit could be presented first, depending on the preferences of teachers. Activities in the Sourcebook focus on the square centimetre before the square metre, as the former is a more manageable size for students to work with.

As students often confuse perimeter and area, time spent helping students understand the concept of area is very beneficial in the long term. No attempt should be made to develop area formulae in symbolic forms in Year 5, but throughout the activities involving rectangular shapes, it is likely that many students will notice the relationships.

After considerable work with square centimetres and square metres, have children use their calculators to explore the relationship between them. This relationship will be formally developed in Year 6. Diagrams and MAB may be used to show that there would be (10 000) 100 x 100 square centimetres in one square metre. It is not intended that students draw up a complete subdivision, but some of it may be done until the pattern becomes obvious.

Year 6

Introduce hectares, develop formula for area of rectangles (squares)

Year 6 Sourcebook pp. 155-166

In Year 6, activities lead to the development of formulae to calculate the areas of rectangles, including squares. However, before a formula is developed and used students should be confident in:

- using tessellating shapes to cover surfaces;
- using tessellating congruent shapes to measure area;
- using standard units such as square centimetres and square metres to estimate and measure areas.

Students are encouraged to use mental calculations, calculators and written forms to calculate areas.

Area - Chapter 19 : 4
Activities involving the use of calculators and mental calculations are particularly useful as such activities require students to think about area, about how area applies to the shapes in question and about the relationship between the length and the width. Many more activities may be completed if calculators and mental calculations are used. When teachers wish students to calculate area using a written procedure, specific lessons should be set aside to practise the procedure.

Introduce the hectare as a more suitable unit for measuring the area of large surfaces.

**Year 7**

**Develop a formulae for area of a triangle and area of a circle**

*Year 7 Sourcebook pp. 119-132*

Year 7 students will continue work on estimating and measuring with arbitrary units, square centimetres, square metres and hectares. They will revise the origin and application of the formulae for the area of rectangles including squares before being introduced to the formula for finding the area of triangles and circles. The latter is quite abstract and relies on the reasoning power of the students. The activities lead students to a better understanding of the formulae which is preferable to immediately asking students to use abstract formulae immediately.

![Diagram of a triangle and a circle](image)

Because students have several years to demonstrate proficiency with the application of these rules, Year 7 should be spent developing understanding of the underlying concepts. Discussion, cutting and pasting of materials, estimation and use of calculators will all be used to help students develop the ideas. Students should be encouraged to use mental calculations, the calculator and written methods to calculate areas.

Activities involving the use of calculators and mental calculations are particularly useful as such activities require students to think about area, about how area applies to the shapes in question and about the relationship between the length and the width. Many more activities may be completed if calculators and mental calculations are used. The more times students use the formula, the better they remember it. When teachers want students to work out area using the written method of setting out, specific lessons should be set aside to practise the procedure.
When a region (or area) has been covered with congruent shapes (same shape, same size) so that there are no gaps or overlaps, the region is said to have been tessellated. The region itself is called a tessellation. Another description of this process is ‘tiling a region’.

A large number of shapes will tessellate, including all triangles and quadrilaterals.

Of the regular polygons (i.e. polygons with all sides equal and all angles equal), only the equilateral triangle, square and hexagon will tessellate by themselves.

Sometimes, a combination of two polygons will tessellate, e.g. octagons and squares; hexagons and diamonds.

By taking a basic shape which tessellates, then altering the outline systematically, it is possible to produce many different designs. Many artists used such techniques in their works. One of the most famous was Mauritz Escher, a Dutchman born in 1898. Escher was one of Europe’s original artists who, in his early life, produced some unusual mathematical artworks on wood using engraving. He had a fantastic eye for detail, as may be seen in many of his works such as ‘Eight Heads’. Escher died in 1972, aged 73.
Square Metre/Square Centimetre (Introduce Year 5)

Square centimetres and square metres are standard units for measuring area and are introduced formally in Year 5. Either unit may be presented first, depending on the preferences of teachers. Activities in the Year 5 Sourcebook focus on square centimetres before the square metre, as the former is a more manageable size for students to work with. Square centimetres and square metres are usually associated with 'square' shapes. Activities should highlight the fact that these areas may take different shapes, but square shapes are usually used because they fit together well. The square metre, for example, may be arranged into different shapes to illustrate conservation aspects.

In Year 5, have students use their calculators to explore the relationship between square metres and square centimetres. This relationship is to be formally developed in Year 6. Diagrams may be used to show that there would be \((10 000) 100 \times 100\) square centimetres in one square metre. It is not intended that students draw up a complete subdivision, but some may be done until the pattern becomes obvious.

Square metres may be written as \(m^2\) and square centimetres as \(cm^2\). This must be read as 'square metres' and 'square centimetres' respectively.

Hectare (Introduce Year 6)

Introduce the hectare as a more suitable unit for measuring the area of large surfaces. Larger units are used to save the use of very large numbers of smaller units. It is more convenient, for example, to refer to the measurement 40 hectares than to 400 000 square metres.

The origin of the word 'hectare' might be explored to help students remember the size of one of the units:
- The 'are' part of the word means 100 square metres.
- The metric prefix 'hecto' means one hundred.
- Hectare translates to 'one hundred ares'.
- One hundred 'ares' is equal to 10 000 square metres.

The shape of one hectare may be as varied as any of the other units already explored, but if hectare is in the shape of square, it has a side of 100 metres. Students may use calculators to explore the dimensions of other shapes which have an area of 10 000 square metres (one hectare).

Some of the possible combinations are:

- \(100 \times 100\) m
- \(25 \times 400\) m
- \(20 \times 500\) m
- \(10 \times 1000\) m
- \(4 \times 2500\) m

Where practical, allow students to measure out some of the above shapes to appreciate the area of one hectare. It may be more reasonable for students to mark out and observe fractions of one hectare, such as one half or one quarter.

The word hectare may be used or it may be symbolized as 'ha' with no full stop, no capital letters and no 's' for plural.
When teaching the area of a rectangle do not immediately ask students to apply abstract formula. Without experiencing appropriate pre-requisite activities, the formula will lack meaning and the calculation will be nothing more than a mechanical task. As a result of children experiencing appropriate pre-requisite activities in earlier grades and the activities on pages 161-163 of the Year 6 Sourcebook, children will develop a much better understanding of the concept involved which may even result in them deducing the formula for themselves.

Children should be provided with opportunities to work in pairs or small groups to apply the discoveries they have made.

Students should be encouraged to use mental calculations, the calculator and written methods to calculate areas. The calculator and mental calculation activities are particularly useful as students must think about area, how it applies to the shapes in question, and also about the relationships between area and the length and width, or base and height. Additionally, many more examples may be completed if these two methods of calculation are employed.

When necessary, written setting out may be explained and demonstrated by the teacher and then practised by the students.

It is important that the written procedures used by students match the way they have been taught to calculate area. Formal representation need only be practised on specific occasions because it is by far the slowest method of finding the area of rectangles. It does, however, assist students to link the formal written procedures with the quicker mental processes.

The following is suggested as a model:

1. Draw a diagram representing the information supplied.
2. Write down what has to be calculated, i.e. area
3. Write the formula. Area = L x W
4. Substitute the data given. = 12 cm x 9 cm
5. Perform the calculation. = 108 cm²
6. Write the answer and consider the type of unit.

Area of a Triangle (Introduce Year 6)

When teaching the area of a triangle do not immediately ask students to apply abstract formula. Without experiencing appropriate pre-requisite activities the formula will lack meaning and the calculation will be nothing more than a mechanical task. As a result of children experiencing appropriate pre-requisite activities in earlier grades and the activities on pages 123-126 of the Year 7 Sourcebook, they will develop a much better understanding of the concept involved which may even result in them deducing the formula for themselves.

Children should be provided with opportunities to work in pairs or small groups to apply the discoveries they have made.

Students should be encouraged to use mental calculations, the calculator and written methods to calculate areas. The calculator and mental calculation activities are particularly useful as students must think about area, how it applies to the shapes in question, and also about the relationships between area and the length and width, or base and perpendicular height. Additionally, many more examples may be completed if these two methods of calculation are employed.
When necessary, written setting out may be explained and demonstrated by the teacher and then practised by the students.

To use the customary formula for area of a triangle, students need to be aware of the terms 'base' and 'perpendicular height'. To draw attention to these terms, use right-angled triangles, isosceles triangles and equilateral triangles. After students are familiar with the terms, they may be applied more generally to scalene triangles. Several activities are recommended to develop an understanding of base and perpendicular, as it is very important that these are fully understood.

As a result of many activities students should be able to verbalise some of the methods:

- half of (the base x perpendicular height);
- base x perpendicular height and halve it;
- half of (the perpendicular height x the base);
- half of base x perpendicular height).

**Area of a Circle (Introduce Year 7)**

When teaching the area of a circle do not immediately ask students to apply abstract formula. Without experiencing appropriate pre-requisite activities, the formula will lack meaning and the calculation will be nothing more than a mechanical task. As a result of children experiencing appropriate pre-requisite activities in earlier grades and the activities on pages 126-127 of the Year 7 Sourcebook, children will develop a much better understanding of the concept involved which may even result in their deducing the formula for themselves.

Children should be provided with opportunities to work in pairs or small groups to apply the discoveries they have made.

Students should be encouraged to use mental calculations, the calculator and written methods to calculate areas. The calculator and mental calculation activities are particularly useful as students must think about area, how it applies to the shapes in question, and also about the relationships between area and the length and width, or base and height.

Additionally, many more examples may be completed if these two methods of calculation are employed.

When necessary, written setting out may be explained and demonstrated by the teacher and then practised by the students.

When students were first asked to find the circumferences of circles in Years 6 and 7, there was a lot of developmental work which led students to deduce that the circumference was just more than three times the diameter. After many mental approximations using this knowledge, students were introduced to \( \pi \). Use of calculators was suggested when more exact calculations were required.

A similar circumstance arises with finding the areas of circles (once more, because of the influence of \( \pi \)). Therefore, students should be led through some developmental activities and then asked to deduce the formula for area, knowing that it also results in an approximate answer. Great importance is placed on estimating the area of circles with mental calculations. If students...
are able to do this and explain their methods, the progression to more accurate written work will be much easier. Students only have to follow the same procedures which were used to calculate mentally, but, instead of multiplying the square of the radius by 3, they will multiply it by 3.14 instead.

Use calculators frequently for these more precise calculations—it is students' recall of the formula that is being practised, not their calculating ability.

The Year 7 Sourcebook outlines two methods to develop the formula to find the area of a circle. Refer to pages 126-127 for an explanation of these methods.

The formula for area of a circle is $\pi r^2$.

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**Tangrams**

A Chinese puzzle made up of a square cut into seven pieces which may be rearranged to make various shapes. *Example:*

Tangrams are very useful for a number of reasons. They help in the development of spatial thinking and observation, give students experience in creating and following spatial patterns, and demonstrate conservation aspects of area.

The use of tangrams for investigating area is advocated in the Sourcebooks and reference is made to them in Year 3, p.248; Year 4, pp.340-341; Year 5, p.207 and Year 6, p.57.

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**Geoboards**

A geoboard is a board studded with nails forming a pattern or grid, usually of squares or equilateral triangles.

Geoboards are used for shape, area and number activities in which elastic bands are arranged around sets of nails.

The use of geoboards for investigating area is advocated by the Sourcebooks and reference is made to them in Year 4, p.341.

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**One Centimetre Plastic Grids**

Some activities investigating area from Year 5 suggest the use of a transparent plastic sheet marked with 1cm grids. The best way to acquire these is to photocopy grid paper onto O.H.Ts. Be sure the O.H.Ts. you use are designed for use in the photocopier.

---

**Dominoes, Trominoes, Tetrominoes, Pentominoes**

Shapes made from two squares, three squares, four squares and five squares are called dominoes, trominoes, tetrominoes and pentominoes respectively and are sketched as shown:

- Domino
- Trominoes
- Tetrominoes
- Pentominoes
Square metres may be written as ‘m$^2$’ but must only be read as ‘square metres’.

Square centimetres may be written as ‘cm$^2$’ but must only be read as ‘square centimetres’.

A Hectare may be symbolized as ‘ha’. Once again there are not full stops, no capital letters and no ‘s’ for plurals. If the word hectare is written in full (3 hectares) it may naturally be plural.

Children should hear area terms used correctly in activities that interest them. Teachers should model the terms correctly and encourage the children to use them appropriately within their own language pattern.

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<th>LANGUAGE</th>
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</tr>
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<td>Include previous language. Spread, overlap, spaces, more, most, last, large, larger, largest, great, greater, greatest, small, smaller, smallest, whole, part.</td>
</tr>
<tr>
<td>3</td>
<td>Include previous language. Inside/outside, area, boundary.</td>
</tr>
<tr>
<td>4</td>
<td>Include previous language. Tessellates, shapes that do tessellate, shapes that do not tessellate.</td>
</tr>
<tr>
<td>5</td>
<td>Include previous language.</td>
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<tr>
<td>6</td>
<td>Include previous language. Hectare, dominoes, trominoes, tetrominoes, pentominoes.</td>
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Year 1

Introduce: • days, weeks, long/short time, fast/slow
• informal o’clock reference

Year 1 Sourcebook pp. 70-82

Year 1 children classify tasks as taking a long or a short time or as being slow or fast. They also estimate, measure and compare the duration of intervals of time with a wide variety of informal units. e.g. humming for a short or long time.

Year 2

Introduce: • hours, minutes, months, years
• o’clock reading of time

Year 2 Sourcebook pp. 113-154

Clocks – Year 2 activities continue to explore informal units of time such as candle and sand clocks. The formal units of ‘hour’ and ‘minute’ are introduced and their durations compared. O’clock times on both analogue and digital clocks are also examined. Other times are not introduced formally but they may still be mentioned informally if they have significance in the children’s routine - e.g. “We have 10 minutes to tidy up before the bell rings”.

Calendars – The regular features of a calendar such as months, dates, weeks and days are identified and a calendar is created for ongoing recording of weather conditions. Yesterday may be introduced as the ‘day before today’ and ‘tomorrow’ as the ‘day after today’. If a date is known, the day may be identified and vice versa. Daily routines, literature and birthdays all present an opportunity to explore time sequences.

Year 3

Introduce: • half/minute past, writing dates, calendars and reading of analogue and digital clocks

Year 3 Sourcebook pp. 141-164

Year 3 children continue to use informal units of time as a means of measuring intervals of time.

Clocks – At Year 3 level, children are expected to understand and be able to read ‘o’clock’, ‘minutes past’ and ‘half past’. They should be able to write the time in standard form (for example 3:35 in the morning/afternoon/evening) and to show the time on both analogue and digital clocks and in diagrams.

Calendars – Calendar conventions such as the last day being recorded at the beginning, abbreviations, holiday dates shaded are identified. Children will be expected to interpret a calendar year fully, write dates formally and identify the ordinal values assigned to the days and the months. Fortnights are introduced and the children learn the poem for remembering the number of days in each month. Refer to section 3c of this chapter.

Timelines – Personal timelines may be constructed.

Year 4

Introduce: • seconds, seasons, quarter hours, decades and centuries
• a.m./p.m., and quarter past reading of clocks

Year 4 Sourcebook pp. 199-218

Clocks – Sourcebook activities up to and including Year 4 only use ‘past’ language for reading times, such as ‘fifteen minutes past seven’, ‘quarter past seven’ or the short form of ‘seven fifteen’. The ‘to’ language for reading analogue clocks is not introduced until Year 5. Year 4 students should be able to write the time using the standard format (e.g. 3:35 in the morning/afternoon/evening) and show the time on both analogue and digital clocks and in diagrams.
Although the concept of minutes was introduced in Year 2, students will need to be involved in a variety of activities to help attain a sound understanding of this unit of time and develop strategies for estimating short durations in minutes. The relationship between seconds and minutes should be discussed and where appropriate, ask students to express the times, minutes and seconds on both. (e.g. 1 min 30 s is the same as 90 s or 1.5 min).

Besides using standard units to record the passing of time (e.g. seconds, minutes, hours) students may use other informal units such as pulse rates, drumbeats, and other musical rhythms. Egg timers, candles, and sundials may also be investigated in relation to the passing of time. Students who have alarms or timers on their watches may use them to mark the passing of intervals of, for example, 5 minutes, 10 minutes or an hour, to focus attention particularly on these amounts of time.

An understanding of the need to write a.m. and p.m. in particular situations should be developed and referred to ‘morning’ and ‘afternoon/evening’.

**Calendars** – Year 4 students will be involved in identifying days, dates and times (in days or months) to or since certain events. Year 4 activities focus on the longer time units of years, decades and centuries. Students will be familiar with years through birthdays, Year levels at school, and other annual events they can relate to personally. They use the terms ‘this year’, ‘last year’ and ‘next year’ quite frequently in their conversations.

Decades and centuries are more difficult concepts for students at this level, as many, if not all students, will not even be ten years old themselves.

All of this work should be treated relatively informally; the emphasis should be on building students’ vocabulary and understanding of these units of time.

**Year 5**

- Introduce: • leap year
  - quarter/minutes to, reading of time, stopwatches and age calculations

*Year 5 Sourcebook pp. 111-126*

In Year 5 the ‘to’ language is introduced for reading analogue clocks. Until Year 5 only ‘past’ language has been used such as ‘fifteen minutes past’, ‘quarter past seven’ or ‘seven fifteen’.

The abbreviations a.m. and p.m. can still be referred to as simply ‘morning’ and afternoon/evening’. Explorations of the relationships between units of time such as minutes, seconds, days, weeks, months and years will continue.

To broaden their understanding of longer durations, students will compile logs and timetables for a week, a month or a year. Timelines illustrating events in history are constructed in this year.

The number of days in each month should be explored and readily recalled. An investigation of the calendars of consecutive years should identify the pattern changes and the reason for leap years. Discuss the relationship among days, weeks and years to consolidate the following facts:

- There are 52 weeks and one day in one year
- There are 365 days in one year
- There are 366 days in one leap year.

In working through estimating and measuring activities on duration of time, the ideas of ‘rate’ and ‘speed’ are likely to be discussed. Encourage this discussion without attempting to formalise the ideas.

Once students are working fluently with seconds include measuring in parts of seconds. Integrate fraction ideas with timing in hundredth and use different durations to have students:

- compare times (compare 2.05 seconds with 2.50 seconds)
- practise rounding (is 10.53 seconds closer to 10 or 11 seconds?)
- show equivalence relationships (3.50 seconds is the same as 3 ½ seconds and 3.5 seconds).
**Year 6**

Introduce: • B.C. and A.D.
- transport timetables

*Year 6 Sourcebook pp. 75-86*

Year 6 is very much a consolidation year of all the formal units of time that have been explored in previous years, although A.D. and B.C. are formally introduced.

The relationship between units of time should be reinforced, using suitable contexts especially those which involve the conversion of one unit to another. Activities should focus on the development and application of problem solving techniques, such as drawing sketches and diagrams, reading times tables, graphs and maps, organising information into lists and tables, and searching for relevant information.

Techniques for calculating with time are explored in Year 6. One of these techniques involves the same renaming and trading concepts as whole number operations. Another technique uses the 'counting on' method often used when counting out change in monetary transactions.

**Clocks** – All students should be proficient with the 'past the hour' and 'to the hour' methods of reading analogue clocks. Year 6 activities focus on the use of stop watches to time durations in seconds and parts of seconds.

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**Year 7**

Introduce: • 24 hour time, lunar months and calculating with time

*Year 7 Sourcebook pp. 65-75*

By Year 7 most students should have a sound understanding of the various units of time, the associated symbols and the relationships between the units. Understanding and applying knowledge of time is an integral part of Year 7 work as children time events, calculate time intervals, and interpret timetables for programs or excursions, etc.

The lunar month and 24 hour time are two concepts formally introduced and explored in Year 7.

The recording of the time of day was introduced in Year 2 using the colon (:), and this form is continued through to Year 7. However, from Year 8, students will be using the accepted standard dot symbol (.) to separate hours and minutes. This symbol was not used in the early and middle primary years to avoid any confusion with the decimal point. Teachers are invited to introduce the dot symbol to Year 7 students if it is considered appropriate.

---

**Time - Chapter 21:5**
Because time is continuous and invisible, it is a difficult concept for young children to understand. The children should be involved in extensive ongoing practical work throughout each year to support their understanding of the concepts.

The teaching of time in the primary school will involve activities of two distinct types:
- understanding of the concept of time;
- telling time.

Understanding of the concept of time includes duration (passing of time) and sequencing events. Telling time involves reading a clock face and recording time.

Children need to acquire some idea of the continuity of time. They may do this by, for example, watching sand run through a timer or listening to advertisements on TV or to hour chimes, noting the beginning and ending. Such activities will help the child appreciate that an event or activity has duration.

The concepts of days of the week, months of the year and minutes of the hour, etc. need to be developed gradually.

The teacher should:
- make daily references to the relevant terms, and reinforce them with appropriate activities;
- encourage the children to be aware of times throughout the day even if the unit of time has not been formally taught.

For example – although minutes are introduced in Year 2 it is advantageous to make comments such as –

"We have only 10 minutes to tidy up before the bell rings".

When making references to durations try to be accurate. I.e. don't say 'in a minute' if you know it is more likely to be in an hour or so.

Students should be capable of the following before they are introduced to quarter hours:
- reading times involving hours, half hours, and minutes past the hour.
- positioning the hands of an analogue clock to show times that they are told, or that they see recorded in digital form.
- saying and recording the time (digitally) that they see on a clock face.

The quarter hour times may be introduced as another way of saying 'fifteen minutes past ...’ Students may use both forms but should realise that ‘quarter past’ and ‘... fifteen’ are the more commonly used terms.

Quarter hours should be explained to students as being related to a quarter turn.
Using the standard clock face, ask students to:
• put the hands of the clock facing the 12
• use the minute hand to show a quarter turn
• show how many quarter turns make one full turn

Ask students to calculate the time (in minutes) represented by each quarter turn (quarter of an hour) based on their knowledge of one hour being 60 minutes.

Set the clock at a time (e.g. 5:10; 10:35, 1:05) and ask students to show where the minute hand would be a quarter of an hour later. Students should be able to count out lots of five minutes as they move the hands.

Note: Students need to be able to recognise when a quarter of an hour has passed any stated time – not just from the ‘o’clock’ position).

Students could draw diagrams (such as those below) representing quarter hour time spans and showing the relationships between quarter hours, half hours and hours.

Note: This time is recorded as 8:15 and may be described as:
15 mins past 8
quarter past 8
eight fifteen.
8:15

Note: The ‘past’ language of reading time is a prerequisite of the ‘to’ language. Therefore the following teaching sequence unfolds.

<table>
<thead>
<tr>
<th>Year 2</th>
<th>Year 3</th>
<th>Year 4</th>
<th>Year 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>o'clock</td>
<td>‘minutes past’</td>
<td>‘quarter past’</td>
<td>‘quarter to’</td>
</tr>
<tr>
<td></td>
<td>‘half past’</td>
<td></td>
<td>‘minutes to’</td>
</tr>
</tbody>
</table>

**Months of the Year (Introduce Year 2)**

Months are identified in Year 1 and recall of how many days in each month is sought during Year 5 only after many exploration activities have been completed. (Refer to Year 5 Sourcebook p. 114 activity 1d).

As a result of exploration activities children may realise that every second month (January, March, May and so on) has 31 days, with the exception of July and August. Both these months have 31 days.
This pattern may be shown by the 'knuckle' method which relates the knuckles on both hands to the months with 31 days, and the depressions in between the knuckles to the other months. The pattern is broken between July and August, as shown in the diagram.

Another aid to remembering the number of days in each month is this well-known rhyme.

Thirty days hath September
April, June and November.
All the rest have thirty-one
Excepting February alone,
Which has twenty-eight days clear
And twenty-nine in each leap year.

Our calendar is based on the ancient Roman calendar, and the names of the months reflect this.

January - The Roman god Janus was depicted as having two faces looking in opposite directions.

February - In this month (on the 15th), the Romans held a festival of purification known as the Februa.

March - Mars was the Roman god of war. In Europe, March is often a stormy month.

April - The Latin word aperio means "I open". In Europe at this time, plants begin to bud, or open.

May - Romulus, the founder of Rome, gave his name in honour of Maia, who was the mother of the Roman god Mercury. In Europe this was a month of merry-making.

June - Juno was the wife of Jupiter, the chief Roman god.

July - Julius Caesar was a famous Roman soldier, statesman and emperor of Rome.

August - Augustus Caesar was a nephew of Julius Caesar.

Septem, octo, novem and decem mean 'seven', 'eight', 'nine' and 'ten' respectively in Latin.

The original Roman calendar was ten months long. Julius Caesar altered the calendar, and January and February became the first two months of the year.

### Seasons (Introduce Year 4, p. 205)

Australia's position on the globe determines which months each of the four seasons occur. A class chart such as the one below may be used to show the months and seasons. Involve students in finding pictures or making drawings of activities which might take place in the community in those seasons.

Students may paste the pictures (e.g. surfing, people with warm clothing, a rodeo, planting crops) next to the appropriate season/month on the chart, and give reasons why certain activities occur in certain seasons.

The chart provides a visual reminder of how the seasons are organised. Class discussions of the seasons could include concepts learned in social studies and science subject areas.
Dates (Introduce Year 3, p. 157)

Dates may be recorded in a variety of ways. All of the following formats are correct.

- 13 June 1990
- 13/6/90
- 13.06.90
- 13 June, 1990
- Monday, 13 June, 1990

Some computer programs adopt the American style of placing the month first: 6/13/90, meaning 13 June, 1990. Students should be aware of the convention.

The Decade (Introduce Year 4, p. 216)

The word 'decade' comes from the Greek word 'deka' which approximately means 'ten'. Although the term 'decade' is often used to refer to specific 10-year periods such as 1971-1980, 1981-1990, it may also be used in a slightly different context to refer to any 10-year span, e.g. a decade ago, a decade from now. Once the actual years involved in a particular decade have been pinpointed, some of the important events which took place during that period could be identified and discussed.

The Century (Introduce Year 4, p. 216)

Century is derived from the Latin work 'centum' meaning 'hundred'. By convention, the first century is taken to begin at the time of Christ's birth. The second century began at the year 10, the third century began at the year 20, and so on. The twentieth century, therefore, would have begun in 190.

To find out what century a particular year is in, the following process might be used: 1845: It may be seen that 1800 years (18 hundred years) have already passed; therefore 1845 is in the nineteenth century.

Lunar Month (Introduce Year 7, pp. 68-69)

The concept of a lunar month is not a simple one, and is not introduced until Year 7. Therefore, Year 7 activities involve students in investigations and observations about the moon, but details are not explained. The main difficulty lies in the fact that the moon takes about 27.3 days to make one revolution of the earth, but, in this time, the earth has moved along in its revolution of the sun. For the moon to reach the same relative position with respect to the sun takes about 29.5 days. These differences will not be explored, but students can make with own observations of data relating to the influence the moon has on the tides around our shores.
24 Hour Time  (*Introduce Year 7, pp. 67-68*)

24 hour time is formally taught in Year 7. Although students will probably have seen the 24 hour format of recording time, it should not be assumed that they understand the concept. When discussing the concept of 24 hour time including the following points:

- It is quite natural because there are 24 hours in a day;
- The format saves having to include a.m. and p.m. or some other indicator which differentiates before noon from after noon;
- When clarity is essential, the format is very useful because there is only one way to interpret this method;
- The format is used by the armed forces and airlines (though airlines use 12-hour times when communicating with the general public).

Because we need to be able to identify when the 24 hour format is being used, 4 digits are always spoken or written. For example, the time 7:30 may be confused because of the lack of information. In the 24 hour format, it would be:

- 0730 if it were in the morning
- 1930 if it were in the afternoon

There is a particular way of saying the times, and students should become familiar with it. The following examples illustrate the type of language which is used:

- 1000 = 'ten hundred hours'
- 1030 = 'ten thirty hours'
- 1300 = 'thirteen hundred hours'
- 1335 = 'thirteen thirty-five hours'
- 0500 = 'oh five hundred hours'
- 0540 = 'oh five forty hours'
- 0050 = 'double-oh fifty hours'  

*NOTE: Twenty-four hours is the average time from midnight one night to midnight the next night. (*At given point not within the Artic or Antartic Circles*) For teachers, reference only.

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a.m. and p.m. (*Introduce Year 4, pp. 210-211*)

a.m. = anti merediem - before noon
p.m. = post merediem - after midday

<table>
<thead>
<tr>
<th>Years</th>
<th>Recording</th>
<th>Reading</th>
</tr>
</thead>
<tbody>
<tr>
<td>Years 2 and 3</td>
<td>8 o'clock in the morning</td>
<td>eight o'clock in the morning</td>
</tr>
<tr>
<td>Years 4 and 5</td>
<td>8:20 a.m.</td>
<td>twenty minutes past eight in the morning</td>
</tr>
<tr>
<td>Years 6 and 7</td>
<td>8:20 a.m.</td>
<td>8:20 a.m.</td>
</tr>
<tr>
<td>Years 7 and 8</td>
<td>8:20 a.m.</td>
<td>8:20 a.m.</td>
</tr>
</tbody>
</table>

---

A.D. and B.C. (*Introduce Year 6*)

A.D. is the abbreviation for the Latin term ‘anno domini’. It means ‘in the year of the Lord’ or sometimes ‘of the Christian era’, i.e. from the date Jesus was born.

B.C. means before Christ, i.e. before Jesus was born.
Because both analogue and digital clocks are used in everyday life, the children need to be able to read the time from both types of display. Differences in the clocks may be identified as early as Year 1. Where possible identify or represent time on both clocks as opportunities arise.

Where possible, students should have access to 'geared' clocks to show times. On these clocks, the hour hand responds automatically when the minute hand is moved. The hour hand does not point directly at the hour number unless the time is on the hour (o'clock). When the time shows 6:30, for example, the hour hand is halfway between the six and seven. When the time is 6:40 the hour hand should be just over halfway between the six and the seven.

Teachers will need to show and explain the movement of the hour hand to students and help them with the positioning of the hand when home-made (i.e. non-geared) clocks are used to show times.

Activities that involve stopwatches provide students with the chance to use the time units such as seconds, parts of seconds, minutes and, possibly, hours, and to apply skills and concepts learned in other topic areas.

These include:
- place-value concepts, associated with decimal fractions;
- addition, subtraction, multiplication or division of decimal fractions;
- gathering of data (times) that may be displayed on tables or graphs;
- calculation of means, medians and modes;
- predictions that may be made on the basis of collected data.

Using the calendar, a number of facts become quite apparent. There are 12 months in a year and through some calculation, it may be established that there are 365 days in the year. (Leave discussion of leap years until Year 5 unless the year happens to be a leap year).

The number of weeks in the year may be found by tallying up the weeks on the calendar or finding the number of groups of 7 days in 365 days (using the calculator). The fact that there are 52 whole weeks in a year may be discussed for interest only, at this Year level.

The students should always be aware of what year it is through writing and discussing the date (i.e. 23.8.87 or 23 August 1987).
There are, of course, many interesting uses for all kinds of calendars. But many students in the middle years will still need help to interpret the information that is coded or abbreviated in various calendars, e.g. names of days represented by single letters; holidays represented by coloured numbers.

Consequently, until the upper primary years, it may be worthwhile to use teacher/student-made calendars, which are constructed month by month, in preference to commercially produced ones.

Two examples of basic calendar pages are given below.

### Standard format

<table>
<thead>
<tr>
<th>M</th>
<th>A</th>
<th>R</th>
<th>C</th>
<th>H</th>
</tr>
</thead>
<tbody>
<tr>
<td>S</td>
<td>M</td>
<td>T</td>
<td>W</td>
<td>T</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>8</td>
<td>9</td>
<td>10</td>
<td>11</td>
<td>12</td>
</tr>
<tr>
<td>15</td>
<td>16</td>
<td>17</td>
<td>18</td>
<td>19</td>
</tr>
<tr>
<td>22</td>
<td>23</td>
<td>24</td>
<td>25</td>
<td>26</td>
</tr>
<tr>
<td>29</td>
<td>30</td>
<td>31</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Linear format

<table>
<thead>
<tr>
<th>MARCH</th>
</tr>
</thead>
<tbody>
<tr>
<td>Date</td>
</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td>Day</td>
</tr>
<tr>
<td>T</td>
</tr>
<tr>
<td>Weather information</td>
</tr>
<tr>
<td>Other information</td>
</tr>
</tbody>
</table>

### Calculating Ages (Introduce Year 5)

If converting age in months to years and months, the following process is recommended.

110 months

110 + 12

think

9 years

(that's 108 months)

110 months - 108 months

= 2 months

9 years 2 months

The inverse would therefore be:

9 years 2 months

think

9 years

(that's 108 months)

108 months + 2 months

= 110 months

9 x 12 months

*Time - Chapter 21 : 12*
Performing written calculations with time requires students to use the same kinds of renaming and regrouping procedures that have been developed with whole number operations.

Subtracting when both times are a.m. (or p.m.)

Example: It is now 1:45 p.m. How long will I have to wait for a television program that begins at 4:10 p.m.?

<table>
<thead>
<tr>
<th>Working</th>
<th>The student thinks or says</th>
</tr>
</thead>
<tbody>
<tr>
<td>h</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>Take 1:45 p.m. away from 4:10 p.m.</td>
</tr>
<tr>
<td>10</td>
<td>10 minutes take away 45 minutes – I need to rename 1 hour.</td>
</tr>
<tr>
<td>-</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>45</td>
<td></td>
</tr>
<tr>
<td>h</td>
<td>The hour becomes 60 minutes. Add that to the 10 I already have ...</td>
</tr>
<tr>
<td>3</td>
<td>makes 70 minutes.</td>
</tr>
<tr>
<td>70</td>
<td></td>
</tr>
<tr>
<td>A</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td></td>
</tr>
<tr>
<td>-</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>45</td>
<td></td>
</tr>
<tr>
<td>h</td>
<td>70 minutes take away 45 minutes leaves 25 minutes.</td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>70</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
<tr>
<td>20</td>
<td></td>
</tr>
<tr>
<td>-</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>45</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>3 hours take away 1 hour leaves 2 hours</td>
</tr>
<tr>
<td>25</td>
<td></td>
</tr>
</tbody>
</table>

Subtracting when both a.m. and p.m. times are involved

A different situation arises when the times involve both a.m. and p.m. If the times are on the same day, the 24-hour format may be used.

Example: A carpenter began working on a cupboard at 7:50 a.m. and finished it at 4:25 in the afternoon. How long was he on this job?

<table>
<thead>
<tr>
<th>Working</th>
<th>The student thinks or says</th>
</tr>
</thead>
<tbody>
<tr>
<td>h</td>
<td></td>
</tr>
<tr>
<td>16</td>
<td>4:25 p.m. may be written as 16:25.</td>
</tr>
<tr>
<td>25</td>
<td>A procedure similar to the proceeding one is followed.</td>
</tr>
<tr>
<td>-</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td></td>
</tr>
<tr>
<td>50</td>
<td></td>
</tr>
</tbody>
</table>

An alternative to the 24 hour format is the add-on procedure. Students may calculate the period of time from 7:50 to 12:00 (4 hours 10 minutes) and then add on 4 hours 25 minutes to give a total of 8 hours 35 minutes.
Adding on Times

This occurs when there is a specific starting time given, a period of elapsed time and the finishing time is required.

Example: The time was 8:45 p.m. when I began to watch a 95 min video. What will be the time when it finishes?

<table>
<thead>
<tr>
<th>Working</th>
<th>The student thinks or says</th>
</tr>
</thead>
<tbody>
<tr>
<td>h min</td>
<td>95 mins is 1 h and 35 min.</td>
</tr>
<tr>
<td>8 45</td>
<td>I have to add that time on to 8:45 p.m.</td>
</tr>
<tr>
<td>+1 35</td>
<td>45 + 35 = 80</td>
</tr>
<tr>
<td>h min</td>
<td>80 min is 1 h and 20 min.</td>
</tr>
<tr>
<td>1 45</td>
<td>Write another 1 in the hours place.</td>
</tr>
<tr>
<td>+1 35</td>
<td>Write 20 in the minutes place.</td>
</tr>
<tr>
<td></td>
<td>Add 1, 8 and 1.</td>
</tr>
<tr>
<td></td>
<td>The time will be 10:20 p.m.</td>
</tr>
</tbody>
</table>

While it is wise to explore this formal method, other methods may be suggested and discussed with students.

For example:
- The 95 min could be added to the 45 min (140 min) and then this number could be renamed to hours and minutes (2 h 20 min). When added to 8:00 p.m., the final time becomes 10:20 p.m.
- A simple timeline may be used with students using up the 95 min as they move along the line. For example, 15 min would take them up to 9:00 p.m., another 60 min takes them to 10:00 p.m., and then there is only 20 min left. Therefore, the video finishes at 10:20 p.m.

Times on different Days

Another technique is necessary when the times are on different days.

In the following example it is not possible to simply subtract the times as they are. The students may draw a simple timeline (such as that which follows) to help them represent the duration of the journey.

```
Midnight
6 7 8 9 10 11 12 1 2 3 4 5 6 7 8 9 10 11 12
p.m.         a.m.
```

Using this model as a guide, the students may calculate the amount of time passed between 6:25 p.m. and 12:00 midnight, then add on the time till the end of the journey (11 h and 30 min).
These situations are seldom encountered but, once students can cope competently with the addition and subtraction procedures, similar renaming methods will allow them to calculate using multiplication and division.

Conversely, the units involved may be converted to a single unit, operated on and then reconverted to other units if necessary. Students should discuss various procedures and decide on the most appropriate one for them.

- A situation involving multiplication is:
  A factory opened for work at 8:10 a.m. One of its machines could finish a product every 22 min. At what time would it finish the 18th product for that day?

- A situation involving division is:
  A computer printed out copies of the same letter from 10:25 a.m. until 11:15 a.m. If each one took 25 s, how many letters were printed?

**Example:** A family began a journey at 6:25 p.m. and arrived at 11:30 a.m. the next morning. How long did the journey take?

<table>
<thead>
<tr>
<th>Working</th>
<th>The student thinks or says</th>
</tr>
</thead>
<tbody>
<tr>
<td>h min</td>
<td>Work out the time up until midnight.</td>
</tr>
<tr>
<td>12 00</td>
<td>Take 6:25 p.m. away from 12:00 midnight.</td>
</tr>
<tr>
<td>- 6 25</td>
<td>Rename 1 h to become 60 min.</td>
</tr>
<tr>
<td>11 60</td>
<td>60 min take away 25 min leaves 35 min.</td>
</tr>
<tr>
<td>12 00</td>
<td>11 h take away 6 h leaves 5 h.</td>
</tr>
<tr>
<td>- 6 25</td>
<td>5 35</td>
</tr>
<tr>
<td>5 35</td>
<td>Add on the 11 h 30 min to the 5 h and 35 min.</td>
</tr>
<tr>
<td>+ 11 30</td>
<td>35 + 30 = 65 min.</td>
</tr>
<tr>
<td>65</td>
<td>This renames to 1 h and 5 min.</td>
</tr>
<tr>
<td>h min</td>
<td>Cross out the 65 and put 1 in the hours place and 5 in the minutes place.</td>
</tr>
<tr>
<td>1 5 35</td>
<td>Add up the hours.</td>
</tr>
<tr>
<td>+ 11 30</td>
<td>The journey takes a total of 17 h and 5 min.</td>
</tr>
<tr>
<td>65</td>
<td></td>
</tr>
<tr>
<td>17 05</td>
<td></td>
</tr>
</tbody>
</table>
This time is recorded as 8:15 and may be described as:

- 15 minutes past 8
- quarter past 8
- eight fifteen
- 8:15

The recording of the time of day was introduced in Year 2 using the colon (·), and this form is continued through to Year 7. However, from Year 8, students will be using the accepted standard dot symbol (.) to separate hours and minutes. This symbol was not used in early and middle primary years to avoid any confusion with the decimal points. Teachers are invited to introduce the dot symbol to Year 7 students if it is considered appropriate.

**a.m. and p.m.**

- **a.m.** (lower case a, full stop, lower case m, full stop)
- **p.m.** (lower case p, full stop, lower case m, full stop)

The following outline will help you develop consistency throughout your school.

<table>
<thead>
<tr>
<th>Years</th>
<th>Recording</th>
<th>Reading</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 and 3</td>
<td>8 o'clock in the morning</td>
<td>eight o'clock in the morning</td>
</tr>
<tr>
<td>4 and 5</td>
<td>8:20 a.m.</td>
<td>twenty minutes past 8 in the morning</td>
</tr>
<tr>
<td>6 and 7</td>
<td>8:20 a.m.</td>
<td>8:20 a.m.</td>
</tr>
<tr>
<td>8</td>
<td>8:20 a.m.</td>
<td>8:20 a.m.</td>
</tr>
</tbody>
</table>

**Dates**

Dates may be recorded in a number of ways:

- Monday, 13 June 1990
- 13 June 1990
- 13/6/90
- 13.06.90

Some computer programs adopt the American style of placing the month first: 6/13/90 meaning June 13, 1990. Students should be aware of this convention.
Note: no plurals and no full stops.
Abbreviations for other time units may be used, but they are not standard and may vary. For example, Tuesday may be abbreviated as Tues. or Tue.

There is a particular way of saying the times, and students should become familiar with it. The following examples illustrate the type of language which is used:

- 1000 ‘ten hundred hours’
- 1030 ‘ten thirty hours’
- 1300 ‘thirteen hundred hours’
- 1335 ‘thirteen thirty-five hours’
- 0500 ‘oh five hundred hours’
- 0540 ‘oh five forty hours’
- 0050 ‘double-oh fifty hours’

Because we need to be able to identify when the 24 hour format is being used, 4 digits are always spoken or written. For example, the time 7:30’ may be confused because of the lack of information. In the 24 hour format, it would be:

- 0730 if it were in the morning
- 1930 if it were in the afternoon
Most children are familiar with, and quite often use, language associated with the measurement of time long before they have an accurate understanding of its meaning. It is important to note that your references to time should be accurate. This means that a statement such as 'in a minute' should mean exactly that.

Teachers should model the following terms and encourage children to use them appropriately within their own language patterns.

The following year level designations are appropriate for every day language usage, and does not necessarily correspond to the formal introduction of the concept. For year level designations of concept introduction refer to Part 1 of this chapter.

<table>
<thead>
<tr>
<th>YEAR 1</th>
<th>YEAR 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>RELATIONAL</td>
<td>TIME</td>
</tr>
<tr>
<td>first/last</td>
<td>daytime</td>
</tr>
<tr>
<td>long/short time</td>
<td>night-time</td>
</tr>
<tr>
<td>longer/shorter time</td>
<td>morning</td>
</tr>
<tr>
<td>longest/shortest time</td>
<td>afternoon</td>
</tr>
<tr>
<td>time</td>
<td>today</td>
</tr>
<tr>
<td>fast/slow</td>
<td>tomorrow</td>
</tr>
<tr>
<td>POSITIONAL</td>
<td>yesterday</td>
</tr>
<tr>
<td>before/after</td>
<td>Sunday</td>
</tr>
<tr>
<td>first, second... tenth</td>
<td>Monday</td>
</tr>
<tr>
<td></td>
<td>Tuesday</td>
</tr>
<tr>
<td></td>
<td>Wednesday</td>
</tr>
<tr>
<td></td>
<td>Thursday</td>
</tr>
<tr>
<td></td>
<td>Friday</td>
</tr>
<tr>
<td></td>
<td>Saturday</td>
</tr>
<tr>
<td></td>
<td>Week</td>
</tr>
<tr>
<td>RELATIONAL</td>
<td>TIME</td>
</tr>
<tr>
<td>next</td>
<td>day</td>
</tr>
<tr>
<td>before/after</td>
<td>weekend</td>
</tr>
<tr>
<td>beginning/end</td>
<td>month</td>
</tr>
<tr>
<td>earlier/later</td>
<td>January</td>
</tr>
<tr>
<td>older/younger</td>
<td>February</td>
</tr>
<tr>
<td>oldest/youngest</td>
<td>March</td>
</tr>
<tr>
<td></td>
<td>April</td>
</tr>
<tr>
<td></td>
<td>May</td>
</tr>
<tr>
<td></td>
<td>June</td>
</tr>
<tr>
<td></td>
<td>July</td>
</tr>
<tr>
<td></td>
<td>August</td>
</tr>
<tr>
<td>POSITIONAL</td>
<td>September</td>
</tr>
<tr>
<td>eleventh to</td>
<td>October</td>
</tr>
<tr>
<td>thirty-first</td>
<td>November</td>
</tr>
<tr>
<td></td>
<td>December</td>
</tr>
<tr>
<td></td>
<td>year</td>
</tr>
<tr>
<td></td>
<td>clockface</td>
</tr>
<tr>
<td></td>
<td>hands</td>
</tr>
<tr>
<td></td>
<td>this year/next year/last year</td>
</tr>
<tr>
<td></td>
<td>calendar</td>
</tr>
<tr>
<td></td>
<td>digital display</td>
</tr>
<tr>
<td></td>
<td>o'clock</td>
</tr>
<tr>
<td></td>
<td>hour</td>
</tr>
<tr>
<td></td>
<td>minute</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>YEAR 3</th>
<th>YEAR 4</th>
<th>YEAR 5</th>
<th>YEAR 6</th>
<th>YEAR 7</th>
</tr>
</thead>
<tbody>
<tr>
<td>fortnight</td>
<td>decade</td>
<td>leap year</td>
<td>A.D.</td>
<td>lunar month</td>
</tr>
<tr>
<td>second</td>
<td>century</td>
<td>'quarter to'</td>
<td>'anno domin'</td>
<td>24 hour time</td>
</tr>
<tr>
<td></td>
<td>seasons</td>
<td>'minutes to'</td>
<td>'in the year of our'</td>
<td>'hundred hours'</td>
</tr>
<tr>
<td></td>
<td>'quarter past'</td>
<td>Lord</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>a.m. and p.m. to be</td>
<td></td>
<td></td>
<td>Refer to 3h, 3i</td>
</tr>
<tr>
<td></td>
<td>read as in the</td>
<td></td>
<td></td>
<td>for elaboration.</td>
</tr>
<tr>
<td></td>
<td>morning, afternoon/evening.</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

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