Years 1 - 10
Mathematics Syllabus

Support Document (Years 1-7)
for
Queensland Primary Schools

Written by
Paula Anderson
### Book 1: Orange Book

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Support Document (Years 1 - 7)

for

Queensland Primary Schools

This document has been designed to be used in Queensland Primary Schools -

- by *teachers* to supplement their Year Level Sourcebook
- by *schools* to produce school policy documents from extracts
- as a *ready-reckoner* for principals, deputy principals and teachers
- as an *inservice* document
- as a *preservice* document
- as a *year level expectation brief* for relief and supply teachers
- as an *overview* for beginning, returning and support teachers
As anyone who has worked with the Years 1 - 10 Mathematics Sourcebooks would agree, they are very comprehensive documents which are invaluable to teachers in many ways - namely orientating teachers with year level expectations and being an excellent source of a large range of appropriately pitched developmental activities. Because valuable concept development information is woven appropriately throughout the sourcebooks, teachers would, in no doubt, benefit from having ready access to the full complement of sourcebooks, rather than just the year level they are teaching.

Unfortunately, this is not practical or financially feasible, therefore, my global objective in writing this Syllabus Support Document has been to extract all the information from the Year 1 to Year 7 Sourcebooks and to collate this to produce an Overview of each major concept for Queensland Primary Schools.

The ‘snapshot’ of concept development presented in a year level sourcebook will be valuably enriched with this document as it presents the ‘total picture’.

Therefore it is proposed that each teacher receives this document in addition to their mathematics syllabus documents.

An ideal complement of Mathematics Syllabus documents for each teacher would include the:

- Syllabus
- Teaching Assessment & Syllabus Guidelines
- Mathematics Sourcebook
- Ideas from the Sourcebook
- Years 1 - 10 Mathematics Syllabus Support Document for Queensland Primary Schools.

This document is based solely on the philosophy and content of Queensland’s Department of Education, Years 1-10 Mathematics Syllabus, the Years 1-10 Mathematics Teaching Curriculum and Assessment Guidelines and Year 1-7 Sourcebooks and makes direct reference to same. It acts as a ready-reckoner enabling teachers to access relevant information quickly and accurately once they understand the organisation of the document.
Throughout, teachers are constantly being referred to appropriate sourcebook activities.

Access is facilitated by the use of a detailed index and symbols. Each concept is dealt with in a chapter under headings such as:

- **Scope and Sequence**

The Scope and Sequence is a summary which indicates the concepts to introduce in each year level. The major objective of this summary is to promote continuity within schools. In some chapters, for example – Time, sections of the scope and sequence are elaborated on in attachments. The scope and sequence for each of the algorithm and number fact chapters have been comprehensively and sequentially detailed for each year level. They are designed to be photocopied and used as diagnostic checklists or as planning component.

- **Year Level Expectations**

The Year Level Expectation section deals with what breadth and depth concepts are to be explored for each year level in greater detail than the Scope and Sequence Chart. Teachers who are new to a year level or returning teachers will find this section particularly useful as it is designed to give the teacher a feel for the year level. It is recommended that teachers also read either side of their year level to enhance their understanding of the expectations. Please note the Sourcebook references for each year level.

- **Glossary of terms**

Each term specific to the concept development aspect is dealt with in detail in this glossary section. The concept development aspect is discussed at the beginning of each glossary. Note the year of introduction is mentioned where appropriate. Each elaboration of a term is a collection of all the information given on this term from all the Sourcebooks from years 1 to 7. No information given is foreign to the Sourcebooks. This is the section of the Volume chapter you would turn to if you wanted to know:

What is the difference between volume and capacity? What does 'milli' mean again?
• **Reading and Recording**

The major objective of this section is to promote continuity of language and setting out within schools and therefore continuity in a child's learning experiences. This is the section you refer to if you wanted to know how to read and write 24 hour time, how to set out algorithms, how to accurately symbolise units of measure and how to read and write the date correctly.

• **Language**

The language section endeavours to present the words which should become part of a child's everyday vocabulary in each year level. It is important to note that some words may be listed a year or two earlier than the introduction of the concept associated with that word. For example, while January may be listed in the Language section for Year 2, the concept of months is not introduced until Year 3. For this reason the year level of introduction as specified in the Scope and Sequence section of a concept chapter will not necessarily correspond with the language year level listings.

*School policy documents* may be readily produced by writing cover notes for extracts.

It is hoped that educators such as principals and deputy principals, practising, supply, relief, preservice and beginning teachers will all benefit from this document in their endeavours to implement Queensland's Department of Education, Years 1 - 10 Mathematics Syllabus.
Access this book by:

A Turning to a Concept Chapter and referring to the appropriate section using the indicators

- Scope and sequence
- Year level expectation
- Concept development
- Glossary of terms
- Reading and recording
- Language

or

B Referring to the detailed index in the book to quickly access a single topic
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A child's ability to count is dependent on an understanding of the following principles.

- Discrete objects may be counted.
- The conventional number name order is required.
- Each item being counted is tagged with one number name (one-to-one correspondence).
- The total number of items in a group will be the same irrespective of the order in which the items are counted.
- The last number counted tells how many items are in the group.
- The number of elements in a small group may be determined by sight.
- Each number in the counting sequence is preceded by a smaller number and succeeded by a larger number.

Children progress through the following stages when learning to count:

**Immature counting**

When beginning to count, children often use their own repeated sequence before learning the conventional sequence - for example, a young child may count four objects as “1, 3, 5, 9”.

**Rote counting**

The child knows the number names in sequence, but may not be able to point to objects while counting them.

**Point counting**

One-to-one correspondence is exhibited, but the child may not understand yet that the last number counted tells how many elements are in the group.

**Rational counting**

All principles of counting are exhibited. Once rational counting to 20 has been mastered, other counting strategies should be included.

**Counting on**

The child may start at any number and begin counting. Such counting leads to the discovery of many valuable patterns in our number system. It is also a useful strategy for addition.

**Counting back**

The child may count back from a given number. This is a useful strategy for subtraction.

**Skip counting**

Children count by twos, fives and tens. In addition to recognising patterns, this provides valuable readiness for multiplication and division. Skip counting on and back also provides a good base for working with money.
Concrete, Verbal and Symbolic

Children need a sound understanding of number concepts and processes so that they may apply them appropriately in unfamiliar situations. In addition to requiring regular practice and frequent reinforcement of number concepts, children also need to approach these concepts from various perspectives, so that links are established between what they see (concrete), say (verbal) and write (symbolic).

Real World Situation

Concrete (counters, etc)

Symbolic (written form)

Verbal (explaining, acting out)

Give the children continual practice throughout the year to establish the links between the oral, written and concrete forms of numbers.

For the larger whole numbers there should continue to be an interrelationship of materials, language and symbolic representations to ensure continuity of knowledge and understanding of the concept. It is important, therefore, for teachers to continue to provide many concrete experiences with real objects, pictures and diagrams. To understand place value, children must realise the significance of a digit's position in a number - that, for example, in the number 364 the values of the digits are:

3 6 4

4 ones
6 tens
3 hundreds

All children, regardless of year level, should be given as much opportunity as possible to verbalise number as it helps them to work through an understanding of mathematical concepts in their own terms.

Year 3, p. 19
The development of a clear understanding of place value must receive major emphasis. Children apply their understanding of place value concepts when they model, compare, record, count and classify the numbers. Each child (some more than others) requires a great deal of hands-on experience with grouping and regrouping concrete materials. Throughout the years, the use of materials should progress gradually from non structured, (sticks and counters) to structured (Unifix cubes and M.A.B.). In doing so children move from familiar and often everyday objects to materials which have been created to represent different mathematical concepts.

A wide variety of non-structured and structured materials should be manipulated by the children so that meaning is not attached to just one type of material.

Materials used at least until Year 5 should obviously demonstrate grouping in tens. That is, an object representing 10 should be 10 times as long, or ten times as wide or ten times as heavy as the object representing one. For this reason, the abacus is not suitable for use until Year 5.

Examples of non structured materials are:

- toy animals
- stones, macaroni, marbles, counters
- iceblock sticks, toothpicks, straws, matchsticks.

Year 2, pp. 18 - 19.
### Concrete Materials for Number

#### Scope and Sequence

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<th>M.A.B.</th>
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</tr>
<tr>
<td>3</td>
<td>✓</td>
<td>✓</td>
<td></td>
<td>Introduce Yr 3, p. 23</td>
<td>Introduce Yr 3, p. 25</td>
<td>✓</td>
<td>✓</td>
<td>bean sticks, paper squares, paddle pop sticks</td>
</tr>
<tr>
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<td>✓</td>
<td>✓</td>
<td>squared paper</td>
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</table>

*Whole Number - Chapter 1:6*
The use of M.A.B. for number work is not recommended until Year 3 as children cannot unbundle or undo M.A.B. tens to make ten ones. Instead, 10 M.A.B. ones must be exchanged or swapped for a M.A.B. ten, thus requiring a higher level of abstraction. However, children who display competence with place value concepts may enjoy exploring the relative value of M.A.B. tens and ones, and converting a number of ones into tens and ones.

Year 2, p. 19

M.A.B. is considered a more efficient way of representing numbers beyond 99 than bundling sticks, hence their introduction in Year 3 where numbers to 999 are studied. By introducing M.A.B. in Year 3 it is hoped children will be able to handle the more complex concept of trading.

Introducing M.A.B.

1. As with all new concrete material - children should be able to experiment with the new materials in their own way before structured play begins.

2. Establish the names of individual pieces. (one, ten, etc).

   Involve the children in building towers containing, for example,
   • 4 ones and 3 tens;
   • 7 ones and 6 tens;
   • 9 tens and 2 ones.

3. Engage children in a variety of structured play activities.

4. Explore relationships between M.A.B. i.e. Trading Game.

5. Represent two digit numbers with M.A.B.

6. Count with M.A.B. It is important that children have many opportunities to count with M.A.B. so that they become familiar with the material. This will enable them to learn various ways of counting, such as in twos.

Year 3, p. 23

When using M.A.B. from Year 3 on, the children should be involved in the trading process, for example:

Put out 12 tens
Are there enough tens to make a hundred?
How many tens are left?
What is the number

The children should also understand the pattern of the trading process:

• 10 ones is the same as 1 ten;
• 10 tens is the same as 1 hundred.
The following represents how a 3 digit number may be modelled with M.A.B. on a place value chart.

1. Model a number for the children with M.A.B., for example 475. Investigate the number by asking the following questions:
   - *How many hundreds did you use to make the number?* (Record the reply on a place-value chart on the chalkboard.)
   - *How many tens did you use to make the number?* (Record the reply on the chart.)
   - *How many ones did you use to make the number?* (Record the reply on the chart.)

   Involve the children in linking the numbers on the chart with the appropriate M.A.B.:
   - Point to the 7 on the chart.
     - *What is the value of this digit?* (The children say "7 tens").
     - *Show me the 7 tens with the M.A.B.*

   - Repeat for many three-digit numbers. Give special attention to numbers with zero tens and/or ones.
   - Write a number on the place-value chart. Have the children model the number with M.A.B.

<table>
<thead>
<tr>
<th>Hundreds</th>
<th>Tens</th>
<th>Others</th>
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</thead>
<tbody>
<tr>
<td>4</td>
<td>7</td>
<td>5</td>
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It is recommended that M.A.B. be described in terms of the respective value of the blocks, rather than by the terms *shorts*, *longs* and *flats* and *blocks*.
Number Expanders (Grouping)  (Introduce Year 3)

The number expander provides a way of demonstrating the language implicit in the place value system: it is also an excellent aid for demonstrating the grouping and regrouping of numbers. It is essential that children have an excellent understanding of place value and be able to regroup numbers in as many ways as possible if they are to group within algorithms with understanding.

**Step 1**

Cut out a rectangular piece of thick paper 36 cm long and 4 cm wide. Shade and print "tens" and "ones" as shown:

```
4 cm  |   tens   | 36 cm
     |   ones   |
```

**Step 2**

Fold together the shaded parts as shown.

```
[Diagram showing folded paper with "tens" and "ones" shaded areas]
```

The number expander should now look like this:

```
[]   []
```

**Step 3**

In light pencil, write any two digits on your folded expander:

```
7   3
```

**Step 4**

Now open out your expander. Do you see the value of each digit you write?

```
7   tens   3   ones
```
The blank squares on the number expander may be covered with Contact or Magic Transparent Tape for durability. The children may then write on the expander with water-based pen. Encourage the children to use the number expander to investigate numbers through a sequence like the following:

**Write the number 89 on the joined blank square of your number expander.**

![Number Expander with 89](image)

*Open your expander right out.*

![Number Expander with Tens and Ones](image)

*How many are in the tens place in 89?*

*Point to the part that tells you the number of tens.*

*How many are there in the ones place in 89?*

*Point to the part that tells you the number of ones.*

*Close your expander.*

*Open your expander as shown below.*

![Number Expander with Ones](image)

*How many ones are there in 89?*

*Link the numbers written on the number expander with modelling by using MAB, and with representing by using calculators.*

*Grouping and regrouping of 526 may be*
  * 526 ones
  * 5 hundred 2 tens 6 ones
  * 52 tens 6 ones
  * 5 hundreds 26 ones

*The number expanders are excellent to help reinforce this regrouping of numbers and are suitable for 2 digit numbers onwards, including decimal fractions.*

**Whole Number - Chapter 1 : 10**
Number Lines *(Introduce Year 3)*

When using number lines make sure the number line is large enough for all the graduations to be clearly visible and that children count spaces and not lines. Relate the use of a ruler to the concept of number line.

Number lines may be used to show the positions of whole numbers and fractions. This positioning of numbers may not be easily understood and carried out by all students.

Ensure that students become competent at marking the halfway point for a number as the halfway point serves as a reference point for marking other numbers on the line. Discuss marks once the halfway point is established.

Before these reference points may be of use, students must understand the size of the number between each major reference mark. In the previous example, the number is 1 000. Have the students mark the "quarter" or "tenth" points using their estimation skills. Each half then will be at 500, each quarter at 250 and each tenth will be at 100. Once students are successful with number lines beginning at zero, introduce number lines which begin at other numbers.
The use of the abacus requires a higher level of abstraction than materials such as M.A.B. because the bead which represents 10 is not 10 times the size of the bead which represents one. With number study reaching 99,999 in Year 5, the need to use the abacus becomes apparent as numbers become too large to be modelled practically with M.A.B.

Until Year 5, materials referred to in the Sourcebooks to show numbers have been of the type that represent "one-for-one" relationships. When M.A.B. is used, for example, each individual piece of wood shows the actual value of the place name. The blocks are of different sizes to represent the different values.

Materials such as the abacus which do not show this direct relationship may be used to help develop place-value ideas. The abacus draws upon the "one-for-many" relationship. Position is critically important when using this device because all beads are the same size. The position of the beads indicates place value.

The abacus may be effectively used to extend students' understanding of larger numbers in our number system. It requires students to use their knowledge of place value to talk about and to write numbers. The abacus may also enhance students' understanding of regrouping. With M.A.B., when blocks are regrouped - for example, 10 ones for 1 ten - physical attributes such as length and size remain the same. On the abacus, however, 10 beads in one place have the same value as one bead in the next place to the left. The physical attributes do not look the same.

This distinction needs to be made very clear to students. After familiarising students with the use of the abacus with smaller numbers, have them represent a variety of numbers in the ten thousands on an abacus.

As a variation, have students make drawings of abacuses and colour in beads to represent given numbers. Numbers could be written in words or figures. To further develop the regrouping idea, use the abacus as a counting device and have students count in tens, hundreds and thousands, regrouping the beads where appropriate.

Note:
Although the use of M.A.B. with five-digit numbers tends to become unwieldy, it is possible to use this resource successfully to introduce such numbers.
Progression through place-value ideas is dependent upon the children's understanding.

Children should display a clear understanding of each place-value idea, and competency in manipulating materials to represent the idea, before moving on to bigger numbers. Only then will children build a firm understanding of this complex concept, which is then applied to progressively larger numbers in each successive year level.

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<th>YEAR</th>
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<tr>
<td>1</td>
<td>1.0-10</td>
<td>1. Concept and language from 0-10</td>
</tr>
<tr>
<td></td>
<td>limits are not specific, the selection of numbers larger than 10 should be dependent upon the children's ability and interest.</td>
<td>3. Numeral from 0-10.</td>
</tr>
<tr>
<td>2</td>
<td>1. Revise 0-10 2. 10, 20, 20, 30, 40 3. 20, 30, 50 4. 21-99 5. 11-19 6. Numbers larger than 99 may be mentioned incidentally but they should not be analysed until children have a firm understanding of two digit numbers.</td>
<td>The following is the suggested developmental sequence for the associated language and representation of numbers to 99. This sequence introduces children to regular number names first, rather than progressing through the whole numbers from one to 99 in that order. 1. Oral and written descriptions of groups of 10. 2. Oral and written descriptions of tens and ones. 3. Use of the number names of the multiples of 10 orally. 4. Use of the number names of the multiples of 10 written in word form. 5. Use of the number names of numbers comprising tens and ones orally. 6. Use of the number names of numbers comprising tens and ones in word form. 7. Use of the number names of numbers comprising tens and ones written in digit form. 8. Use of the number names of numbers comprising tens and ones (for the numbers 11 to 19, and numbers with 0 and 1 in the ones place) orally and written in word then digit forms.</td>
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<td>Year 2, p. 18</td>
<td>Year 2, p. 20</td>
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</table>

The numbers from 11 to 19 are investigated separately because of the following unique features:
• They all contain two digits but are single-word names.
• Their names do not begin with the number of tens they contain.
• Their names end with 'teen' instead of 'ty', except for 11 and 12. |

Year 2, p. 34
<table>
<thead>
<tr>
<th>YEAR</th>
<th>OVERVIEW</th>
<th>IN DETAIL</th>
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</thead>
<tbody>
<tr>
<td>3</td>
<td>1. Revise 0-99</td>
<td>The teen numbers are treated as a separate activity because of language inconsistencies which present difficulties for some children. At all times emphasis must be given to relating the verbal and symbolic aspects of these numbers. Numbers with internal zero are treated last because the children need to thoroughly understand place-value concepts to consider zero as a place holder in a number. Some children ignore it altogether and read a number such as 306 as thirty-six.</td>
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<tr>
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<td>2. Concept of 100</td>
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<td></td>
<td>3. Multiples of 100</td>
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<td>4. 120-199, 220-299, 320-399, ... 920-999</td>
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<td>5. 110-119, 210-219, 310-319, ... 910-919</td>
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<td>6. 101-109, 201-209, 301-309, ... 901-909</td>
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<td>4</td>
<td>1. Revise numbers to 999</td>
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<td>2. Numbers to 9,999</td>
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<td>5</td>
<td>1. Revise numbers to 9,999</td>
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<tr>
<td></td>
<td>2. Numbers to 99,999</td>
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<tr>
<td>6</td>
<td>1. Revise numbers to 99,999 beyond</td>
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</tr>
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<td></td>
<td>2. Numbers to 1,000,000</td>
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<td>7</td>
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Whole Number - Chapter 1 : 14
Children, while developing their understanding of numbers to 10 should develop their ability to identify numbers represented by patterns and pictures by sight alone. Yrs 1, pp. 148-149

Some recommended patterns are:

- natural three patterns:

- Montessori patterns:

- tally marks:

- domino five patterns:

- pictures:

The numbers from zero to four may be identified without counting or partitioning. Numbers larger than four need to be counted or partitioned to be identified. Use the cards showing the numbers from zero to four as flashcards. The children should be able to identify the number on each card as the sheets are quickly flipped. Alternatively, ask the children to hold up their copy of card you display, or to hold up a card showing the same number from a set displaying a different pattern.
The preferred style

798    seven hundred and ninety-eight
2 039   two thousand and thirty-nine
3 001   three thousand and one
203 930 two hundred and three thousand, nine hundred and thirty
120 058 one hundred and twenty thousand and fifty-eight

To maximise students' understanding of number, it is imperative that structured materials be used to illustrate the relative value of each digit. To understand place value, students must realise the significance of a digit's position in a number, e.g. in the number 264 573 the values of the digits are:

```
            3 ones (3)
              7 tens (70)
                5 hundreds (500)
                  4 thousands (4 000)
                    6 ten thousand (60 000)
                      2 hundred thousand (200 000)
```

When writing numbers of four digits or greater, a 'space' rather than a comma is to be used between the hundreds and thousands digits, e.g. 2 468.

Initially, students could use place-value headings to help in writing larger numbers. Emphasise how the spacing of the digits separates the thousands from the rest of the digits and helps the numbers to be read more easily. In the number '14 325', for example, the grouping to the left of the space is read "fourteen thousand" and the grouping to the right of the space, 'three hundred and twenty-five'.

Year 5, p. 17
‘Milli’ means a thousand. The word million came from the Italian word ‘mille’ meaning a large thousand, i.e. a thousand thousand. Students may be interested to know that the term million was first used in Italy in the 1400s to indicate the value of 10 barrels of gold. Until that time there had been no need to use such a large number.

The concept of a million (or more) objects is difficult even for adults to grasp; materials are still used to show large numbers.

Because confusion may result from the inappropriate use of the terms ‘billion’ and ‘trillion’, do not use them if at all possible. Either use the relevant decimal factor, or write out the amount in full, as in the following table, which shows the approved definitions used in Australia and the United Kingdom.

<table>
<thead>
<tr>
<th>Term</th>
<th>Significance</th>
<th>Corresponding power of ten</th>
</tr>
</thead>
<tbody>
<tr>
<td>million</td>
<td>thousand x thousand</td>
<td>$10^6$</td>
</tr>
<tr>
<td>billion</td>
<td>million x million</td>
<td>$10^{12}$</td>
</tr>
<tr>
<td>trillion</td>
<td>million x billion</td>
<td>$10^{18}$</td>
</tr>
<tr>
<td>quadrillion</td>
<td>million x trillion</td>
<td>$10^{24}$</td>
</tr>
</tbody>
</table>

A different convention is used in the USA where:
‘million’ signifies a thousand thousands ($10^6$)
‘billion’ signifies a thousand millions ($10^9$)
‘trillion’ signifies a million millions ($10^{12}$)

Point out to the students that the media in Australia tend to follow the USA convention for defining a billion (i.e. 1 000 000 000) rather than the internationally approved definitions. Review with students the place-value patterns for describing each number:

1 000 000 is the same as
1 thousand thousands
10 hundred thousands
100 ten thousands
1 000 thousands
10 000 hundreds
100 000 tens
1 000 000 ones
Look at patterns with more than one in a place:

500 000 is the same as
5 hundred thousands
50 ten thousands

When introducing 1 000 000:

Review with students the 'grouping' pattern upon which our numeration system is based. Use a place-value chart to show the ones group, thousands group and millions group.

Some students find it easier to describe place values if they use a number expander or abacus.

26 892 457

Put several abacuses side by side to obtain the required number of places. In order to provide interesting and relevant contexts for the activity, use numbers from population statistics, distances or similarly everyday applications.

Have students write numbers in expanded form and vice versa. For example:

1 389 246 = 1 000 000 + 300 000 + 80 000 + 9 000 + 200 + 40 + 6

To vary the translation of the expanded form to standard form, allow students to enter the 'parta' (e.g. millions, tens) into their calculators in order to make up the given number.
# Number Subgroup Scope and Sequence

<table>
<thead>
<tr>
<th>Year 1</th>
<th>Year 2</th>
<th>Year 3</th>
<th>Year 4</th>
<th>Year 5</th>
<th>Year 6</th>
<th>Year 7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ordinal Number</td>
<td>To tenth</td>
<td>1. To thirty-first for calendar days 2. To ninety-ninth (word &amp; digit forms)</td>
<td>Year 2, p. 63-69</td>
<td>Increased in accordance with counting limits</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Odd/even numbers</td>
<td>Introduce</td>
<td>Year 2, p. 49</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Equal &amp; not equal</td>
<td>Introduce</td>
<td>Year 3, p. 46</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Multiple &amp; factors</td>
<td>Introduce</td>
<td>Year 4, p. 28</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>&gt; &lt; greater than less than</td>
<td>Introduce</td>
<td>Year 5, p. 18</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Prime &amp; composite</td>
<td>Introduce</td>
<td>Year 5, p. 21</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Square numbers</td>
<td>Introduce</td>
<td>Year 5, p. 21</td>
<td>Relate square &amp; triangular numbers</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Factor trees</td>
<td>Introduce</td>
<td>Year 5, p. 21</td>
<td>Year 6, p. 18</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Palindromic numbers</td>
<td>Introduce</td>
<td>Year 5, p. 21</td>
<td>Year 6, p. 23</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Triangular numbers</td>
<td>Introduce</td>
<td>Year 5, p. 21</td>
<td>Year 6, p. 19</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Exponential notation</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Square root</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Introduce</td>
</tr>
</tbody>
</table>

Whole Number - Chapter 1: 19
Ordinal number expresses both order in position - for example, second place - and order in time - for example, second turn. Before identifying an ordinal number, a starting point must be established. Children should gain experience in using different starting points and in following different counting directions, such as left to right, right to left, top to bottom, bottom to top, clockwise and anticlockwise.

The range of ordinal numbers is gradually increased in accordance with the current counting limit - in word form and in digit form.

Odd and Even Numbers  *(Introduce Year 2, p. 49)*

Pictorial representation of numbers may help children understand odd and even numbers, for example:

Montessori pattern

\[
\begin{array}{cccccccccc}
1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\
\bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\
\end{array}
\]

These patterns could be displayed around the classroom.

Number lines may be used to further illustrate the pattern of odd and even numbers, thus:

\[
\begin{array}{cccccccccc}
20 & 21 & 22 & 23 & 24 & 25 & 26 & 27 & 28 \\
\end{array}
\]

Remind the children that _odd and even_ numbers are only ever whole numbers.
Equal and Not Equal Numbers *(Introduce Year 3, p. 46)*

From Year 3, discuss with the children the concept of equal, meaning the same value, and not equal (unequal), meaning not the same value. Situations such as the following could be discussed:

- an equal number of children in Year 3A and Year 3B;
- unequal numbers of boys and girls in Year 3A;
- unequal numbers of male and female teachers at our school.

The children could discuss and then represent the situation to verify the equality or inequality. For example:

<table>
<thead>
<tr>
<th>Girls</th>
<th>Boys</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>5</td>
</tr>
</tbody>
</table>

Multiples and Factors *(Introduce Year 4, p. 28)*

**Multiple**

A multiple of a given number is the given number multiplied by an integer.

Example: Multiples of 2 are 2, 4, 6, 8, 10, ...

**Factors**

Factors are all the whole numbers that may be divided exactly into another number.

Example: 1, 2, 4, and 8 are factors of 8.

8 ÷ 1, 8 ÷ 2, 8 ÷ 4, 8 ÷ 8

'Greater than' and 'Less than' > < *(Introduce Year 5, p. 21)*

The symbols > and <

The symbols of inequality (>, <) are introduced in the Year 5 Sourcebook as ways of writing 'greater than' and 'less than'. Remind students who have difficulty remembering what the symbols represent that the 'arrowhead' should point to the smaller number.
Composite Number - A Composite Number is a whole number which has factors other than itself and one. A composite number is not a prime number.

Prime Number - Prime numbers are numbers that have no factors other than themselves and 1. The smallest odd prime number is 3. The only even prime number is 2. Note: 0 and 1 are usually considered to be neither prime nor composite.

The composite numbers all have a special property which may be easily demonstrated using concrete materials. When a composite number of objects (cubes, counters) is arranged as an array of equal rows, it is possible to make two or more arrangements. For example, the composite number 6 may be represented as:

- $1 \times 6$
- $2 \times 3$

Explain that numbers which may only be shown as two arrays are called prime numbers.

Lead students to generalise that:
- a prime number is a whole number that has only two factors (itself and one);
- a composite number is a whole number that has more than two factors.

Students should realise that only one arrangement may be made with one counter. During discussion ensure the following points are made:
- 1 has only one factor - itself
- prime numbers have only two factors
- composite numbers have more than two factors
- no arrays at all may be made to represent zero

Lead children to conclude that 1 and 0 are special numbers that cannot be classified as prime or composite.

Square Numbers - Square numbers are generated by multiplying numbers by themselves, for example, $2 \times 2 = 4$ so four is a square number.

Patterns related to square numbers help students learn multiplication facts and gain a better understanding of the number system. Students must realize that the number of objects used to represent a square number must be able to be arranged in a square shape.
Factor Trees (Introduce Year 6, p.18)

Introduce students to the idea of a "factor tree" as a way of representing the prime factors of a number. Show that as factors are identified, the tree will "branch out" until prime numbers appear at the bottom as the "roots" of the tree. These "roots" of the tree are the prime factors of the number.

Factor trees may also be used to consolidate the relationship between factors and products, i.e. the tree may be built by starting from the "roots" or factors. Have students build factor trees from their "roots".

After a number of trees have been built, involve the students in comparing and discussing the bottom rows of a number of trees. Students should be able to conclude that if trees share the same number at the top, they should always have the same set of factors in the bottom row i.e. prime factors.

Palindromic Numbers (Introduce Year 6, p. 23)

A palindrome is a word or number that reads the same backwards as forwards. Introduce and discuss palindromic words, such as "mum", "radar", "pup", and compare these to palindromic numbers e.g. 44, 7667, 34 543 and 2 683 862. A palindromic number may be formed from any number by following the procedure:

- Write down a number (or use a calculator).
- Reverse the order of the digits.
- Write down that number beneath the original number.
- Add the two numbers.
- Repeat the procedure until a palindromic number is formed.

\[
\begin{align*}
48 + 84 &= 132 \\
132 + 231 &= 363
\end{align*}
\]
Triangular numbers are formed by adding consecutive whole numbers with the number "1" being the first triangular number.

Through their investigations students should discover that the difference between each consecutive pair of triangular numbers is a consecutive whole number.

Triangular numbers may be related to square numbers. Allow students to investigate their relationship by having them:

- build square numbers using counters or pegs and pegboards;
- divide each square into two triangles using string and explain their findings;
- create a sequence of square numbers;
- explain a relationship between square numbers and triangular numbers from the sequence;
- investigate numbers beyond 100 to find out if the pattern still applies.
Exponential notation uses index figures (exponents) to show the power to which a number is raised. The exponent is a short way of showing the number of times the base number is multiplied to give the product.

\[ 5 \times 5 \times 5 \times 5 = 5^4 \]

5 is a factor

base

exponent

Read as "five to the fourth power", "the fourth power of five" or "five to the fourth".

When writing a number in exponential notation, the exponent should be about half the size of the base figure and placed slightly above and to the right of it. Only positive and zero exponents should be considered at this level.

Powers of ten

Introduce students to the convention of writing powers of ten (1, 10, 100, 1000,...) using a base figure of 10 and an exponent to indicate the number of times 10 is a factor of the number.

For example: \[ 100 = 10 \times 10 \]
10 is a factor of 100 twice and may 100 be expressed as \[ 10^2 \]
Zero power

(i) Many children do not develop this concept readily. It should be introduced by way of an investigation which helps students to determine the idea for themselves.

Firstly, establish on the calculator that:

\[ 5^4 = 5 \times 5 \times 5 \times 5 = 625 \]

Use the following steps to lead to the idea that \( 5^0 = 1 \).

<table>
<thead>
<tr>
<th>What does ( 5^4 ) equal?</th>
<th>5^4</th>
<th>625</th>
</tr>
</thead>
<tbody>
<tr>
<td>Divide by 5. Record the results</td>
<td>5^3</td>
<td>125</td>
</tr>
<tr>
<td>Divide by 5 again, and again, and again.</td>
<td>.....</td>
<td>.....</td>
</tr>
</tbody>
</table>

(ii) When 10 is used as the base, a special pattern occurs when it is raised to various powers. Comment on ten as a special base in that the number of zeros relates to the power or exponent.

Note the following:

\[ 1000000 = 10 \times 10 \times 10 \times 10 \times 10 = 10^6 \]
\[ 100000 = 10 \times 10 \times 10 \times 10 \times 10 = 10^5 \]
\[ 10000 = 10 \times 10 \times 10 \times 10 = 10^4 \]
\[ 1000 = 10 \times 10 \times 10 = 10^3 \]
\[ 100 = 10 \times 10 = 10^2 \]
\[ 10 = 10 = 10^1 \]
\[ 1 = 1 = 10^0 \]

Note the following:

\[ 1000 = 10^3 \] (3 zeros, exponent 3)
\[ 100000 = 10^5 \] (5 zeros, exponent 5)
\[ 1 = 10^0 \] (0 zeros, exponent 0)

Note:
Any number to the zero power is one; any number to the power of one is equal to that number itself.

i.e. \( 2^0 = 1 \)
\( 2^1 = 2 \)

\( 10^2 \) is read as "ten to the second power", "ten squared" or "square of 10"

\( 10^3 \) may be read as "ten to the third power" or "ten cubed"

Use the M.A.B. flat to investigate \( 10^2 \) and the M.A.B. block to investigate \( 10^3 \)
The language and notation for square roots may be introduced in Year 7. It is important to establish the relationship between knowing the square of a number and finding the square root. Discuss how the idea of square roots may be useful if we know the area of a square region and want to find the length of the sides of the region.

\[ \begin{align*}
1^2 &= 1 & \sqrt{1} &= 1 \\
2^2 &= 4 & \sqrt{4} &= 2 \\
3^2 &= 9 & \sqrt{9} &= 3 \\
4^2 &= 16 & \sqrt{16} &= 4 \\
5^2 &= 25 & \sqrt{25} &= 5
\end{align*} \]

When students are calculating square roots beyond the limits of their basic facts, opt for estimation and calculator procedures in preference to pen and paper factorisation methods.
## Approximating Numbers

### Scope and Sequence

<table>
<thead>
<tr>
<th>Year</th>
<th>Activity</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Year 1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Year 2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Year 3</td>
<td>Numbers to the nearest 10</td>
<td>Year 3, pp. 48-52</td>
</tr>
<tr>
<td>Year 4</td>
<td>Numbers to the nearest 10 and 100</td>
<td>Year 4, pp. 32-36</td>
</tr>
<tr>
<td>Year 5</td>
<td>Approximating numbers to the nearest 10, 100, 1000 and 10000</td>
<td>Year 5, pp. 24-26</td>
</tr>
<tr>
<td>Year 6</td>
<td>Approximating whole numbers as appropriate</td>
<td>Year 6, pp. 20-22</td>
</tr>
<tr>
<td>Year 7</td>
<td>Approximating as required by practical situations</td>
<td>Year 7, pp. 22-26</td>
</tr>
</tbody>
</table>

**Note:** Some approximation and rounding strategies cannot be taught in isolation from the operations, e.g., compatible numbers.
Children should always be encouraged to estimate before counting from Year 1. Children need to develop a "feel" for numbers and establish a basis for "sensing" where a certain number lies in relation to those around it, realizing, for example, that 128 is closer to 130 than to 120.

Practical applications of approximating numbers in all grades should be carried out so that the activity becomes *a means to an end and not an end in itself*. Efforts should be made to provide realistic applications and reasons for rounding numbers. In isolation, rounding numbers provides little incentive beyond mere practice of rules. Most practice should be done in conjunction with algorithm and calculator work, estimation, applications with money and measures or in conjunction with some games or puzzles.

To further develop the idea of rounding, discuss the concept of "halfway point" in relation to the following diagram. An aircraft which develops mechanical trouble at or beyond the halfway reached the "halfway point" it will return to the place where it began its journey. A similar situation exists with the rounding of numbers. The "halfway point" corresponds to "5", "50" or "500" when rounding tens, hundreds and thousands. Highlight the convention of always rounding the first five numbers down and the last five numbers up.

<table>
<thead>
<tr>
<th>Departure Point</th>
<th>Engine trouble here, so turn back</th>
<th>Destination</th>
</tr>
</thead>
<tbody>
<tr>
<td>Halfway</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Departure Point</td>
<td>Point of no return</td>
<td>Destination</td>
</tr>
<tr>
<td>Halfway</td>
<td>Engine trouble here, so continue to destination</td>
<td></td>
</tr>
</tbody>
</table>

Students should understand that approximating numbers is essential to the process of estimating. It is important to emphasise that approximating is a very quick mental procedure which maintains the magnitude of original numbers.

Activities selected need to show that approximations should only be used if they are:
- easy to work out
- easier to comprehend than the original data
- capable of being held in the mind (even if only for a short time)

If the above points cannot be achieved, the original data should be used.

The importance of teaching approximations effectively and well is reflected by the fact that far more estimations are done in real life than actual precise calculations. *Year 6, p. 20*

A number that is a multiple of another is referred to as a *compatible number*. Depending on the situation, a number may be approximated to a more compatible one. If 31 495 is to be shared among eight people, the procedure is as follows:
- Look at 31 495
- Decide that 30 000 maintains the original size.
- Round the number to 32 000 because it is easier to divide by 8.
Point out to children that, for example, when rounding off to the nearest 100, digits to the right of tens column may be ignored and when rounding to the nearest 1000, digits to the right of the hundreds column may be ignored.

Also encourage children to decide which two multiples the number to be rounded off lies between. For example, when rounding to the nearest 100, students must first decide which two multiples of a hundred the number lies between. 789 lies between 700 and 800. 2 361 lies between 2 300 and 2 400.

Although numbers may be approximated by round them to the nearest 10, 100, 1 000 or 10 000, students should understand that this is not always the case. Discuss some real-life situations that involve approximations which do not strictly adhere to 'rounding' rules. Some of these examples, such as the following, might include fractions.

- When approximating a person's age we might say the person is 'in her twenties' when her age is 28.
- Attendance at a concert might be estimated at 'close to 150' when there were 127 people who attended.
- A sporting match which lasted 2 hours 18 minutes may be said to have lasted two and a quarter hours.
- A cost of $3 300 could be rounded off to three and a half thousand dollars.
- When estimating 38c × 16, 38c could be rounded to 50c.
- When approximating the crowd at a sporting fixture, for example, 22 475 could be expressed as 'over 22 000'.

Lead students to recognize when to round and how. The activities in the sourcebooks are designed to give students confidence in making adequate approximations quickly.

Draw attention to the fact that the size of a number tends to govern how it is approximated. Large numbers, for example, change little in complexity if rounded to tens or hundreds and so should be rounded to larger places. The more places there are in the number, the less difference 'chopping' places makes to the approximation. It makes relatively little difference, for example, to approximate 381 862 as 380 000, by 'chopping' the last 4 digits.

Have students discuss this 'chopping' aspect with numbers of different sizes by asking them how many digits could be safely ignored with three-digit, four-digit, five-digit or six-digit numbers.

In year 6 the rounding process may be connected to the 'chopping' process. Once significant figures have been decided for approximating a number, rounding may be used to further approximate the number – for example, 28 494 may be approximated to 28 000 to maintain the original magnitude, then rounded to 30 000.

This should be a rapid mental process but will need to be carried out verbally for some time until students are familiar with the pattern of thinking.
Number lines are useful to help discuss the rounding of numbers. For example - to round 5 400 to the nearest 1 000, have students:

- Mark the halfway point.
- Mark 5 400 on the number line.
- Discuss whether 5 400 is closer to 5 000 or 6 000. (Relate to the half-way point.)
- Mark points which represent 5 410, 5 490, 5 450, 5 403.
- Discuss why there is no difference in the rounded answers.

Note that you round down for the first four numbers and round up for the last four numbers.

```
Round down   Round up
0 1 2 3 4 5 6 7 8 9
```

Some students may find number expanders helpful as they round numbers. For example, to round 632 298 to thousands:

Direct students to look at this part of the number. Because 298 is less than half way (500), the approximation becomes 632 thousand.

Discuss with the children how rounding money works in practice. When shopping for a 54 cent item, it is not appropriate to round to 50c and only take that amount. Rounding is useful only for approximate figures. When purchasing, it is necessary to meet the exact amount, so it is safer always to round up. Whether it is appropriate to round up or to round according to the usual rules when dealing with money depends on the circumstances. Explore some of these different circumstances with the children, for example trading, buying, selling or giving change in cents or whole dollars. Ask them to work mentally, with written algorithms and also with calculators.

Year 3, p. 51
By investigating other systems of numeration, students may gain a better appreciation of our own system. In the Year 6 Sourcebook the Roman, Egyptian and Babylonian systems were studied. Included in the Year 7 book are ideas for investigating an oriental system and one of the many Aboriginal systems.

There is no need for children to memorise the various symbols and names - rather understand the principles of the systems.

**The Roman Number System (introduce Year 6, p. 22)**

**Discussing the Roman System**

When introducing this system, some of the historical background for the development of the symbols should be given. Discuss the following aspects:

- The symbols I, II, III, IV, V for one, two, three, four, five probably originated to reflect how these amounts could be shown using fingers.

With the later introduction of the principle of deducting a number from a greater number (subtractive principle), four was recorded as IV.

**Note:** The majority of watch manufacturers today prefer the III symbol for watches which use a Roman numeral display.

- The origin of the symbol for ten (X) is not clear and a number of explanations are possible:
  - When both hands are used to show 2 fives, the thumbs may cross to form the X for ten.
  - 2 five symbols, one on top of another, form X.
  - The tally system, used for grouping in tens may have been abbreviated.
The principle of addition was initially used for the symbols six to nine, i.e. VI, VII, VIII, VIII. Later the 'subtractive' principle was used, and nine was represented as IX and four as IV.

The symbol C used to represent 100 comes from the first letter of 'Centum', the Roman word for hundred.

The symbol M for 1000 comes from 'Mille', the Roman word for 1000.

Although the Roman system did not have place value, it did have a grouping scheme – for example, Arabic 323 is equal to Roman CCCXXIII.

A special feature of the Roman system is a symbol for halfway point of main number groupings, i.e. V = 5, L = 50, D = 500. This is similar to keeping a reference point when counting on the fingers of one hand and has the important advantage of making numerals easier to recognise.

Year 6

A chart similar to the following should be available to students as they are undertaking these activities. Making such a chart could be a class activity.

<table>
<thead>
<tr>
<th></th>
<th>Roman</th>
<th>Egyptian 7b</th>
<th>Babylonian 7c</th>
<th>Arabic</th>
</tr>
</thead>
<tbody>
<tr>
<td>Zero</td>
<td></td>
<td></td>
<td></td>
<td>0</td>
</tr>
<tr>
<td>One</td>
<td>I</td>
<td>I</td>
<td>Y</td>
<td>1</td>
</tr>
<tr>
<td>Two</td>
<td>II</td>
<td>II</td>
<td>II</td>
<td>2</td>
</tr>
<tr>
<td>Three</td>
<td>III</td>
<td>III</td>
<td>YYYY</td>
<td>3</td>
</tr>
<tr>
<td>Four</td>
<td>IV</td>
<td>III</td>
<td>YYYY</td>
<td>4</td>
</tr>
<tr>
<td>Five</td>
<td>V</td>
<td>III</td>
<td>YYYY</td>
<td>5</td>
</tr>
<tr>
<td>Six</td>
<td>VI</td>
<td>III III</td>
<td>YYYY</td>
<td>6</td>
</tr>
<tr>
<td>Seven</td>
<td>VII</td>
<td>III III III</td>
<td>YYYY</td>
<td>7</td>
</tr>
<tr>
<td>Eight</td>
<td>VIII</td>
<td>III III III</td>
<td>YYYY</td>
<td>8</td>
</tr>
<tr>
<td>Nine</td>
<td>IX</td>
<td>III III III</td>
<td>YYYY</td>
<td>9</td>
</tr>
<tr>
<td>Ten</td>
<td>X</td>
<td></td>
<td>X</td>
<td>10</td>
</tr>
<tr>
<td>Twenty</td>
<td>XX</td>
<td></td>
<td>XX</td>
<td>20</td>
</tr>
<tr>
<td>Fifty</td>
<td>L</td>
<td></td>
<td>L</td>
<td>50</td>
</tr>
<tr>
<td>One hundred</td>
<td>C</td>
<td></td>
<td>9</td>
<td>100</td>
</tr>
<tr>
<td>Five hundred</td>
<td>D</td>
<td></td>
<td>500</td>
<td>500</td>
</tr>
<tr>
<td>One thousand</td>
<td>M</td>
<td></td>
<td>1000</td>
<td>1000</td>
</tr>
<tr>
<td>Ten thousand</td>
<td></td>
<td></td>
<td>10 000</td>
<td>10 000</td>
</tr>
</tbody>
</table>
The Egyptian Numeral System *(Introduce Year 6, p. 23)*

About 2000 B.C. the ancient Egyptians used hieroglyphics (picture writing) to write numerals as shown below:

1 / Stroke  
10  Arch  
100 Coiled Rope  
1000 Lotus Flower  
10000 Finger  
100000 Tadpole

This system was based on 10 but it did not include a zero symbol, nor did it use the principal of place value. The Egyptians formed numerals by putting basic systems together.

For example, they wrote the numeral 1326 like this:

1326

With this system, Egyptians could put symbols in any order, because the value of the system did not depend on its position - that is, there is no place value in these systems.

The Hindu-Arabic System *(Discuss Year 6, p. 23)*

The numerals 1 to 9 that are used today are thought to have originated in India about 2000 years ago. The system was adopted by Arab traders journeying to India. The Arabs made further modifications to the symbols and also introduced the symbol for zero.

Mathematicians regard the Hindu-Arabic system as one of the world's greatest inventions. Its greatness lies in the principle of place value and the use of zero. These two ideas make it easy to represent numbers and perform mathematical operations that would be difficult with any other kind of system.

Have students research the origins and history of the Hindu-Arabic system to the present time.
This number system does not contain a zero, but it does have place value. Specific symbols are used to indicate place value for tens, hundreds and thousands, but there is no extra symbol when ones only are involved.

For each place value greater than ones, two symbols are necessary:

<table>
<thead>
<tr>
<th>Number symbols</th>
<th>Place-value symbols</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 is =</td>
<td>10 is †</td>
</tr>
<tr>
<td>2 is =</td>
<td>100 is ‡</td>
</tr>
<tr>
<td>3 is =</td>
<td>1000 is §</td>
</tr>
<tr>
<td>4 is</td>
<td></td>
</tr>
<tr>
<td>5 is</td>
<td></td>
</tr>
<tr>
<td>6 is</td>
<td></td>
</tr>
<tr>
<td>7 is</td>
<td></td>
</tr>
<tr>
<td>8 is</td>
<td></td>
</tr>
<tr>
<td>9 is</td>
<td></td>
</tr>
</tbody>
</table>

In this system 284 is written as follows:

\[ \equiv \text{ godera } \text{ burla godera, burla godera } \]

Point out that each written symbol is spoken when the number is read out loud. Compare this to our system where the name of the place is not spoken. Also point out that Oriental numbers are often written vertically.

The system outlined here originates from the Koko-Yidimir language which is spoken along the Queensland coastline from the Annan and Endeavour rivers to the northern side of Cape Flattery, although it is understood considerably beyond these limits.

While this language has names for the numbers one, two and three, counting beyond those numbers is done in pairs. As with the English language, this system uses terms which describe quantity in a general way.

For information relating specifically to Aboriginal groups in Queensland, the following reference of original ethnographic source material is recommended:


Note: When number systems are compared, students must understand that the digits are not the most important feature, but rather the place-value ideas. Hindu-Arabic numeration has become predominant because of its system of place value.
"Language plays an essential part in the formulation and expression of mathematical ideas" according to the Cockcroft Report. [Cockcroft W.H. 1982 Mathematics Counts HMSO Lowden]

During number activities teachers should revise and introduce number terms by modelling them and encouraging children to use them appropriately within their language patterns.

Year 1 recommendations are as follows.

### Year 1 Number Language

<table>
<thead>
<tr>
<th>Relational</th>
<th>Quantitative</th>
<th>Operational</th>
<th>Money</th>
</tr>
</thead>
<tbody>
<tr>
<td>before/after</td>
<td>many/not many, lots/a few, all/none, enough/not enough/too many, more/less, most/least, altogether, pair, double, dozen, how many, one/two more than, one less than, zero, one, two ...10</td>
<td>match/does not, match, same as, count, count on, and/add, too/also/as well/another, put/join together, altogether, plus, take away, left, each, share</td>
<td>money, coin/note, cent, cost/price, trade/swap/exchange</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Positional</th>
<th>Quantitative</th>
<th>Operational</th>
</tr>
</thead>
<tbody>
<tr>
<td>first, second...ninety-ninth, last</td>
<td>more/most/less/least, large/larger/largest/greater/greatest, small/smaller/smallest/fewer/fewest, altogether, groups of/bundles of tens/ones, one/two, three...99, even/not, even/odd</td>
<td>group, share, each, left over, unbundle/undo, repeat, count/count on/count back</td>
</tr>
</tbody>
</table>

Whole Number - Chapter 1 : 36
Language is, however, much more than vocabulary. Attention must be given to syntax and context. Children will need to use and analyse complex forms, such as:

"The number 537 is thirty-seven larger than 500."

"The number 747 is smaller than 777 because 4 tens is smaller than 7 tens."
Although there is a mathematical distinction between the words "Number" and "Numeral", the difference is not of relevance to the primary school student.

Number is a measure of quantity, and a numeral is a symbol used to represent a number. Cardinal number answers the question "How many?"

Number involves the \textit{fiveness} of the number five, the abstract idea of the quantity of 5, whereas one of the numerals of five is the symbol '5'.

Matching

Matching is the physical pairing of objects i.e. match the elements of two groups to determine if they have the same or a different number of elements.

Year 1 Languages includes: Matches, does not match, each, none, all, enough, not enough, too many, less, same, different.

Conservation of Number

Conservation of number means the number of items in a group is not changed by rearranging the items. Activities involving the conservation of number are appropriate for all numbers except zero and one:

e.g. 

\begin{itemize}
  \item \item \item \item \item \item 
  \item \item \item \item \item 
  \item \item \item \item \item 
\end{itemize}

\textit{Year 1, pp.150, 155-156}
Year 2, p. 25
The ability to write numerals is a handwriting skill and in no way reflects a child's understanding of particular numbers. Its assessment therefore should be included in handwriting rather than mathematics.

Reversal of some or all numerals is common with immature writers. Consistently modelling correct forms, giving immediate feedback on each child's attempts and encouraging continued effort will solve this problem for most children. As fine motor control develops and confidence increases, displays of attempts may become more permanent.

Encourage children to join in reciting the following rhymes which describe pencil movements, as they practise writing numerals.

<table>
<thead>
<tr>
<th>Numeral</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>Start at the top, and left we go, All the way round and it's zero.</td>
</tr>
<tr>
<td>1</td>
<td>Tall and straight but leaning over, Number one is a tall soldier.</td>
</tr>
<tr>
<td>2</td>
<td>Around and down, across we go, Number two is made just so.</td>
</tr>
<tr>
<td>3</td>
<td>Around and around, like a little bee, Sniffing the flowers is number three.</td>
</tr>
<tr>
<td>4</td>
<td>Slanted down to near the line, Turn right and cross - a four so fine.</td>
</tr>
<tr>
<td>5</td>
<td>His head is first, his body's so fat, Don't forget, on top- his hat.</td>
</tr>
<tr>
<td>6</td>
<td>Straight little back and big round tummy, Number six looks oh so funny.</td>
</tr>
<tr>
<td>7</td>
<td>Left to right and down we zoom, Seven is standing in the room.</td>
</tr>
<tr>
<td>8</td>
<td>Make an 's' and do not stop, Until you reach the very top.</td>
</tr>
<tr>
<td>9</td>
<td>Along and round, up and down, Number nine is a real clown.</td>
</tr>
</tbody>
</table>
Contents

Scope and Sequence ........................................1
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Count on .......................................................4a
The doubles cluster
Doubles .......................................................4b
Near doubles
Doubles plus one or neighbours .....................4c
Neighbours but one ....................................4d
The addition facts to 10 cluster
Addition facts to 10 ......................................4e
Near 10 .........................................................4f
Adding 9 .......................................................4g
Remaining facts (think 10) .........................4h

Language ..................................................5
# Scope and Sequence

## Addition Facts

<table>
<thead>
<tr>
<th>RECALL STRATEGY</th>
<th>YEAR 1</th>
<th>Sum to 9 then 10</th>
<th>YEAR 2 All facts to 19</th>
</tr>
</thead>
<tbody>
<tr>
<td>A  Count on One and their turnarounds</td>
<td>1 2 1 3 1 4 1 5 1</td>
<td>+ 1 + 1 + 2 + 1 + 3 + 1 + 4 + 1 + 5</td>
<td>G Reinforce 10  1 + 1 + 10</td>
</tr>
<tr>
<td>B  Count on Two and their turnarounds</td>
<td>1 2 2 3 2 4 2 5 2</td>
<td>+ 2 + 1 + 2 + 2 + 3 + 2 + 4 + 2 + 5</td>
<td>H Reinforce 9  2 + 9 + 2 + 10 + 10</td>
</tr>
<tr>
<td>C  *Count on Three and their turnarounds</td>
<td>1 3 2 3 3 4 3 5 3</td>
<td>+ 3 + 1 + 3 + 2 + 4 + 3 + 3 + 4 + 3 + 5</td>
<td>I Reinforce 8  3 + 3 + 8 + 3 + 9 + 9</td>
</tr>
<tr>
<td>D  Count on Zero and their turnarounds</td>
<td>1 0 2 0 3 0 4 0 5</td>
<td>+ 0 + 1 + 0 + 2 + 0 + 3 + 0 + 4 + 0</td>
<td>J Reinforce 10  0 + 0 + 10</td>
</tr>
<tr>
<td>E  Doubles</td>
<td>1 2 3 4 5</td>
<td>+ 1 + 2 + 3 + 4 + 5</td>
<td>K Reinforce 6  7 + 8 + 9 + 9</td>
</tr>
<tr>
<td>F  Addition Facts to 10 and their turnarounds</td>
<td>1 9 2 8 3 7 4 6 5</td>
<td>+ 9 + 1 + 8 + 2 + 7 + 3 + 6 + 4 + 5</td>
<td>L Reinforce 10  10 + 10 + 0</td>
</tr>
</tbody>
</table>

**Please Note:**
- This Scope and Sequence must be used in conjunction with the "Additions Facts Information Sheet".
- Denotes a strategy which is not mentioned in the syllabus. Count on 3 and Neighbours but 1 are not advocated by the syllabus but are suggested here because many teachers successfully use these strategies and because they provide a means for recalling the more troublesome facts.
- Denotes those addition facts which have been taught using a previous strategy. Children should choose the strategy with which they are more comfortable.

**Year 1**
- Introduce the strategies according to the ordinal numbers A to F as indicated. Only introduce the next strategy as children become confident with and competent in using previous strategies.

**Year 2**
- In Year 2 you will need to follow the ordinal indicators from A to Q. The year should progress as follows.

Reinforcement A to F  Additional facts of those reinforced G to L  New Strategies M to Q

**Addition Facts - Chapter 2 : 2**
<table>
<thead>
<tr>
<th>RECALL STRATEGY</th>
<th>Year 2 - All facts to 19</th>
</tr>
</thead>
<tbody>
<tr>
<td>Near Doubles and their turnarounds.</td>
<td>M</td>
</tr>
<tr>
<td>Also known as Doubles plus 1 and Neighbours</td>
<td>1 2 2 3 3 4 4 5 5 6</td>
</tr>
<tr>
<td></td>
<td>+2 +2 +3 +2 +4 +3 +5 +4 +6 +5</td>
</tr>
<tr>
<td></td>
<td>6 7 7 8 8 9 9 10 +7 +6 +8 +7 +9 +8 +10 +9</td>
</tr>
<tr>
<td>Near 10 and their turnarounds</td>
<td>N</td>
</tr>
<tr>
<td></td>
<td>One more than 10</td>
</tr>
<tr>
<td></td>
<td>1 2 2 3 3 4 4 5 5 6</td>
</tr>
<tr>
<td></td>
<td>+2 +1 +3 +2 +4 +3 +5 +4 +6 +5</td>
</tr>
<tr>
<td></td>
<td>6 7 7 8 8 9 9 10 +7 +6 +8 +7 +9 +8 +10 +9</td>
</tr>
<tr>
<td>Adding 9</td>
<td>O</td>
</tr>
<tr>
<td></td>
<td>9 1 9 2 9 3 9 4 9 5</td>
</tr>
<tr>
<td></td>
<td>+1 +9 +2 +9 +3 +9 +4 +9 +5 +9</td>
</tr>
<tr>
<td></td>
<td>9 6 9 7 9 8 9 +6 +9 +7 +9 +8 +9 +9</td>
</tr>
<tr>
<td>*Neighbours but one</td>
<td>P</td>
</tr>
<tr>
<td></td>
<td>1 3 2 4 3 5 4 6 5 7</td>
</tr>
<tr>
<td></td>
<td>+3 +1 +4 +2 +5 +2 +6 +4 +7 +5</td>
</tr>
<tr>
<td></td>
<td>6 8 7 9 +8 +6 +9 +7</td>
</tr>
<tr>
<td>Remaining Facts</td>
<td>Q</td>
</tr>
<tr>
<td></td>
<td>7 4 8 4 8 5</td>
</tr>
<tr>
<td></td>
<td>+4 +7 +4 +8 +5 +8</td>
</tr>
</tbody>
</table>

**Please Note:**
This Scope and Sequence must be used in conjunction with the "Additions Facts Information Sheet".

* Denotes a strategy which is not mentioned in the syllabus. Count on 3 and Neighbours but 1 are not advocated by the syllabus but are suggested here because many teachers successfully use these strategies and because they provide a means for recalling the more troublesome facts.

Denotes those addition facts which have been taught using a previous strategy. Children should choose the strategy with which they are more comfortable.

**Year 1**
Introduce the strategies according to the ordinal numbers A to F as indicated. Only introduce the next strategy as children become confident with and competent in using previous strategies.

**Year 2**
In Year 2 you will need to follow the ordinal indicators from A to Q. The year should progress as follows.

Reinforcement A to F Additional facts of those reinforced G to L New Strategies M to Q
Addition Facts

Year Level Expectations

Year 1
Introduction to addition number facts
Year 1 Sourcebook pp. 169-181
After children have explored the numbers to 10, have built up an understanding of the addition concept, are able to read and write addition statements, activities for developing recall facts may begin.

To be consistent with the children's explorations of numbers to 10, only addition facts to 10 are examined in Year 1. Year 1 children are expected to record addition facts to 10 only. Subtraction facts are not introduced until Year 2.

Year 2
Further development and consolidation
Year 2 Sourcebook pp. 74-87
In Year 2 the strategy facts of Year 1 are revised and extended to the sum of 19. Other strategies which were not encountered in Year 1 because of number limits may now be investigated.

Year 3
Consolidation
Year 3 Sourcebook pp. 69-92
In Year 3, the children should be consolidating the strategies introduced in Years 1 and 2. Quick and accurate recall of addition facts will enable the children to perform mental and written calculations efficiently.

The starting point for helping Year 3 children with addition facts is to find out what the children already know and provide them with opportunities to practise using these facts. Teachers also need to ascertain what the children don't know and to teach them strategies for remembering these facts.

Year 4
Consolidation leading to mastery
Year 4 Sourcebook pp. 41-66
At this stage, students need quick and accurate recall of addition facts for them to be able to efficiently perform mental and written calculations. Students who do not have recall of all facts should practise the relevant strategies. Speed and accuracy may be enhanced through games and computer programs.

The suggested process for practising the strategies comprises the following pages. Students should concentrate only on those facts needing more practice. There may be a small number of students who will still need considerable help. Teachers may refer to earlier Sourcebooks for a more detailed account of suitable teaching methodology, but modify the presentation to take into account the child's age, interest and previous experience.

Year 5
Mastery
Year 5 Sourcebook pp. 40-42
Undoubtedly Year 5 calls for complete mastery of facts for most students for the majority of time.
**Strategy Clusters**
Essentially, there are three clusters of strategies for memorising and recalling addition facts. The first cluster uses counting on by zero, one, two and three. The second cluster involves doubling numbers. The double plus one (neighbours) and neighbours but one are included in this strategy. The third cluster involves building upon the addition facts for 10.

**Addition Recall Strategy Clusters**

<table>
<thead>
<tr>
<th>Count on Cluster</th>
<th>Year 1</th>
<th>Year 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Count on One</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>Count on Two</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>Count on Zero</td>
<td>✓</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Double Cluster</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Doubles</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Near Doubles (Doubles Plus One or Neighbours)</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Neighbours but One</td>
<td>✓</td>
<td>✓</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Addition Facts to 10 Cluster</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Addition Facts to 10</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Near 10</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Adding 9</td>
<td></td>
<td>✓</td>
</tr>
<tr>
<td>Think Ten (Remaining Facts)</td>
<td></td>
<td>✓</td>
</tr>
</tbody>
</table>

**Turnarounds/Commutative Principle**
The idea of "turnarounds" or the commutative principle should be emphasised in each strategy. If children understand that $6 + 4$ is the same as $4 + 6$, then the number of facts to be learned may be halved.
About the Strategy

The Count On Strategy means children count on from a given number without having to recount the entire group. (Year 1, p.177). The "count on one" strategy may be modified and used for other count on strategies, mix the practice examples.

The count on strategy may be used when one of the addends is 0, 1, 2 or 3. The children are encouraged to identify the larger number and quickly count on to arrive at the answer.

Language

"Count On"
"One More Than". "Two More Than". "Three More Than".
"Start Big and Count On"
For Zero, the situation will dictate the language
e.g. "none of", "no eggs had hatched", "no more of", "not one of".

Examples

Count One

<table>
<thead>
<tr>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>+1</td>
<td>+1</td>
<td>+1</td>
<td>+1</td>
<td>+1</td>
<td>+1</td>
<td>+1</td>
<td>+1</td>
</tr>
</tbody>
</table>

Count on Two

<table>
<thead>
<tr>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>+2</td>
<td>+2</td>
<td>+2</td>
<td>+2</td>
<td>+2</td>
<td>+2</td>
<td>+2</td>
<td>+2</td>
<td>+2</td>
</tr>
</tbody>
</table>

Count on Zero

<table>
<thead>
<tr>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>+0</td>
<td>+0</td>
<td>+0</td>
<td>+0</td>
<td>+0</td>
<td>+0</td>
<td>+0</td>
<td>+0</td>
<td>+0</td>
</tr>
</tbody>
</table>

Reference

Year 1 pp. 177-179
Year 2 pp. 76-77
Year 3 p. 46

Addition Facts - Chapter 2 : 6
About the Strategy

Doubles involve identifying objects which occur in pairs, couples or doubles - for example, twins, pairs of gloves, double ice-creams or double-decker buses. Discuss the meaning of the terms "pair", "couple" and "double". Apply these terms to addition facts by having the children identify the doubles facts.

Use real world pictures such as those below to assist children with understanding and recall.

Language

"Pair" "couple" "double" "twins"

3 wickets
+ 3 wickets.
6 wickets

"Double 3 is 6"

Examples

\[
\begin{array}{cccccccc}
1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\
+1 & +2 & +3 & +4 & +5 & +6 & +7 & +8 & +9 \\
\end{array}
\]

Sourcebook Reference

Year 1 - pp. 179-180
Year 2 - pp. 77-78
Year 3 - p. 72
Year 4 - p. 46
Doubles Cluster - Near Doubles  *(Introduce Year Two)*  

**(Neighbours or Doubles plus One)**

**About the Strategy**

Once double facts are well known, one of the numbers may be increased by 1 to make one more than a double (double plus one).

**Language**

"Double 2 is 4 and one more makes 5"

"4+5 is one more than double 4 which is 8, so the answer is 9"

**Examples**

<table>
<thead>
<tr>
<th></th>
<th>+2</th>
<th>+3</th>
<th>+4</th>
<th>+5</th>
<th>+6</th>
<th>+7</th>
<th>+8</th>
<th>+9</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
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<td></td>
<td></td>
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<td>5</td>
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<td></td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
<td></td>
<td></td>
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</tbody>
</table>

Teach these with their turnarounds

**Year of Introduction**

Year 2

**Sourcebook Reference**

Year 2 p.78

Year 3 p.77

Year 4 p.47

---

**Doubles Cluster - Neighbours but One  *(Introduce Year Two)*  

**About the Strategy**

Although not a strategy suggested by any of the sourcebooks, "Neighbours but One" does provide a suitable strategy for some of the harder facts. Assist children in seeing that by taking 1 off 7 and giving it to 5 you simply have double 6 which is the number inbetween 5 and 7.

**Language**

"If we take one from the 7 and give it to the 5 we have double 6".

**Examples**

<table>
<thead>
<tr>
<th></th>
<th>+3</th>
<th>+4</th>
<th>+5</th>
<th>+6</th>
<th>+7</th>
<th>+8</th>
<th>+9</th>
<th>+10</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
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</tbody>
</table>

Teach these with their turnarounds

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*Addition Facts - Chapter 2:8*
About the Strategy

The rainbow diagram or the bead frame are excellent for identifying all the pairs of addends which add to 10.

Language
"Making tens"

Examples
\[
\begin{array}{ccccccc}
1 & 9 & 2 & 8 & 3 & 7 & 4 & 6 & 5 \\
+9 & +1 & +8 & +2 & +7 & +3 & +6 & +4 & +5 \\
\end{array}
\]

Sourcebook Reference
Year 1 p. 180
Year 2 p. 78
Year 3 p. 77
Year 4 p. 46

---

About the Strategy

The Near 10 strategy involves assisting children to identify the facts that are close to being a fact for 10 - for example 4 + 7 is close to 4 + 6.

Compare these facts to the tens facts to identify how much more or less than the tens facts they are.

Language
4 + 7 is one more than 4 + 6, which is 10. So 4 + 7 must be 11.
4 + 5 is one less than 4 + 6 which is 10. So 4 + 5 must be 9.

Examples
\[
\begin{array}{cccc}
2 & 3 & 4 & 5 \\
+9 & +8 & +7 & +5 \\
\end{array}
\]

Teach these with their turnarounds

One less than 10.
\[
\begin{array}{cccc}
1 & 2 & 3 & 4 \\
+8 & +7 & +6 & +5 \\
\end{array}
\]

Teach these with their turnarounds

Sourcebook Reference
Year 2 pp. 78-79
Year 3 pp. 77-78
Year 4 p. 47
About the Strategy

It is appropriate to introduce this addition strategy only when children have investigated the effect of making a number 10 more than or 10 less than. Discuss the fact that the result of adding nine is one less than adding 10, because nine is one less than 10. Therefore, the answer on the 100 board is one place before the number directly beneath the original number.

Language

"Three is here. If we add 10 to 3 the answer is 13, which is straight underneath. When we add 9, the answer is 12 which is one before 13. Note: Some children will not need the 100 board to assist them in thinking through the answer.

"All I do is add on 10 and then I make the number smaller by one."

"Add ten-less one."

Examples

9 9 9 9 9 9 9 9 9
+1 +2 +3 +4 +5 +6 +7 +8
Teach these with their turnarounds

Sourcebook Reference
Year 2 pp. 79-80
Year 3 p. 77
Year 4 p.47

The Remaining Facts - Think 10 (Introduce Year Two)

About the Strategy

After all previous facts and their turnarounds have been investigated and practised, there are only a small number of remaining addition facts. One way that the children may think through these facts is to "make the 10 and then add the extra."

Language

i.e. 8+5 "8 and 2 make ten and 3 more is 13"

Examples

8 8
+4 +5
Teach these with their turnarounds

Sourcebook Reference
Year 2 p.80
Year 3 pp. 77-78
Year 4 p.47

Addition Facts - Chapter 2 : 10
Even though children may experience both vertical and horizontal forms of recording calculations, vertical recording is recommended in preference to the horizontal form. Using the vertical form helps children avoid problems associated with interpreting the equal sign and provides a firm basis for developing the written algorithm with larger numbers.

When children encounter the equals sign on the calculator, it may be described as the answer key.

Addition algorithms are read from the ‘top down.’

e.g. \[ \begin{array}{c} 4 \\ + 3 \\ \hline \end{array} \quad \text{‘4 ones and 3 more ones makes 7 ones altogether’} \]

or ‘4 ones and 3 ones makes 7 ones altogether’

or ‘4 ones add 3 ones makes 7 ones altogether’

\textit{before using}

‘4 ones plus 3 ones makes 7 ones altogether’
Language provides the link between the manipulation of materials and symbolic representations, as well as between the past and present experiences of children. Initially, children will use everyday language to describe what thoughts and actions they are thinking and doing. Gradually, teachers may include the following operational terms in discussions, explaining meanings and modelling. Children may be encouraged to include these terms appropriately within their own language patterns.

It is suggested that, in developing children's understanding of addition, a variety of language patterns be modelled so that the children realise that terms such as "altogether", "added to", "joined" and "as well" imply addition in certain contexts.

During Year 3 the language of addition will become more formalised. The children will use their own language when describing situations but as addition becomes more internalised, the children's language should become the same as the formal language.

Difficult Language
Care must be taken to ensure that the words in this vocabulary list are used appropriately in context. For example, though the word "also" is often associated with the addition operation, children should not believe that "also" is a cue word that always means "add". Similarly, the word "more" may be associated with any of four operations, depending on the sentence structure:

- "23 is 4 more than what number?"
- "I have 4 lollies and am given 15 more. How many do I have then?"
- "After the school camp, each of the 20 children had to pay $2.00 more to cover the cost of the bus. How much extra did the bus cost?"
- "Take 75 sticks and bundle them in tens. Will you have more than 7 bundles?"

The word "by" is one that will require careful analysis in context. Consider the following uses:

- "42 is larger than 10 by 32."
- "42 is larger than ....by 32."

Addition Facts - Chapter 2 : 12
Contents

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Glossary of Terms ..................................................2
Year Level Expectations ...........................................3
Recall Strategies for Subtraction Facts ............4

Easy Facts:
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- Count Back .......................................................4b
- Doubles .............................................................4c
- Zeros .................................................................4d
- Tens Facts ..........................................................4e

Harder Facts:
- One more than Facts ...........................................4f
- Near Doubles ......................................................4g
- One more than Ten ..............................................4h
- Remaining Facts ................................................4i

Reading and Recording.................................5
Language ............................................................6
The main strategy for learning a subtraction fact is to relate it to a known addition fact. Addition facts must therefore be well known, although some students may know the addition facts but might not be able to relate them to subtraction facts at this stage. For these students, the five-step teaching plan outlined below should be particularly helpful.

The plan to teach new facts involves:
1. Identifying a strategy to determine the set of facts to be learned.
2. Finding the corresponding addition facts.
3. Matching the subtraction partners.
4. Building the family of facts.
5. Practising the facts.

#### Thinking strategies for subtraction facts
The first four thinking strategies are the easier ones. How often these strategies must be practised before the facts are fully memorised will vary from child to child. A new thinking strategy should be introduced only when the children use the previous ones.

<table>
<thead>
<tr>
<th></th>
<th>Year 2 - Easy Facts</th>
<th>Year 3 - All Facts</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Count On&lt;br&gt;Where the numbers are close simply count on from the smaller to the larger number.</td>
<td>2-1, 3-2, 4-3, 5-4, 6-5, 7-6, 8-7, 9-8, 10-9 3-1, 4-2, 5-3, 6-4, 7-5, 8-6, 9-7, 10-8, 11-9</td>
</tr>
<tr>
<td>B</td>
<td>Count Back&lt;br&gt;When the part you take is small you quickly count back from the larger number.</td>
<td>Count back one. 2-1, 3-1, 4-1, 5-1, 6-1, 7-1, 8-1, 9-1, 10-1, 11-1 Count back two. 3-2, 4-2, 5-2, 6-2, 7-2, 8-2, 9-2, 10-2, 11-2 Count back three. 4-3, 5-3, 6-3, 7-3, 8-3, 9-3, 10-3, 11-3, 12-3</td>
</tr>
<tr>
<td>C</td>
<td>Doubles</td>
<td>2-1, 4-2, 6-3, 8-4, 10-5, 12-6, 14-7, 16-8, 18-9</td>
</tr>
<tr>
<td>D</td>
<td>Zeros</td>
<td>Where all is taken. 1-1, 2-2, 3-3, 4-4, 5-5, 6-6, 7-7, 8-8, 9-9 Where nothing is taken. 1-0, 2-0, 3-0, 4-0, 5-0, 6-0, 7-0, 8-0, 9-0</td>
</tr>
<tr>
<td>Recall Strategy</td>
<td>Year 2 - Easier Facts</td>
<td>Year 3 - All Facts</td>
</tr>
<tr>
<td>-----------------</td>
<td>-----------------------</td>
<td>-------------------</td>
</tr>
<tr>
<td><strong>E</strong> Tens Facts</td>
<td>10-1, 10-2, 10-3, 10-4, 10-5, 10-6, 10-7, 10-8, 10-9</td>
<td></td>
</tr>
</tbody>
</table>

### Harder Facts

#### One More Than Facts

<table>
<thead>
<tr>
<th><strong>F</strong> Doubles Plus One</th>
<th>3-1, 5-2, 7-3, 9-4, 11-5, 13-6, 15-7, 17-8, 19-9</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>G</strong> Near Doubles</td>
<td>3-2, 5-3, 7-4, 9-5, 11-6, 13-7, 15-8, 17-9</td>
</tr>
<tr>
<td><strong>H</strong> One More than a Ten</td>
<td>11-1, 11-2, 11-3, 11-4, 11-5, 11-6, 11-7, 11-8, 11-9</td>
</tr>
</tbody>
</table>

#### Remaining Facts

<table>
<thead>
<tr>
<th><strong>I</strong> Take All the Ones and One More</th>
<th>11-2, 12-3, 13-4, 14-5, 15-6, 16-7, 17-8, 18-9</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>J</strong> Taking Nine from the Ten</td>
<td>11-9, 12-9, 13-9, 14-9, 15-9, 16-9, 17-9, 18-9</td>
</tr>
<tr>
<td><strong>K</strong> Taking Eight from the Ten</td>
<td>11-8, 12-8, 13-8, 14-8, 15-8, 16-8, 17-8</td>
</tr>
<tr>
<td><strong>L</strong> Others</td>
<td>8-5, 12-4, 12-5, 13-5, 14-6, 9-6</td>
</tr>
</tbody>
</table>

**Note:**

- denotes those subtraction facts which have been taught using a previous strategy. Children should choose the strategy with which they are more comfortable.

**Year 2**

Introduce the strategies according to the ordinal numbers A to E as indicated. Only introduce the next strategy as children become confident with and competent in using previous strategies.

**Year 3**

Follow the ordinal indicators from A to L so that reinforcement of the earlier facts occurs before the new strategies are introduced.
Glossary of Terms
Subtraction Facts

Means to an End Not an End in Themselves

To assist children with their basic addition facts to 10, memorisation strategies are suggested. These strategies provide children with the means for calculating facts quickly and easily. It is important to note that these strategies are a means to an end and not an end in themselves. Instant recall from memory and understanding remains the ultimate goal.

Strategy Clusters

Strategies for the recall of subtraction facts do, to some extent, reflect many of the addition recall strategies. They are loosely grouped below according to the addition strategies they reflect.

<table>
<thead>
<tr>
<th>Count on</th>
<th>Year 2</th>
<th>Year 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Count on</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>Count back</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>Doubles</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Doubles</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>Doubles plus one</td>
<td></td>
<td>✓</td>
</tr>
<tr>
<td>Near doubles</td>
<td></td>
<td>✓</td>
</tr>
<tr>
<td>Tens Facts</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tens facts</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>One more than ten</td>
<td></td>
<td>✓</td>
</tr>
<tr>
<td>Take all the ones and one more</td>
<td></td>
<td>✓</td>
</tr>
<tr>
<td>Taking nine from the ten</td>
<td></td>
<td>✓</td>
</tr>
<tr>
<td>Taking eight from the ten</td>
<td></td>
<td>✓</td>
</tr>
<tr>
<td>Zeros</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Zeros where all is taken</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>Zeros when nothing is taken</td>
<td>✓</td>
<td></td>
</tr>
</tbody>
</table>
Relating Subtraction to Addition: A Five Step Plan

The main strategy for learning a subtraction fact is to relate it to a known addition fact. Therefore, before a subtraction fact is introduced, children must know the corresponding addition fact.

Both the conceptual understanding and mastery of the subtraction facts depend on an understanding of the relationship between addition and subtraction. The language used with missing addend examples helps develop this understanding. For example, the solution to 9 - 8 can be reached by asking, “What can I add to eight to make nine?” If the children know what 8 + 1 = 9, then they will know that 9 - 8 + 1.

There may be some children however who know their addition facts but are unable to relate them to subtraction facts at the introductory stage. For these students, the five step teaching plan outlined below should be particularly helpful.

The plan to teach new facts involves:

(i) Identifying a strategy to determine the set of facts to be learned

Sometimes there will be more than one appropriate strategy and children will be able to choose the strategy they prefer.

(ii) Finding the corresponding addition facts

Children should try to associate each subtraction fact with its corresponding addition fact – for example:

\[
\begin{array}{c}
16 \\
-9 \\
\hline
9 \text{ and what are 16?} \\
9 \text{ and 7 are 16}
\end{array}
\]

(iii) Matching the subtraction partners

Children should practise matching the subtraction partners. For example, the partner to 17 would be 17 because 9

\[
\begin{array}{c}
-8 \\
-9 \\
+8 \\
\hline
17
\end{array}
\]

Number cards and calculators can be used to practise and reinforce these ideas. A good understanding of subtraction partners can cut the number of facts to be learned in half.

(iv) Building the family of facts

Children are given a fact – for example:

\[
\begin{array}{c}
4 \\
+9 \\
\hline
13
\end{array}
\]

or a number with its addends – for example:

\[
\begin{array}{c}
13 \\
4 \\
9 \\
\hline
\end{array}
\]

and are asked to write a family of facts, using these numbers – for example:

\[
\begin{array}{cccc}
4 & 9 & 13 & 13 \\
+9 & +4 & -4 & -9
\end{array}
\]

(v) Practising the facts

Each set of new facts can be practised through drill, puzzles, games and using appropriate computer programs.

Subtraction Facts - Chapter 3 : 5
When More Than One Strategy Applies

It should be noted that in some instances, more than one strategy can be employed to memorize or recall a subtraction fact. For example, 2 - 1 can be calculated by applying either the count on, count back or the doubles strategy. In such cases children should choose a strategy with which they are more comfortable.

When To Introduce New Strategies

Teachers should only introduce new strategies when children are confident with and competent in using previous strategies. Frequent practice is recommended for confidence and competence to develop. When children are proficient with each strategy, mix the practice examples so that children are required to select suitable strategies as well as use them.

The Subtraction Concept

Subtraction situations can be identified by the need to compare the values of the numbers. The three different types of situations that represent subtraction are called 'take away', 'comparison' and 'missing addend'. Children need to know that subtraction concepts can be represented in these three ways. The term 'missing addend' need not be used by the children.

<table>
<thead>
<tr>
<th>Type of subtraction</th>
<th>Word problem</th>
<th>Diagram</th>
</tr>
</thead>
<tbody>
<tr>
<td>'take away'</td>
<td>There were 8 people at a party and 3 went home. How many people were left?</td>
<td></td>
</tr>
<tr>
<td>'comparison'</td>
<td>Janice has 9 books and Marni has 3. How many more books does Janice have?</td>
<td></td>
</tr>
<tr>
<td>'missing addend'</td>
<td>I need 7 rolls of wallpaper for the lounge walls. I only have 4 rolls. How many more do I need? (This term is not for use by students).</td>
<td></td>
</tr>
</tbody>
</table>

Missing Addend/Subtraction Facts

Before a subtraction fact is introduced, children must know the corresponding addition fact. Both the conceptual understanding and mastery of the subtraction facts depend on an understanding of the relationship between addition and subtraction.

Because many subtraction facts are learned by relating them to known addition facts, the 'missing addend' model is often used. Knowledge of addition facts is important because the strategy of thinking of the related addition fact is a most useful aid to memorisation.
For example:
12
-7
think of the addition fact
or what can I add to 5 to get 12?

7 and what are 12?
7 + 5 are 12

Take away/Subtraction Algorithm

When the algorithm is being taught and/or revised, the children must rely on the 'take away' model and associated language to build the process and to find the solution. The operation is represented symbolically in a vertical format and read from the top down, for example:

7
-3
takeaway/take 3
makes 4, or leaves 4, or is 4.

Introduce the subtraction concept by using stories about the children's classroom and the outside environment.

Since sequential development of the written algorithm and basics facts is desirable, it is very important to ensure that each child is proficient with one stage before progressing to the next as knowledge of the prior stages is usually a prerequisite. The obligation to students is not to rush them through stages so that they can keep up with their peers, but to ensure that they have the opportunity to develop understanding and skill before being asked to attempt a more difficult task.

Since considerable overlap occurs throughout subtraction activities, children can be practising strategies for basic facts and estimation when they are also applying written algorithms, mental calculations, estimations and using calculators. Their skills should not be taught in isolation of one another. They should be presented simultaneously throughout the program and not consecutively to isolated units.

To help students better understand the operations, they should represent each operation in as many ways as possible. Teachers are encouraged to present the full range of situations in activities which require students to identify, explain and represent each operation.

Real World Situation
‘When I went to the park for a picnic, I had 9 apples to share with my friends. Only eight of them were eaten. How many apples did I take home from the picnic?’

Concrete Modelling the situation using concrete materials e.g. Unifix

Symbolic
9
-8

Verbal Explaining the situation and acting it out.

Note: this is most important that students be able to analyse problems or tasks then identify and apply the mathematical operation involved rather than being presented with an isolated, meaningless algorithm.

It is therefore important that children have many opportunities to apply and discuss a real life problem while working with concrete materials and making symbolic recordings. By the time only numbers are used, the method of working and thinking their way through the subtraction algorithm should be well established.
Year Level Expectations
Subtraction Facts

Year 1

Only the concept and associated language of subtraction is introduced in Year 1. Formalizing the concept with this sign, recording and memorization is reserved for Year 2. Subtraction is to be introduced as the inverse of addition. When two numbers are added together, one can be taken away again to leave the other addend. In this way, the conservation of number is reinforced.

e.g. “Six birds and three more birds makes nine birds altogether. How many birds would there be if three birds flew away again?”

The subtraction concept can also be investigated by covering one part of the whole and their finding out what is hidden or missing e.g. “This is a picture of six cricket wickets”.

“How many can you see?”

“How many am I hiding?”

“Let’s see if you are right.”

(Year 1 p.p. 180-181)

Year 2

In Year 2 the concept of subtractions formalized to include the symbol (−), recording and memorization of facts.

This formalization involves children in examining the three forms of subtraction - namely take away, missing addend and comparison. (Refer to page __ for further information about the three types of subtraction).

Each of the three types of subtraction involves children in using different language and thinking, so it is recommended that the new dimensions are only introduced when children clearly understand the previous one.

In Year 2 the easier subtraction strategies to aid memorization and instant recall are investigated and practised. These include:

Count on or back
Doubles
Zeros
Tens facts

(Refer to pages __ for more information on strategies).

Year 3

All subtraction facts are to be taught by the end of Year 3.

In Year 2 the easier thinking strategies are investigated. After revising these strategies, introduce the others. (Of course, the strategies focused on here are not the only ones that could be used.) Some children will continue to use these strategies for a considerable time.

Though these strategies are of central importance, drill will still form part of the learning process.

The vertical format is recommended, and fact charts, whether for the class or for individuals,
should use this format. A grid displayed in the classroom could show which facts have been covered by the class.

**Prerequisites**

There are certain prerequisites to learning the number facts for this level:
- addition concept and easy facts;
- concept of subtraction (whole/part, comparison of numbers);
- relationship between addition and subtraction;
- ability to count on/back;
- numeration of teens.

These prerequisites will have to be reviewed at the beginning of Year 3 before the more difficult subtraction facts are taught.

**Year 4**

Many students will already have recall of most subtraction facts by Year 4, whereas other students will still need a lot of practice. The teacher will have to find out which facts are known/not known before implementing this teaching plan.
Recall Strategies
Subtraction Facts

The following outlines thinking strategies for memorising and recalling subtraction facts. The amount of practice that children require to memorise facts using any of these strategies will vary from child to child. A new strategy should only be introduced when children are confident in using previous ones. Investigating such strategies is not intended to be an end in itself. The ultimate goal remains instant recall of these facts.

### Count on - Easy Facts (Introduce Year 2)

<table>
<thead>
<tr>
<th>Count on 1</th>
<th>Count on 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>take</td>
<td>take</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
</tr>
<tr>
<td>4</td>
<td>6</td>
</tr>
<tr>
<td>3</td>
<td>7</td>
</tr>
<tr>
<td>5</td>
<td>8</td>
</tr>
<tr>
<td>4</td>
<td>9</td>
</tr>
<tr>
<td>6</td>
<td>10</td>
</tr>
<tr>
<td>5</td>
<td>11</td>
</tr>
</tbody>
</table>

**About the Strategy**
Count on one and count on two can be used when the numbers in the fact are close. Children are encouraged to quickly count on from the smaller to the larger number.

**Language/Thinking**
- I have 6... 7... 8.
- That's Z.

**Examples:**

### Count back - Easy Facts (Introduce Year 2)

<table>
<thead>
<tr>
<th>Count back 1</th>
<th>Count back 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>take</td>
<td>take</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>5</td>
<td>6</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>6</td>
<td>7</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>7</td>
<td>8</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>8</td>
<td>9</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>9</td>
<td>10</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>10</td>
<td>11</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
</tr>
</tbody>
</table>

**About the Strategy**
Count back one and count back two are strategies which can be used when the part you 'take' is small. The children can quickly count back from the larger number.

**Language/Thinking**
- Begin at 8,
- count back 2
- 7... 6

**Examples:**

Subtraction Facts - Chapter 3 : 10
Doubles - Easy Facts (introduce Year 2)

About the Strategy
Encourage children to think of the 'doubles' pictures used for addition double facts. The known doubles addition facts can be used to teach the unknown subtraction facts.

<table>
<thead>
<tr>
<th>Doubles</th>
<th>take</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>6</td>
<td>3</td>
</tr>
<tr>
<td>8</td>
<td>4</td>
</tr>
<tr>
<td>10</td>
<td>5</td>
</tr>
<tr>
<td>12</td>
<td>6</td>
</tr>
<tr>
<td>14</td>
<td>7</td>
</tr>
<tr>
<td>16</td>
<td>8</td>
</tr>
<tr>
<td>18</td>
<td>9</td>
</tr>
</tbody>
</table>

Language/Thinking

Examples:

Zeros - Easy Facts (introduce Year 2)

About the Strategy
To reinforce their understanding, ask the children to act out situations where all or none are taken away.

Language
Where none are taken away.
For zero, the situation will dictate the language, e.g. nothing, none.
"If you take none away you'll still have the same number that you started with."
Where all are taken away.
"If you have 6 and take all 6 away - you'll have none left."

<table>
<thead>
<tr>
<th>When none are taken away</th>
<th>take</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>0</td>
</tr>
<tr>
<td>6</td>
<td>0</td>
</tr>
<tr>
<td>7</td>
<td>0</td>
</tr>
<tr>
<td>8</td>
<td>0</td>
</tr>
<tr>
<td>9</td>
<td>0</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>When all are taken away</th>
<th>take</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td>7</td>
<td>7</td>
</tr>
<tr>
<td>8</td>
<td>8</td>
</tr>
<tr>
<td>9</td>
<td>9</td>
</tr>
</tbody>
</table>

Tens Facts - Easy Facts (introduce Year 2)

About the Strategy
Children rely on their knowledge of facts adding to 10 and the diagrams which give visual association, such as:

6 + 4 is represented on this 10 frame

The rainbow diagram links the addends.

Subtraction Facts - Chapter 3:11
Language
“You know 6 and 4 makes 10. 10 take 6 leaves 4.”

Examples:

<table>
<thead>
<tr>
<th>Tens Facts</th>
</tr>
</thead>
<tbody>
<tr>
<td>take</td>
</tr>
<tr>
<td>10</td>
</tr>
<tr>
<td>10</td>
</tr>
<tr>
<td>10</td>
</tr>
<tr>
<td>10</td>
</tr>
<tr>
<td>10</td>
</tr>
<tr>
<td>10</td>
</tr>
<tr>
<td>10</td>
</tr>
<tr>
<td>10</td>
</tr>
<tr>
<td>10</td>
</tr>
</tbody>
</table>

One More Than Facts - Hard Facts (introduce Year 3)

About the Strategy
Knowledge of addition doubles should help children identify their subtraction facts.

Language/Thinking  13
-6
“Twelve take 6 leaves 6 and one more makes 7.”

Examples:

<table>
<thead>
<tr>
<th>Doubles plus one</th>
</tr>
</thead>
<tbody>
<tr>
<td>take</td>
</tr>
<tr>
<td>3</td>
</tr>
<tr>
<td>5</td>
</tr>
<tr>
<td>7</td>
</tr>
<tr>
<td>9</td>
</tr>
<tr>
<td>11</td>
</tr>
<tr>
<td>13</td>
</tr>
<tr>
<td>15</td>
</tr>
<tr>
<td>17</td>
</tr>
<tr>
<td>19</td>
</tr>
</tbody>
</table>

Near Doubles - Hard Facts (introduce Year 3)

About the Strategy
Knowledge of addition doubles should help children identify their subtraction facts.

Language/Thinking  11
-6
“12 take 6 is 6 and one less is 5”.

Examples:

<table>
<thead>
<tr>
<th>Near Doubles</th>
</tr>
</thead>
<tbody>
<tr>
<td>take</td>
</tr>
<tr>
<td>3</td>
</tr>
<tr>
<td>5</td>
</tr>
<tr>
<td>7</td>
</tr>
<tr>
<td>9</td>
</tr>
<tr>
<td>11</td>
</tr>
<tr>
<td>13</td>
</tr>
<tr>
<td>15</td>
</tr>
<tr>
<td>17</td>
</tr>
</tbody>
</table>
About the Strategy
Knowledge of “addition to ten facts” should help children identify their subtraction facts.

Language/Thinking  
11
-7
“10 take 7 is 3 and one more is 4”.

Examples:

<table>
<thead>
<tr>
<th>One more than 10</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>take</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>6</td>
<td></td>
</tr>
<tr>
<td>13</td>
<td>7</td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>8</td>
<td></td>
</tr>
<tr>
<td>17</td>
<td>9</td>
<td></td>
</tr>
</tbody>
</table>
### Remaining Facts - Hard Facts (introduce Year 3)

- Take all the ones and one more
- Take 9 from the ten
- Take 8 from the ten
- Others

<table>
<thead>
<tr>
<th>Strategy</th>
<th>Language/Thinking</th>
<th>Examples</th>
</tr>
</thead>
<tbody>
<tr>
<td>Take all the ones and one more</td>
<td>15 &quot;15 take 5 is 10 and -6 1 less is 9&quot;</td>
<td>Take all the ones and one more take</td>
</tr>
<tr>
<td></td>
<td></td>
<td>11___ 2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>12___ 3</td>
</tr>
<tr>
<td></td>
<td></td>
<td>13___ 4</td>
</tr>
<tr>
<td></td>
<td></td>
<td>14___ 5</td>
</tr>
<tr>
<td></td>
<td></td>
<td>15___ 6</td>
</tr>
<tr>
<td></td>
<td></td>
<td>16___ 7</td>
</tr>
<tr>
<td></td>
<td></td>
<td>17___ 8</td>
</tr>
<tr>
<td></td>
<td></td>
<td>18___ 9</td>
</tr>
<tr>
<td>Taking nine from the ten</td>
<td>13 &quot;13 is 10 + 3&quot;</td>
<td>Taking nine from the ten take</td>
</tr>
<tr>
<td></td>
<td>-2 &quot;10 take 9 is 1 and 3 more makes 4.&quot;</td>
<td>11___ 9</td>
</tr>
<tr>
<td></td>
<td></td>
<td>12___ 9</td>
</tr>
<tr>
<td></td>
<td></td>
<td>13___ 9</td>
</tr>
<tr>
<td></td>
<td></td>
<td>14___ 9</td>
</tr>
<tr>
<td></td>
<td></td>
<td>15___ 9</td>
</tr>
<tr>
<td></td>
<td></td>
<td>16___ 9</td>
</tr>
<tr>
<td></td>
<td></td>
<td>17___ 9</td>
</tr>
<tr>
<td></td>
<td></td>
<td>18___ 9</td>
</tr>
<tr>
<td>Taking eight from the ten</td>
<td>14 &quot;14 take 10 is 4 and -8 2 more makes 6.&quot;</td>
<td>Take eight from the ten take</td>
</tr>
<tr>
<td></td>
<td>or &quot;10 take 8 is 2 and 4 more makes 6.&quot;</td>
<td>11___ 8</td>
</tr>
<tr>
<td></td>
<td></td>
<td>12___ 8</td>
</tr>
<tr>
<td></td>
<td></td>
<td>13___ 8</td>
</tr>
<tr>
<td></td>
<td></td>
<td>14___ 8</td>
</tr>
<tr>
<td></td>
<td></td>
<td>15___ 8</td>
</tr>
<tr>
<td></td>
<td></td>
<td>16___ 8</td>
</tr>
<tr>
<td></td>
<td></td>
<td>17___ 8</td>
</tr>
<tr>
<td>Other</td>
<td>*12 &quot;take the two and -4 two more from the ten - leaves 8.&quot;</td>
<td>Other take</td>
</tr>
<tr>
<td>No formal strategy is recommended for these. I have made a few suggestions.</td>
<td>*8 and 9 Could be done as a 'count back 3'</td>
<td>8___ 5</td>
</tr>
<tr>
<td></td>
<td>*5 -6</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>12___ 4</td>
</tr>
<tr>
<td></td>
<td></td>
<td>12___ 5</td>
</tr>
<tr>
<td></td>
<td></td>
<td>13___ 5</td>
</tr>
<tr>
<td></td>
<td></td>
<td>14___ 6</td>
</tr>
</tbody>
</table>

* Denotes strategies not mentioned in the syllabus.
As numerous examples of take away, missing addend and comparison are presented, a gradual progression through the following forms of representation is recommended. The three types of subtraction differ until they reach the word and digit representation.

<table>
<thead>
<tr>
<th>Take Away</th>
<th>Missing Addend</th>
<th>Comparison</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Five boys were on the bus then three got off. How many boys were left on the bus?</td>
<td>• Debbie needs 5 stickers for her collection but only has three. How many more stickers does she need?</td>
<td>• Jim had five blue marbles and three red marbles. How many more blue marbles than red marbles does Jim have?</td>
</tr>
<tr>
<td>• Five boys take away three, leaves two boys.</td>
<td>• Five stickers are needed by she only has three, so we need two more stickers.</td>
<td>• Five marbles, take away three, leaves two marbles.</td>
</tr>
</tbody>
</table>

- using words and digits  
  '5 take 3 leaves 2'  
- using a vertical format e.g.

```
  5  - five  
  take 3 - take away/take 3  
  2  - makes 2/leaves 2/is 2
```

- using the "-" sign

```
  5  - five  
  -3 - take away/take 3  
  2  - makes 2/leaves 2/is 2
```

*Note:* The subtraction operation is represented in a vertical format and read from the top down.

- Be careful how ‘comparison’ and ‘missing addend’ situations are presented to the children as they are not simple ‘take away’ situations.

- For a careful analysis of how each situation should be presented to the children, please consult the Year 2 Sourcebook on pages 88 to 93.
Language forms the link between the children's manipulation of materials and their symbolic representation of this activity, that is, using words, digits or signs. It also provides a link between the children's past and present experiences. The children will initially use everyday language to describe their thinking and doing. Gradually, teachers can include the following operational terms in discussions, explain their meaning and model their use. Then the children can be encouraged to use them appropriately within their own language patterns.

The children will need many experiences with everyday situations that suggest subtraction, even though all the information necessary to find an answer may not be present, for example:

- Neville is missing 10 stamps. Peta picked 3 lemons.
- Nicole lost 2 hairpins. Howard gave away 6 marbles.

This form of language will be transformed into 'take away', difference and 'how many more' as the children's concept of subtraction develops.

### Year 1
- take away, left

### Year 2

<table>
<thead>
<tr>
<th>Take Away</th>
<th>Missing Addend</th>
<th>Comparison</th>
</tr>
</thead>
<tbody>
<tr>
<td>left</td>
<td>more</td>
<td>difference</td>
</tr>
<tr>
<td>take away</td>
<td>extra</td>
<td></td>
</tr>
<tr>
<td>away</td>
<td>too many</td>
<td></td>
</tr>
<tr>
<td>how many</td>
<td>enough</td>
<td></td>
</tr>
<tr>
<td>leaves</td>
<td>not enough</td>
<td></td>
</tr>
<tr>
<td>remaining</td>
<td></td>
<td></td>
</tr>
<tr>
<td>minus</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Year 3
- As for Year 2. Include equals and subtract.
Multiplication Facts

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This scope and sequence, together with the ‘Recall Strategies – Multiplication Facts’ part 4 of this Chapter provides the teacher with one path through the recall strategies for multiplication as advocated by the syllabus. It is inclusive in that upon completion one or more strategies has been presented for all the multiplication facts to $9 \times 9 = 81$.

<table>
<thead>
<tr>
<th>Type of Facts</th>
<th>Easier Facts – Introduce Year 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>The Twos Facts</td>
<td>2 2 2 2 2 2 2 2 2 2</td>
</tr>
<tr>
<td>and their</td>
<td>$x0$ $x1$ $x2$ $x3$ $x4$ $x5$ $x6$ $x7$ $x8$ $x9$</td>
</tr>
<tr>
<td>their turnarounds</td>
<td>0 1 2 3 4 5 6 7 8 9</td>
</tr>
<tr>
<td>and their</td>
<td>$x2$ $x2$ $x2$ $x2$ $x2$ $x2$ $x2$ $x2$ $x2$ $x2$</td>
</tr>
<tr>
<td>The Fives Facts</td>
<td>5 5 5 5 5 5 5 5 5 5</td>
</tr>
<tr>
<td>and their</td>
<td>$x0$ $x1$ $x2$ $x3$ $x4$ $x5$ $x6$ $x7$ $x8$ $x9$</td>
</tr>
<tr>
<td>their turnarounds</td>
<td>0 1 2 3 4 5 6 7 8 9</td>
</tr>
<tr>
<td>and their</td>
<td>$x5$ $x5$ $x5$ $x5$ $x5$ $x5$ $x5$ $x5$ $x5$ $x5$</td>
</tr>
<tr>
<td>The Nines Facts</td>
<td>9 9 9 9 9 9 9 9 9 9</td>
</tr>
<tr>
<td>and their</td>
<td>$x0$ $x1$ $x2$ $x3$ $x4$ $x5$ $x6$ $x7$ $x8$ $x9$</td>
</tr>
<tr>
<td>their turnarounds</td>
<td>0 1 2 3 4 5 6 7 8 9</td>
</tr>
<tr>
<td>and their</td>
<td>$x9$ $x9$ $x9$ $x9$ $x9$ $x9$ $x9$ $x9$ $x9$ $x9$</td>
</tr>
<tr>
<td>The Zeros Facts</td>
<td>0 0 0 0 0 0 0 0 0 0</td>
</tr>
<tr>
<td>and their</td>
<td>$x0$ $x1$ $x2$ $x3$ $x4$ $x5$ $x6$ $x7$ $x8$ $x9$</td>
</tr>
<tr>
<td>their turnarounds</td>
<td>0 1 2 3 4 5 6 7 8 9</td>
</tr>
<tr>
<td>and their</td>
<td>$x0$ $x0$ $x0$ $x0$ $x0$ $x0$ $x0$ $x0$ $x0$ $x0$</td>
</tr>
<tr>
<td>The Ones Facts</td>
<td>1 1 1 1 1 1 1 1 1 1</td>
</tr>
<tr>
<td>and their</td>
<td>$x0$ $x1$ $x2$ $x3$ $x4$ $x5$ $x6$ $x7$ $x8$ $x9$</td>
</tr>
<tr>
<td>their turnarounds</td>
<td>0 1 2 3 4 5 6 7 8 9</td>
</tr>
<tr>
<td>and their</td>
<td>$x1$ $x1$ $x1$ $x1$ $x1$ $x1$ $x1$ $x1$ $x1$ $x1$</td>
</tr>
</tbody>
</table>


Denotes those multiplication facts which have been taught previously using another strategy.
The Remaining Facts - Harder Facts

After the twos, fives, nines, zeros and ones facts and their turnarounds have been taught, there are only 15 remaining facts to be learned.

Since 5 of the remaining 15 facts are square number facts and another 7 are built on facts, the following progression through the 15 remaining facts is recommended by the author who has rearranged those presented on pages 114 and 115 of the Year 4 Sourcebook so that as many remaining facts as possible occur in sets of facts which have some strategy.

Refer to 'Recall Strategy - Multiplication Facts' in section 4 of this chapter for an outline of the strategies.

<table>
<thead>
<tr>
<th>Type of Facts</th>
<th>Harder Facts - Also introduce Year 3/4</th>
</tr>
</thead>
<tbody>
<tr>
<td>The Square Number Facts</td>
<td>0 1 2 3 4 5 6 7 8 9</td>
</tr>
<tr>
<td></td>
<td>x0 x1 x2 x3 x4 x5 x6 x7 x8 x9</td>
</tr>
<tr>
<td>The Threes Facts</td>
<td>3 3 3 3 3 3 3 3</td>
</tr>
<tr>
<td></td>
<td>x0 x1 x2 x3 x4 x5 x6 x7 x8 x9</td>
</tr>
<tr>
<td></td>
<td>0 1 2 3 4 5 6 7 8 9</td>
</tr>
<tr>
<td></td>
<td>x3 x3 x3 x3 x3 x3 x3 x3 x3 x3</td>
</tr>
<tr>
<td>The Sixes Facts</td>
<td>6 6 6 6 6 6 6 6 6 6</td>
</tr>
<tr>
<td></td>
<td>x0 x1 x2 x3 x4 x5 x6 x7 x8 x9</td>
</tr>
<tr>
<td></td>
<td>0 1 2 3 4 5 6 7 8 9</td>
</tr>
<tr>
<td></td>
<td>x6 x6 x6 x6 x6 x6 x6 x6 x6 x6</td>
</tr>
<tr>
<td>The Fours Facts</td>
<td>4 4 4 4 4 4 4 4 4 4</td>
</tr>
<tr>
<td></td>
<td>x0 x1 x2 x3 x4 x5 x6 x7 x8 x9</td>
</tr>
<tr>
<td></td>
<td>0 1 2 3 4 5 6 7 8 9</td>
</tr>
<tr>
<td></td>
<td>x4 x4 x4 x4 x4 x4 x4 x4 x4 x4</td>
</tr>
<tr>
<td>The Eights Facts</td>
<td>8 8 8 8 8 8 8 8 8 8</td>
</tr>
<tr>
<td></td>
<td>x0 x1 x2 x3 x4 x5 x6 x7 x8 x9</td>
</tr>
<tr>
<td></td>
<td>0 1 2 3 4 5 6 7 8 9</td>
</tr>
<tr>
<td></td>
<td>x8 x8 x8 x8 x8 x8 x8 x8 x8 x8</td>
</tr>
<tr>
<td>The Sevens Facts</td>
<td>7 7 7 7 7 7 7 7 7 7</td>
</tr>
<tr>
<td></td>
<td>x0 x1 x2 x3 x4 x5 x6 x7 x8 x9</td>
</tr>
<tr>
<td></td>
<td>0 1 2 3 4 5 6 7 8 9</td>
</tr>
<tr>
<td></td>
<td>x7 x7 x7 x7 x7 x7 x7 x7 x7 x7</td>
</tr>
</tbody>
</table>

Denotes those multiplication facts which have been taught previously using another strategy.
Formal development of multiplication and the introduction of the multiplication sign takes place in Year 3. Although the limit set on multiplication facts for Year 3 is $9 \times 9 = 81$, some children may not be able to recall all facts to $9 \times 9$ by the end of the year. Year 4 will be a particularly important consolidation year along with Years 5 and 6. Mastery of multiplication and division is expected by Year 7.

Teachers should only introduce new facts when children are confident with and competent in using the previous facts. Some children may, for example, require deferment of the teaching of the harder facts until the easier facts are further consolidated. Frequent practice is recommended for confidence and competence to develop. The activities suggested in the sourcebooks are excellent for this.

When children are proficient with each set of new facts, mix these with the previous facts to assist with consolidation and recall.
Year 1

In Year 1 children investigate the numbers to 10 by adding, subtracting, multiplying and dividing these numbers in order to compare and analyse the number's size. Children also create their own number stories involving these numbers and operations. However, addition is the only operation which is formally recorded and developed in Year 1.

Year 2

Informal Exploration of the Multiplication Concept
Year 2 Sourcebook pp. 99-100

In the Year 2 topic 'Number study', children arrange materials into small groups and then into groups of tens and ones to study the place value of numbers to 99. These grouping activities give children experience with multiplication and division processes.

In Year 2, problems from school, home and environmental situations are investigated. The focus is on developing a variety of strategies to solve such problems, discussing the problems using the appropriate language and representing them accurately with materials. Even though no formal development of multiplication is intended until Year 3 children may wish to record these problems and their solutions using a combination of pictures, words and digits.

The associated concept of odd and even will be introduced in Year 2.

Year 3

Formal Development of Multiplication
Year 3 Sourcebook pp. 111-133

The formal development of multiplication and the introduction of the 'x' sign takes place in Year 3. The Year 3 activities give the children opportunities to investigate the concept and associated language of multiplication by identifying and representing multiplication symbol and multiplication facts and playing games to consolidate multiplication skills. Work on the associated concept of odd and even will continue in Year 3.

Although the limit set on multiplication facts for Year 3 is $9 \times 9 = 81$ some children may not be able to recall all facts to $9 \times 9$ by the end of the year.

The following sequence is recommended for developing multiplication in Year 3.

[The multiplication concept is dealt with informally in Years 1 & 2].

The following sequence is recommended for developing multiplication:

1. Multiplication concept
   Use language, unstructured and structured materials, pictures and diagrams to model and explain multiplication.

2. Recording multiplication
   - Use symbols and everyday language to represent multiplication, for example: 3 rows of 4; 3 groups of 4.
   - Introduce the symbol for multiplication:
     \[
     \begin{array}{c}
     4 \\
     \times 3 \\
     \hline
     12
     \end{array}
     \]
3. Multiplication facts

- Use the 'x' sign to record multiplication: \[ \begin{array}{c}
6 \\
\times \ 2 \\
\end{array} \]
\[ \begin{array}{c}
\phantom{6 \times 2} \\
12 \\
\end{array} \]

- Develop thinking strategies to assist recall.

A suggested sequence for developing the multiplication facts in Year 3 is:

**Easier Facts**
- twos facts
- fives facts
- nines facts
- zero facts
- ones facts

**Harder Facts**
- Remaining facts from the threes, sixes, fours, eights and sevens.

**Year 4**
Continue work on harder facts
*Year 4 Sourcebook pp. 101-132*

For some children Year 4 work will be a continuation of the Year 3 introductory process while for others it will be a year of consolidation. (Refer to the Scope & Sequence for the introductory process of Multiplication Facts).

The Year 4 Sourcebook deals with introductory process and provides games and activities for consolidation. Work on the associated concepts of odd and even will continue along with the introduction of multiples and factors.

**Year 5**
Consolidation
*Year 5 Sourcebook pp. 44-46*

Consolidation of multiplication work will continue in Year 5. Work on associated concept such as odd, even, multiples and factors will also continue along with the introduction of prime, composite and square numbers and the investigation of the 'Sieve of Eratosthenes' p.23

**Year 6**
Consolidation
*Year 6 Sourcebook pp. 17-19, 29-31.*

Consolidation of multiplication facts will continue in Year 6. Work on associated concepts such as odd, even, factors, multiples, prime, composite and square numbers will continue along with the introduction of factor tree work.

**Year 7**
Mastery
*Year 7 Sourcebook pp. 17-18, 21.*

Mastery of addition and subtraction facts is expected by Year 4 and mastery of multiplication and division facts is expected by Year 7. Work on associated concepts such as odd, even, factors, multiples, prime composite, square numbers and factor trees will continue along with the introduction of the square root and divisibility rules.
Glossary of Terms
Multiplication Facts

Multiplication builds on addition

Because multiplication facts build on addition, it is very important for the children to have a good understanding of addition and ready recall of addition facts before multiplication facts are learned. Some of the strategies for multiplication facts rely on adding numbers mentally.

To calculate 6 sevens, for example, students may rely on the fives facts and say '5 sevens are 35 and another 7 makes 42'. If 35 + 7 cannot be added mentally and quickly, the strategy will be of little value.

It is also important for students to be able to double numbers 'in their heads' for the doubling strategy to be useful.

Three Models of Multiplication

There are three different ways of modelling multiplication: grouping, length and area.

1. Grouping (set) model
   Example: Show 4 teams with 5 people in each team.

2. Linear or length model (sometimes called a measurement model)
   Example: There were 4 skipping ropes each 2 m long.

3. Area (array) model
   Example: Show 4 rows of chairs with
   6 chairs in each row.

   It is important that students model and explain multiplication involving all three types of situations in order to develop a full understanding of the concept - however continued emphasis should be placed on the array model and the word 'by' as both emphasis commutativity (4 x 3 = 3 x 4) much better than the other models.

   Teachers are encouraged to use a consistent approach that involves the same language, model and symbolic representation from the beginning of the concept through to the final stages of the algorithm.

   Whatever model is used, the children should see and hear it in a variety of forms before the symbol 'x' is introduced. Thus, when discussing the number of tyres on three motor cars, the question may be expressed in these ways:

   4
   4
   +4
   _12

   3 groups of 4;

   4 put out 3 times; 3 fours.

   Although the children will be exposed to the three different multiplication models, continued emphasis may be placed on the array model and the work 'by'. Both emphasise commutativity (4 x 3 = 3 x 4) much better than the other models. The work 'times' does not provide as meaningful a context as 'by'.

Multiplication Facts - Chapter 4 : 7
Introducing the Multiplication Symbol *(Introduce Year 3)*

Children are formally introduced to the multiplication symbol (\times) in Year 3, although many of them will already be familiar with it.

Discuss with the children the difficulty of writing out all the words they use to describe an arrangement. Include in the discussion a description of how the symbols + and - relate to the addition and subtraction concepts. Introduce the use of the multiplication symbol (\times) to replace the words.

You may like to discuss the origin of the symbol with the children. In the late fifteenth century, an Italian mathematician, Pacioli, introduced a method called cross multiplication. For example, 76 \times 28 was set out as:

\[
\begin{array}{c}
7 \\
6 \\
\hline
2 \\
8
\end{array}
\]

Ask the children if they may see how the multiplication symbol came to be used; you may explain that, because setting crossed lines was difficult, printers often used the letter x instead.

The history of mathematics provides a wealth of ideas for discussion at all Year levels.

When recording the problem, use the vertical setting out, with the amount given (that is, the number in each row) being the top number and the number of groups (that is, the number of rows) the bottom number. Avoid the use of the word ‘times’.

When the children read the recording, it is preferable if they say, for example:

\[
\begin{array}{c}
2 \\
\cdot \cdot \\
\cdot \cdot \\
\cdot \cdot \\
\cdot \cdot \\
\cdot \cdot \\
\end{array}
\begin{array}{c}
2 \text{ in each row} \\
\times \text{5 rows} \\
\end{array}
\begin{array}{c}
2 \text{ multiplied by 5’} \\
\text{or} \\
2 \text{ by 5’} \\
\text{or} \\
‘5 twos’
\end{array}
\]

The children will need to develop familiarity with the symbolisation. The following activities will help them do this.

- Match a picture with a symbolisation.
  - ‘Two groups with three butterflies in each group’.

\[\text{The symbolisation would be: } 3 \times 2\]

(2 threes or 3 multiplied by 2).

- Draw a picture for this symbolisation:

\[\begin{array}{c}
4 \\
\times 2 \\
\end{array}\]
**Turnarounds/Commutative Principle**

The idea of 'turnarounds' or the commutative principle should be emphasised in each strategy. If the children understand that $6 \times 4$ is the same amount as $4 \times 6$ then the number of facts to be learned may be halved.

The array model leads to an informal analysis of the commutative principle of multiplication and may be used for all facts except zeros facts.

---

**The Array Model**

The array model has considerable potential for teaching multiplication facts and their turnarounds and should therefore be one of the strategies.

An informal analysis of the commutative principle of multiplication for $3 \times 4$ and $4 \times 3$ is explored below.

<table>
<thead>
<tr>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>⋅</td>
<td>⋅</td>
</tr>
<tr>
<td>⋅</td>
<td>⋅</td>
</tr>
<tr>
<td>⋅</td>
<td>⋅</td>
</tr>
<tr>
<td>⋅</td>
<td>⋅</td>
</tr>
<tr>
<td>and</td>
<td></td>
</tr>
<tr>
<td>3 rows of 4</td>
<td></td>
</tr>
<tr>
<td>4 put out 3 times</td>
<td></td>
</tr>
<tr>
<td>4 used 3 times</td>
<td></td>
</tr>
<tr>
<td>4 in each row</td>
<td></td>
</tr>
<tr>
<td>× 3 rows</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
<tr>
<td>× 3</td>
<td></td>
</tr>
<tr>
<td>and</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>× 4</td>
<td></td>
</tr>
<tr>
<td>4 in each row</td>
<td></td>
</tr>
<tr>
<td>× 4 rows</td>
<td></td>
</tr>
<tr>
<td>4 rows of 3</td>
<td></td>
</tr>
<tr>
<td>4 put out 4 times</td>
<td></td>
</tr>
<tr>
<td>3 used 4 times</td>
<td></td>
</tr>
</tbody>
</table>

Both have an answer of 12

Children are routinely exposed to arrays in the form of egg cartons, chocolate bars, crates, etc. Use these familiar objects to reinforce the idea of multiplication. Some work will be needed to develop the notion of rows and columns.

Discuss the idea of a row with the children so that they understand its meaning e.g.: 'Your arms are in a row when you hold them out to the sides'.

*Multiplication Facts - Chapter 4: 9*
Recall Strategies
Multiplication Facts

All multiplication facts may be taught using the array model. Many sets of facts have other strategies which are also suitable. The twos have the 'double pictures', the fives have the 'clock face' and the nines have the finger pattern etc.

The number of facts to be learned may be halved of the idea of 'turnarounds' or the commutative principle is explored for each set of facts. The array model greatly assists this principle. Calculators may also be used to check children's inferences about 'turnarounds'.

**The Twos Facts (introduce Year 3)**

About the Strategies

- Exploring the twos facts using arrays.
  Students may develop the twos facts for themselves by building up an array for each fact and recording the fact. For 2 twos, students may draw a 2 by 2 array:
  
  - Another row of dots may be added to represent 3 two and the fact may be recorded. This process may be continued for the rest of the facts. Alternatively, a new array may be drawn for each new fact.

  Students should be encouraged to see the numbers as 'turnarounds'. If the arrays were turned around or rotated, they would suggest another fact, e.g. 4 rows of 2 would look like 2 rows of 4, therefore 4 twos, i.e.

  \[
  \begin{array}{ccc}
  2 & \vdots & 2 \\
  \vdots & \vdots & \vdots \\
  4 & \vdots & \vdots \\
  \vdots & \vdots & \vdots \\
  \end{array}
  \]  
  \[2 \text{ rows of } 4\]

  Both give an answer of 8

- Exploring the twos facts using doubles.
  The 'turnarounds' to the twos facts may be verified by the addition doubles pictures, e.g. the picture for double 3 is:

  (Pictures may be changed to ones that the students prefer). Students may see that 2 threes is the same as double 3. When students are confronted with a two fact, they may think of the 'turnaround' that relates to the doubles picture. The suggested pictures for the doubles are:
In other words, students should realise that when a two is one of the numbers in the calculation, the answer may be found by doubling the other number. Plenty of opportunities should be given to students to practise doubling numbers, because this skill is important in learning some of the harder multiplication facts. To double a two-digit number (26), for example, students could use this procedure:

**Double 26, think ...**

### Language

<table>
<thead>
<tr>
<th>The Twos Facts</th>
<th>and</th>
<th>Its' Turnaround</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 fours</td>
<td>and</td>
<td>4 twos</td>
</tr>
<tr>
<td>4 lots of / sets of / groups of 2</td>
<td>and</td>
<td>2 lots of / sets of / groups of 2</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>2</th>
<th>and</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td></td>
<td>2</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

| 2 put out 4 times | and | 4 put out 2 times |
| 2 used 4 times    | and | 4 used 2 times    |

| '4 rows of 2'     | and | '2 rows of 4'     |
|                   |     |                  |

| 2 in each row     | and | 4 in each row     |
| x 4 rows          |     | x 2 rows          |

<table>
<thead>
<tr>
<th>2</th>
<th>and</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>x 4</td>
<td></td>
<td>x 2</td>
</tr>
</tbody>
</table>

| 2 multiplied by 4 | and | 4 multiplied by 2 |
| 2 by 4            | and | 4 by 2            |
| 4 twos            | and | 2 fours           |

Both have an answer of 8

### Examples:

<table>
<thead>
<tr>
<th>The Two Facts</th>
<th>and</th>
<th>their turnarounds</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 2 2 2 2 2 2 2 2 2</td>
<td>x 0 x 1 x 2 x 3 x 4 x 5 x 6 x 7 x 8 x 9</td>
<td>0 1 2 3 4 5 6 7 8 9</td>
</tr>
</tbody>
</table>
| 2 2 2 2 2 2 2 2 2 2 | x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 |}

References:

- **Year 3 Sourcebook** pp. 119-122
- **Year 4 Sourcebook** pp. 108-111
- **Year 5 Sourcebook** p. 45

*Multiplication Facts - Chapter 4: 11*
The Fives Facts (Introduce Year 3)

About the Strategy

- Exploring the fives facts using arrays.

Students may develop the list of fives facts to be learned by drawing arrays and recording the facts as shown below.

<table>
<thead>
<tr>
<th>Array</th>
<th>Recording</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 fives</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td>x 2</td>
</tr>
<tr>
<td></td>
<td>10</td>
</tr>
<tr>
<td>3 fives</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td>x 3</td>
</tr>
<tr>
<td></td>
<td>15</td>
</tr>
</tbody>
</table>

Another row of 5 dots may be added each time to represent a new fact, or separate arrays may be drawn for each fact. After each fact is developed, the array should be ‘turned around’ so students may see that the number of dots remains the same regardless of the order of the numbers in the fact.

These ‘turnaround’ facts may also be recorded and explained.

- Exploring the fives facts using the clock face.

The analogue clock is very useful for providing a mnemonic for the five facts. Before it is used to teach the facts, students must be very familiar with the number of minutes represented by each number on the clock when the minute hand is pointing to it.

The following steps are suggested for working with students who do not have this knowledge.

- Draw a large clock fact on the black board.
- Write in the numbers with the help of the students.
- Discuss with students the minute strokes and how many minutes would have passed (starting from the 12) if the minute hand pointed to the 1, 2, 3 ... 10). (11 and 12 may be included, but these are not needed for the facts). Write the minutes past on the outside of the clock, emphasising the minutes represented by the quarter hour marks.
- Practise until students know that the 3, 6 and 9 on the clock represent 15, 30 and 45 minutes respectively.
- Other facts may be worked out from here, e.g.
  - 2 fives - learned as a double
  - 4 fives - 3 fives are 15 and one more 5 is 20
  - 5 fives - 6 fives are 30. 5 before 30 is 25 (or 5 mins before half past)
  - 7 fives - 6 fives are 30 and one more 5 is 35 (5 mins after half past)
  - 8 fives - 10 mins after half past (or 5 mins before the 45 minute mark)
To practise the facts teachers may begin by keeping all the numbers outside the clock visible, and gradually rub off some of the numbers as the lesson progresses. Keep the quarter hour markers visible for the longest time as they may act as reference points. Continually relate the numbers to the five facts.

Point to the numbers inside the clock and ask students to say the minutes represented. Remind students of the strategies they may use to work out some of the answers. At the end of the lesson, rub off all outside numbers and repeat the activity. Students learn these facts through continued practice and teachers are therefore encouraged to give short practice lessons each day until a high level of proficiency is reached.

Throughout the lessons the 'turnarounds' should also be recorded and practised. Students should realise that when one of the numbers in a multiplication fact is a 5, they may think of the clockface, to help them remember the answer.

### Language

<table>
<thead>
<tr>
<th>The Fives Facts</th>
<th>and</th>
<th>Its Turnaround</th>
</tr>
</thead>
<tbody>
<tr>
<td>3 lots of/sets of/groups of 5</td>
<td>and</td>
<td>5 lots of/sets of/groups of 3</td>
</tr>
<tr>
<td>5</td>
<td>and</td>
<td>3</td>
</tr>
<tr>
<td>3</td>
<td>and</td>
<td>5</td>
</tr>
</tbody>
</table>

| 5 put out 3 times | and | 3 put out 5 times |
| 5 used 3 times | and | 3 used 5 times |

| 3 rows of 5' | and | '5 rows of 3' |
| 5 in each row | and | 3 in each row |
| x 3 rows | and | x 5 rows |

| 5 | and | 3 |
| x 3 | and | x 5 |

| 5 multiplied by 3 | and | 3 multiplied by 5 |
| 5 by 3 | and | 3 by 5 |
| 3 fives | and | 5 threes |

Both have an answer of 15
Examples:

<table>
<thead>
<tr>
<th>The Fives Facts</th>
<th>5</th>
<th>5</th>
<th>5</th>
<th>5</th>
<th>5</th>
<th>5</th>
<th>5</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>and</td>
<td>x 0</td>
<td>x 1</td>
<td>x 2</td>
<td>x 3</td>
<td>x 4</td>
<td>x 5</td>
<td>x 6</td>
<td>x 7</td>
</tr>
<tr>
<td>their turnarounds</td>
<td>x 5</td>
<td>x 5</td>
<td>x 5</td>
<td>x 5</td>
<td>x 5</td>
<td>x 5</td>
<td>x 5</td>
<td>x 5</td>
</tr>
</tbody>
</table>

References:

*Year 3 Sourcebook pp. 122-124*
*Year 4 Sourcebook pp. 111-112*
*Year 5 Sourcebook p. 45*

Once students have developed confidence with the fives facts, the twos facts may be included in the activities thereby requiring students to discriminate between facts and strategies.

Students should be encouraged to identify those facts that may be calculated by more than one strategy, and decide which strategy/strategies they prefer to use in those cases.

(Refer to the grey shaded areas of the Scope and Sequence - Multiplication Facts in this chapter).

*E.g.*: A child may prefer to learn \( \frac{2}{5} \) and \( \frac{5}{2} \) as a double (twos fact strategy) than by using the clock fact (fives fact strategy).

---

**The Nines Facts (Introduce Year 3)**

About the Strategies

- **Exploring the Nines facts using array model**

Students may develop the list of nines facts to be learned as they did for the twos and fives facts, by drawing arrays and recording the facts, as shown below.

<table>
<thead>
<tr>
<th>Array</th>
<th>Recording</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 nines</td>
<td>9 ( \times 2 ) ( = 18 )</td>
</tr>
</tbody>
</table>

Students should be encouraged to identify the 'turnarounds' to these facts by looking at the arrays from a different direction. The above array of 9 by 2 may be turned around (rotated) to show a 2 by 9 array, thereby representing the 'turnaround' fact of 9 twos.

- **Exploring the nines facts using the nines pattern.**

<table>
<thead>
<tr>
<th>x 9</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 18</td>
</tr>
<tr>
<td>3 27</td>
</tr>
<tr>
<td>4 36</td>
</tr>
<tr>
<td>5 45</td>
</tr>
<tr>
<td>6 54</td>
</tr>
<tr>
<td>7 63</td>
</tr>
<tr>
<td>8 72</td>
</tr>
<tr>
<td>9 81</td>
</tr>
</tbody>
</table>

With the help of the students, list the nines facts in a table as shown below and ask students to look for patterns. An obvious pattern is the ascending and descending counting numbers displayed down the columns, but this would not help a student answer a fact in isolation.

Some students may see that the first digit of the answer is always one less than the number of nines, e.g. 8 nines \((9 \times 8)\) are seventy ... something. 7 is one less than 8.
Another pattern shows that both digits in the answer add to 9. Eight nines would therefore be 72, because 7 + 2 = 9.

Think

■ Exploring the nines finger pattern

Number each of the fingers from 1 to 10 beginning with the left thumb (see figure below)

Using this method, when the answer to three nines is required, the third finger is bent. The fingers on the left of that finger represent the tens and the fingers on the right represent ones. The answer is 2 tens and 7 ones (27).

Conversely, when the answer to $9 \times *** = 27$ is required, the number 27 (2 tens 7 ones) is represented by separating the first 2 fingers from the last 7 fingers by a bent finger. The number of the bent finger (3) gives the answer. This method works for the desired range (to $9 \times 9$) and is unique to the nines facts.

■ Exploring the nines facts using ten.

The nines facts may be found by using 10 to estimate. Since 9 is close to 10, any nine fact will be close to the equivalent ten fact, for example:

$$9 \times 5 \quad \text{are 1 five less than} \quad 10 \times 5$$

Other patterns should be investigated. The children should be able to give approximate conclusions to the nines facts and decide on the range within which the answer will lie, e.g.:

<table>
<thead>
<tr>
<th>FACT</th>
<th>ESTIMATE</th>
<th>RANGE</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 nines</td>
<td>just less than 2 tens</td>
<td>in the teens</td>
</tr>
<tr>
<td>3 nines</td>
<td>just less than 3 tens</td>
<td>in the twenties</td>
</tr>
<tr>
<td>4 nines</td>
<td>just less than 4 tens</td>
<td>in the thirties</td>
</tr>
<tr>
<td>5 nines</td>
<td>just less than 5 tens</td>
<td>in the forties</td>
</tr>
</tbody>
</table>

Show the children how to find the exact answer, for example:

* 2 nines are 20 subtract 2;
* 4 nines are 40 subtract 4;
* 3 nines are 30 subtract 3;
* 5 nines are 50 subtract 5…

Multiplication Facts - Chapter 4:15
### Language

<table>
<thead>
<tr>
<th>The Nines Fact</th>
<th>and</th>
<th>Its Turnaround</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ \times 2 ]</td>
<td>and</td>
<td>[ \times 9 ]</td>
</tr>
</tbody>
</table>

- 2 lots of / sets of / groups of 9 and 9 lots of / sets of / groups of 2

<table>
<thead>
<tr>
<th>9</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>9</td>
</tr>
<tr>
<td>[ \times 2 ]</td>
<td>[ \times 9 ]</td>
</tr>
</tbody>
</table>

- 9 put out 2 times and 2 put out 9 times
- 9 used 2 times and 2 used 9 times
- 2 rows of 9 and 9 rows of 2
- 9 in each row \[ \times 2 \text{ rows} \] and 2 in each row \[ \times 9 \text{ rows} \]

<table>
<thead>
<tr>
<th>9</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ \times 2 ]</td>
<td>[ \times 9 ]</td>
</tr>
</tbody>
</table>

- 9 multiplied by 2 and 2 multiplied by 9
- 9 by 2 and 2 by 9
- 2 nines and 9 twos

**Both have an answer of 18**

### Examples:

<table>
<thead>
<tr>
<th>The Nines Facts</th>
<th>[9 \times 0 \times 1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8 \times 9]</th>
</tr>
</thead>
<tbody>
<tr>
<td>and their turnarounds</td>
<td>[9 \times 9 \times 9 \times 9 \times 9 \times 9 \times 9 \times 9 \times 9 \times 9]</td>
</tr>
</tbody>
</table>

### References:
- Year 3 Sourcebook pp. 122-124
- Year 4 Sourcebook pp. 112-113
- Year 5 Sourcebook p. 45

Once students have developed confidence with the nines facts, the twos and fives facts may be included in the activities along with the nines facts, thereby requiring students to discriminate between facts and strategies. Students should be encouraged to identify those facts that may be calculated by more than one strategy, and decide which strategy / strategies they prefer to use in those cases.

*Multiplication Facts - Chapter 4 : 16*
The Zeros Facts *(Introduce Year 3)*

**About the Strategy**

Although there is no real strategy for developing the zeros facts, the Year 3 Sourcebook outlines some activities designed to assist with understanding.

When introducing the multiplication concept of zero, be aware that the children will not have encountered many real world examples involving zero.

The children should discover that they cannot draw any diagrams to represent a zeros fact and its turnaround.

Children may be lead to the generalisation that:
- multiplying a number by zero gives an answer of zero
- everytime zero is one of the numbers to be multiplied, the answer will be zero.

**Language**

The zeros facts and their turnarounds may be recorded as:

\[
\begin{array}{c}
0 \\
x \times 7 \\
\times 0
\end{array}
\]

that is, 7 zeros or 7 nothings. That is zero sevens, or no sevens.

The turnaround for zero is difficult for children to understand if the ‘rows of’ or any other language is used. It is recommended that the children use the language - ‘The Turnaround is the same - zero’.

**Examples:**

<table>
<thead>
<tr>
<th>The Zeros Facts</th>
<th>0</th>
<th>0</th>
<th>0</th>
<th>0</th>
<th>0</th>
<th>0</th>
<th>0</th>
<th>0</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>and</td>
<td>x0</td>
<td>x1</td>
<td>x2</td>
<td>x3</td>
<td>x4</td>
<td>x5</td>
<td>x6</td>
<td>x7</td>
<td>x8</td>
</tr>
<tr>
<td>their turnarounds</td>
<td>x0</td>
<td>x0</td>
<td>x0</td>
<td>x0</td>
<td>x0</td>
<td>x0</td>
<td>x0</td>
<td>x0</td>
<td>x0</td>
</tr>
</tbody>
</table>

**References:**

*Year 3 Sourcebook pp. 125-126 - with activities*
*Year 4 Sourcebook p. 114*
*Year 5 Sourcebook p. 45*

Once students have developed confidence with the zero facts, the twos, fives and nines facts may be included in the activities along with the zero facts, thereby requiring students to discriminate between facts and strategies.
# The Ones Facts (Introduce Year 3)

## About the Strategy

- **Exploring the ones facts using the array model.**
  Students should develop the ones facts as they did for the twos, fives and nines facts by:
  - drawing the arrays for the ones facts and recording the facts e.g.
    
    | Array | Recording |
    |-------|-----------|
    | 6 ones | \[ \begin{array}{c} \bullet \\ \bullet \\ \bullet \\ \bullet \\ \bullet \end{array} \] | \[ \times 6 \] |
  - identifying the 'turnarounds' to the facts and recording them.

  Lead children to generalise that:
  - multiplying a number by one does not change the number
  - whenever 1 is one of the numbers to be multiplied, the answer will always be the other number.

## Language

<table>
<thead>
<tr>
<th>The Ones Facts</th>
<th>and</th>
<th>Its Turnaround</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ \times 6 ]</td>
<td></td>
<td>[ \times 1 ]</td>
</tr>
<tr>
<td>6 lots of</td>
<td>and</td>
<td>1 lot of</td>
</tr>
<tr>
<td>/sets of</td>
<td>and</td>
<td>/set of</td>
</tr>
<tr>
<td>/groups of 1</td>
<td>and</td>
<td>/group of 6</td>
</tr>
</tbody>
</table>

| 1 | and | 6 |
| 6 |     |   |

| 1 put out 6 times | and | 6 put out once |
| 1 used 6 times | and | 6 used once |

| '6 rows of 1' | and | '1 row of 6' |
| 1 in each row | and | 6 in each row |
| \[ \times 6 \times 1 \] | and | \[ \times 1 \times 1 \] |

| 1 multiplied by 6 | and | 6 multiplied by 1 |
| 1 by 6 | and | 6 by 1 |
| 6 ones | and | 1 six |

Both have an answer of 6
Examples:

<table>
<thead>
<tr>
<th>The Ones Facts</th>
<th>1</th>
<th>1</th>
<th>1</th>
<th>1</th>
<th>1</th>
<th>1</th>
<th>1</th>
<th>1</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>x 0</td>
<td>x 1</td>
<td>x 2</td>
<td>x 3</td>
<td>x 4</td>
<td>x 5</td>
<td>x 6</td>
<td>x 7</td>
<td>x 8</td>
</tr>
</tbody>
</table>

and their

turnarounds

|               | x 1 | x 1 | x 1 | x 1 | x 1 | x 1 | x 1 | x 1 | x 1 |

Once students have developed confidence with the ones facts, the twos, fives, nines and zeros facts may be included in the activities along with the ones facts, thereby requiring students to discriminate between facts and strategies.

Students should be encouraged to identify those facts that may be calculated by more than one strategy and decide which strategy/strategies they prefer to use in those cases.

**The Remaining Facts (Introduce Year 3/4)**

By encouraging children to colour or tick off facts studied on their multiplication chart as they progress, it will become apparent that, at this stage, there are only 15 facts remaining. These facts fall within the threes, sixes, fours, eights and sevens facts and are as shown.

You will note that 5 of these 15 remaining facts are square number facts. For this reason you may now choose to teach a strategy to work out square numbers.

In addition to a strategy for square number facts, strategies for the threes, sixes and fours facts are also outlined below. The author has rearranged the remaining facts that are presented on pages 114 and 115 of the Year 4 Sourcebook so that as many of the remaining facts as possible occur in facts which have some strategy.

Since the very great majority of children should now be proficient with all the multiplication facts other than these 15 remaining facts – the children should be encouraged to investigate and discuss some strategies of their own. It is possible that the children’s strategies will be just as legitimate and user friendly as those advocated by the sourcebooks. Remember, the child should be encouraged to use the strategy which he/she prefers. Also keep in mind that these strategies provide children with the means for calculating multiplication facts quickly and easily. It is important to note that these strategies are a means to an end and not an end in themselves.

Instant recall from memory and understanding remains the ultimate goal.
The Square Number Facts  (Introduce Year 3/4)

About the Strategy:

The children may prefer to think of the grid pattern for any square number. For example, in the case of $4 \times 4$, they would think of a pattern of 4 rows with 4 in each row. The total number of squares in the grid is 16.

Square numbers may easily be generated on the calculator; for example:

\[ \text{for } 5 \times 5, \text{ press } '5 \times = ' \text{ (gives 25)} \]

Discuss the arrangement of the square numbers on the grid, that is, the diagonal setting out. Have the children consider what happens to 'turnarounds' – that they are the same.

References:
- Year 3 Sourcebook pp. 128-129
- Year 4 Sourcebook p. 115
- Year 5 Sourcebook p. 46

The Threes Facts  (Introduce Year 3/4)

The threes facts are build-on facts because they may be obtained by adding an amount to a known fact. The threes facts may be obtained by building onto the twos facts, therefore the twos facts must be well known.

Mary arrays to show how to build onto the facts to obtain the threes facts, for example:

\[
\begin{array}{cccc}
2 & \cdot & \cdot & \cdot \\
\times 4 & \cdot & \cdot & \cdot \\
\end{array}
\]

Add another row to make:

\[
\begin{array}{cccc}
3 & \cdot & \cdot & \cdot \\
\times 4 & \cdot & \cdot & \cdot \\
\end{array}
\]

The children should be able to explain that 2 rows of four are 8, and another four are 12. So 3 rows of four are 12. Allow the children to examine the turnarounds and decide which order they prefer when working out the answer.

\[
\begin{array}{cccccc}
3 & 3 & 3 & 3 & 3 \\
\times 3 & \times 4 & \times 6 & \times 7 & \times 8 \\
\end{array}
\]

References:
- Year 3 Sourcebooks p. 129
- Year 4 Sourcebooks p. 116
- Year 5 Sourcebooks p. 46

Multiplication Facts - Chapter 4 : 20
The Sixes Facts  (*Introduce Year 3/4*)

The sixes facts are also build-on facts because they may be obtained by adding on to a known amount. For this reason ensure the children may recall the fives facts quickly and in either order, for example: '5 fours' or '4 fives'.

Involve the children in making arrays to see how to obtain the sixes facts by building onto the fives facts, for example:

\[
\begin{array}{cccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccco
This pair of number facts, although possibly the last multiplication facts to be learned - do have a strategy which is unique to them.

For 7 eights students may use the number sequence 5, 6, 7, 8 to derive 56 = 7 eights. 8 sevens could then be taught as the turnaround for 7 eights.

Reference:
Year 4 Sourcebook p. 116
Year 5 Sourcebook p. 46
It is important that the teachers and students use correct and consistent language, models and symbolic representation from the introduction of the multiplication concept right though the grades.

The following language, models and symbolism is that which is advocated by the syllabus and should be used in all classrooms, in all schools throughout Queensland to promote continuity in each child’s learning experience, especially because of the high rate of student mobility between schools.

When recording the problem, use the vertical setting out, with the amount given, that is, the number in each row, being the top number and the number of groups, that is, the number of rows, the bottom number.

Avoid the use of the word ‘times’.

2 in each row 2
x 5 rows x 5

When numbers are recorded vertically in this manner, they may be read downwards as ‘2 x 5’ meaning ‘2 multiplied by 5’ or may be read upwards as ‘5 twos’ to be consistent with the multiplication algorithm which reads upwards. Students may use the procedure for saying and recording multiplication facts e.g. the twos facts may be:

<table>
<thead>
<tr>
<th>Conceptual Development</th>
<th>recorded as</th>
<th>x 0</th>
<th>x 1</th>
<th>x 2</th>
<th>x 3</th>
<th>x 9</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>and read upwards as</td>
<td>‘0 twos’</td>
<td>‘1 two’</td>
<td>‘2 twos’</td>
<td>‘3 twos’</td>
<td>‘9 twos’</td>
</tr>
<tr>
<td>or downwards as</td>
<td>‘2 multiplied by 0’</td>
<td>‘2 multiplied by 1’</td>
<td>‘2 multiplied by 2’</td>
<td>‘2 multiplied by 3’</td>
<td>‘2 multiplied by 9’</td>
<td></td>
</tr>
</tbody>
</table>

Mastery

| and finally as         | ‘0 twos’ | ‘1 two’ | ‘2 twos’ | ‘3 twos’ | ‘9 twos’ |

Refer to ‘Recall Strategies - Multiplication Facts’, Section 4 of this chapter for the correct language, models and symbolism for each strategy.
Language provides a link between children’s manipulation of materials and their symbolic representations, and between their past and present experiences.

Initially children will use their own everyday language to describe what they are thinking and doing, but as addition becomes more internalized, the childrens’ language should become the same as the formal language.

<table>
<thead>
<tr>
<th>Year 2</th>
<th>Year 3</th>
<th>Year 4</th>
<th>Year 5</th>
<th>Year 6</th>
<th>Year 7</th>
</tr>
</thead>
<tbody>
<tr>
<td>groups of</td>
<td>as for Year 1</td>
<td>as for previous years</td>
<td>as for previous years</td>
<td></td>
<td></td>
</tr>
<tr>
<td>lots of</td>
<td>arrange</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>sets of</td>
<td>rows of</td>
<td>9 twos, etc</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>each</td>
<td>bundles</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>every</td>
<td>multiply by</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>equal</td>
<td>put out</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Please note:** The word ‘times’ does not provide as meaningful a context as ‘by’ and is discouraged by the syllabus.

**Related language**

<table>
<thead>
<tr>
<th>Year 2</th>
<th>Year 3</th>
<th>Year 4</th>
<th>Year 5</th>
<th>Year 6</th>
<th>Year 7</th>
</tr>
</thead>
<tbody>
<tr>
<td>odd/even</td>
<td>multiples facts</td>
<td>prime composite</td>
<td>square numbers</td>
<td>square root</td>
<td></td>
</tr>
</tbody>
</table>
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      The Five Facts...............................4b
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      The Zeros Facts..............................4d
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      The Calendar Facts.........................4f (III)
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This scope and sequence, together with 'Recall Strategies - Division Facts' part 4 of this Chapter provides the teacher with one path through the recall strategies for division as advocated by the syllabus. It is inclusive in that upon completion one or more strategies has been presented for all division facts to 9/81.

<table>
<thead>
<tr>
<th>Type of Facts</th>
<th>Easier Facts – Introduce Year 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>The Twos Facts and their partners</td>
<td>2</td>
</tr>
<tr>
<td>The Fives Facts and their partners</td>
<td>1</td>
</tr>
<tr>
<td>The Nines Facts and their partner</td>
<td>9</td>
</tr>
<tr>
<td>The Zeros Facts (No partners)</td>
<td>1</td>
</tr>
<tr>
<td>The Ones Facts and their partners</td>
<td>1</td>
</tr>
</tbody>
</table>

Denotes those division facts which have been taught using an earlier strategy.
The Remaining Facts - Harder Facts

After the twos, fives, nines, zeros and ones facts and their partners have been taught, there are only 15 remaining facts to be learned. Since 5 of the remaining 15 facts are square number facts and another 7 are built on facts, the following progression through the 15 remaining facts is recommended by the author who has rearranged those presented on page 143 of the Year 4 Sourcebook so that as many remaining facts as possible occur in sets of facts which have some strategy. Refer to 'Recall Strategy - Division Facts' in section 4 of this chapter for an outline of the strategies for the number fact groups mentioned below.

<table>
<thead>
<tr>
<th>Type of Facts</th>
<th>Harder Facts - Introduce Year 4, continue Year 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>The Square Number Facts</td>
<td>1</td>
</tr>
<tr>
<td>The Threes Facts and their partners</td>
<td>3</td>
</tr>
<tr>
<td>The Sixes Facts and their partner</td>
<td>6</td>
</tr>
<tr>
<td>The Fours Facts and their partners</td>
<td>4</td>
</tr>
<tr>
<td>The Eights Facts and their partners</td>
<td>8</td>
</tr>
<tr>
<td>The Sevens Facts and their partners</td>
<td>7</td>
</tr>
</tbody>
</table>

Denotes those division facts which have been taught using an earlier strategy.
In Year 1 children’s investigations of the numbers to 10 lead them to adding, subtracting, multiplying and dividing these numbers in order to compare and analyse the numbers’ size. Children also create their own number stories involving these numbers and operations. In Year 1 however, addition is the only operation which is formally recorded and developed.

In the Year 2 topic “Number Study”, children arrange materials into small groups and then into groups of tens and ones to study the place value of numbers to 99. These grouping activities give children experience with multiplication and division processes.

In Year 2, number problems from school, home and environmental situations are investigated. The focus is on developing a variety of strategies to solve such problems, discussing the problems using the appropriate language and representing them accurately with materials. Even though no formal development of division is intended until Year 3, children may wish to record these problems and their solutions using a combination of pictures, words and digits. The associated concept of odd and even will be introduced in Year 2.

In Year 3, children are given considerable opportunity to develop a solid understanding of the division concept by representing and exploring division using physical materials and diagrams. Many children find division more difficult than addition, subtraction and multiplication. This is because they do not have a clear understanding of the division concept before they are introduced to division symbolism and the algorithm. Other reasons for their difficulty include failure to make links between multiplication and division, a too rapid move to symbolism and the use of language that does not convey a clear meaning (for example, “6 goes into 12 ...”). Year 3 teachers must be most diligent in their efforts to facilitate full understanding of the division concept. Introduction to the division algorithm and use of the symbol is Year 4 work.

In Year 3 division can be recorded as 12 shared among 4.

Although division in the real world can be categorised as two main types: partition (sharing) and quotition (masking groups) Year 3 exploration activities focus upon partition because:

- it occurs more often in real life and is therefore more meaningful;
- it can be demonstrated more effectively with concrete materials;
- it is used as the basis for further development of the concept in Year 4.

(Refer to Part 3, Glossary of Terms in this chapter).

Year 3 activities focus on giving the children ample opportunity to use a variety of materials to explore the concept and language associated with division.
Formal Development of Division Facts

Year 4 activities continue the development of the division concept through the use of physical materials, diagrams and language.

Once students are comfortable with the recognition of situations involving division, and with the drawing of diagrams to represent them, the following symbolic recording can be introduced.

<table>
<thead>
<tr>
<th>3</th>
<th>15</th>
</tr>
</thead>
<tbody>
<tr>
<td>3 (\frac{\text{can be read as:}}{\text{15 divided by 3; or}})</td>
<td>15 shared among 3</td>
</tr>
</tbody>
</table>

To work out answers to these situations, the students will need a strategy which enables them to interpret and understand the given problem. They should be advised to change the division into a multiplication which involves known facts, e.g. say “3 whats are 15?”

In doing this, the students are relating division to multiplication, a relationship which will always be very useful.

It is important to keep language consistent. The development of the written algorithm in Year 5 depends heavily on a consistent language pattern that explains the sharing out of physical materials. It is inconsistent, for example, to ask children to read

\[
3 \div 15
\]
as “how many threes in 15?” This could lead to confusion when the written algorithm is developed.

After students have had considerable practice representing “sharing” situations with diagrams and symbols, “grouping” situations can be included. The students should realise that the recording is the same for both situations and that both can be solved by relating the division fact to the known multiplication facts. (Refer to Part 3, Glossary of Terms, in this chapter).

Year 4 students can be practising strategies for learning division facts. The main strategy for learning a division fact is to relate it to known multiplication fact, e.g.

say: ‘6 whats are 30?’

For \(6 \div 30\)

‘6 fives are 30’

‘So 30 divided by 6 is 5’

Another strategy is to rely on counting, i.e. in the above example, counting off groups of six until 30 is reached, but in the long-term, counting is too slow. It could however, be used as a back-up to the preferred method.

There is little value in asking students to learn division facts unless the corresponding multiplication facts are known. Some students, therefore, may not be introduced to division facts until later in the year, when they should have a good knowledge of multiplication facts. Students have until the end of Year 7 before they must be proficient with division facts (although it is expected that many will show proficiency well before then).

The following sequence for the introduction and development of division facts has been designed by the author and varies slightly from that presented in the sourcebooks. This is hopefully a preferred sequence as it may be necessary in some cases to leave some or all of the harder facts until Year 5. It is hoped that this will ease possible confusion created by the conflicting sequences presented on page 135 of the Year 3 sourcebook and page 138 of the Year 4 sourcebook. Endorsement of this scope and sequence will be required at school level.
| Year 2 & 3 | 1. Division Concept  
Using materials, pictures and language to represent division; the sharing process is emphasised. |
| --- | --- |
|  | 2. Recording Division  
Initially division can be recorded as 12 'shared among' 4 |
| Year 4 | 3. Recording division  
12 'divided by' 4. |
|  | 4. Division Facts -  
• Recording the facts e.g. $4 \div 12$  
4a Easier Facts  
• Developing thinking strategies to assist recall  
The Twos Facts  
The Fives Facts  
The Nines Facts  
The Zeros Facts  
The Ones Facts |
| Year 4 & 5 | 4b Harder Facts  
The Square Facts  
The Threes Facts  
The Sixes Facts  
The Fours Facts  
The Eights Facts  
The Sevens Facts |
There are two methods of attack that a school can use.

Either:
A. Wait until the children know all the multiplication facts well before beginning division facts;
B. After children are proficient with one set of multiplication facts, practice the corresponding division facts.

The different methods may suit different situations, but it is important to have consistency throughout the school. At State School it is school policy to use Method A/B.

The facts are presented in a sequence progressing from the easier facts to the more difficult ones. The entire sequence is shown in Part 1 of this chapter. The matching multiplication Scope and Sequence can be found in Part 1 of the Multiplication Facts chapter.

**YEAR 5**  
Continuation of work on harder facts and Consolidation of all facts  
*Year 5 Sourcebooks pp. 46 – 49*

Since students should have a good knowledge of the twos, fives, nines, ones and zero facts and their partners before beginning a study of the remaining facts, it may be necessary in some cases to leave the remaining facts until Year 5. A sound knowledge of the corresponding multiplication facts is essential.

Although mastery of multiplication and division facts is not expected until Year 7, some children may obtain mastery by Year 5.

Work on associated concepts such as odd, even, factors, multiples, prime, composite and square numbers will continue along with the investigation of the Sieve of Eratosthenes. Refer to page 23 of the Year 5 Sourcebook.

**YEAR 6**  
Consolidation  
*Year 6 Sourcebook pp. 29 – 31*

Consolidation of division facts will continue in Year 6.

Work on associated concepts such as odd, even, factors, multiples, prime, composite and square numbers will continue along with the introduction of factor tree work.

**YEAR 7**  
Mastery  
*Year 7 Sourcebook pp. 17 – 18*

Mastery of addition and subtraction facts is expected by Year 4 and mastery of multiplication and division facts is expected by Year 7.

Work on associated concepts such as odd/even, factors/multiples, prime/composite, square numbers and factor trees will continue along with the introduction of the square root and the exploration of divisibility rules.
The main strategy for learning a division fact will be to relate it to a known multiplication fact, e.g.

say: '6 whats are 30?'

For $6 \div 30$

'6 fives are 30'

'So 30 divided by 6 is 5'.

Another strategy is to rely on counting, i.e. in the above example, counting off groups of six until 30 is reached, but in the long-term, counting is too slow. It could however, be used as a back-up to the preferred method.

There is little value in asking students to learn division facts unless the corresponding multiplication facts are known. Some students, therefore, may not have a good knowledge of multiplication facts. Students have until the end of Year 7 before they must be proficient with division facts (although it is expected that many will show proficiency well before then).

There are two methods of attach that a school can use. Either:

A. Wait until the children know all the multiplication facts well before beginning division facts;

or

B. After children are proficient with one set of multiplication facts, practice the corresponding division facts.

The different methods may suit different situations, but it is important to have consistency throughout the school. At State School it is school policy to use Method A/B.

The facts are presented in a sequence progressing from the easier facts to the more difficult ones. The entire sequence is shown in Part 1 of this chapter. The matching multiplication Scope and Sequence can be found in Part 1 of the Multiplication Facts chapter.
## Two Models of Division – Partition & Quotition

### Introduce Partition Initially

### Introduce Quotation Year 4

Although there is one global view of division, i.e. division is the breaking up of a number quantity into equal parts, division in the real world is of two main types: partition (sharing) and quotition (making groups).

<table>
<thead>
<tr>
<th><strong>Partition</strong></th>
<th><strong>Quotition</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Summary:</strong></td>
<td>The number in each group is given. You have to find the number of groups.</td>
</tr>
<tr>
<td><strong>Situation:</strong></td>
<td>I have 12 boys to put into 3 equal teams</td>
</tr>
<tr>
<td><strong>Question:</strong></td>
<td>How many in each team?</td>
</tr>
<tr>
<td><strong>Diagram:</strong></td>
<td><img src="image" alt="Diagram" /></td>
</tr>
<tr>
<td><strong>Language:</strong></td>
<td>12 shared among 3</td>
</tr>
<tr>
<td><strong>Answer:</strong></td>
<td>4 boys in each team</td>
</tr>
</tbody>
</table>

- **Note:** With quotition examples, the “names” in the situation are always the same (e.g. 12 boys, teams of 3 boys).

In Year 2, 3 and initially Year 4, greater emphasis is placed on the partition form because:
- It is closer to “real life” and therefore more meaningful for students;
- It can be demonstrated more effectively with concrete materials; and
- It is used to develop the algorithm and associated language in Year 5.

After students have had considerable practice representing “sharing” situations with diagrams and symbols, “grouping” situations can be included. The students should realise that the recording is the same for both situations and that both can be solved by relating the division to known multiplication facts.

Throughout students should be provided with opportunities to explain situations involving division, and to draw appropriate diagrams. These kinds of activities play a crucial role in helping students firstly to recognise situations involving division, and secondly to become familiar with the different types of diagrams necessary to illustrate the “sharing” and “marking groups” situations.

---

*Division Facts - Chapter 5: 9*
The following word problems may be used as models for the construction of both partitive ("sharing") and quotitive ("group") examples:

- In my exercises, I intend to run 36 kms over 4 days. How far should I run every day so that I run the same number of kilometres each day? ("Sharing")
- Mario caught 12 fish that he packed into two ice boxes. How many in each box? ("Sharing")
- Susan had to pack 24 apples into bags, each holding 6. How many bags did she need? ("Groups")
- Stickers are 5c each. How many can Lea buy for 30c? ("Groups")
- 21 children were selected for 3 tunnel ball teams. How many in each team? ("Sharing")
- Mum had to cut pieces of ribbon 8 cm from a 56 cm length. How many pieces can Mum cut? ("Groups")
- Farmer Cho planted 24 lettuces in 4 rows. How many were in each row? ("Sharing")
- The tyre market had 28 tyres to fit some cars. Each car has 4 tyres. How many cars can have new tyres? ("Groups")
- There are 30 calculators to share evenly among 5 classes. How many should each class be given? ("Sharing")
- I was given an allowance of $18 which had to last for 3 weeks. How much is that per week? ("Sharing")
- 24 children are being taken to sport in 6 cars of the same size. How many children should go in each car? ("Sharing")

**The Division Concept**

Many students find the division algorithm more difficult than the algorithms for additions, subtraction and multiplication. Reasons often given include the following:

- Many students do not have a good understanding of the division concept before they are introduced to the algorithm;
- Students are often asked to "learn" the algorithm in an abstract manner which does not reinforce understanding;
- Approaches and language used vary from teacher to teacher;
- Much of the language used does not convey meaning (e.g. 6 goes into 12)?

For these reasons, students should be given the opportunity to develop a good understanding of the division concept by representing and explaining division using physical materials and diagrams before they are introduced to the algorithm. When students are introduced to the written algorithm in the Year 5 Sourcebook, emphasis is placed on understanding the procedure through the use of physical materials and appropriate language.

![Real-world problem](image1)

![Physical representation](image2)

![Verbal representation](image3)

![Representation in mathematical symbols](image4)

The above model shows the interrelationship of concrete materials, language and symbols within the division concept. This is the same model as that used for addition, subtraction and multiplication.

*Division Facts - Chapter 5 : 10*
In addition to specific strategies that only apply to one set of division facts, there are general strategies and games which can apply to all sets of facts. For the exception of the square facts and the fours, all harder facts do not have specific strategies of their own so have to depend on general strategies and games such as those outlined following.

**General Strategies**

- **Corresponding facts**
  If the students are given a twos division fact, they must be able to associate this fact with the corresponding multiplication fact.
  Example: 
  \[ \begin{array}{c|c}
  5 & 40 \\
  \hline
  \text{think} & \text{40 shared amongst 5} \\
  \end{array} \]
  \[ 5 \times 5 = 25 \]
  \[ 5 \text{ eights are 40.} \]
  These ideas can be checked using materials, diagrams and calculators to reinforce students' observations and assumptions.

- **Finding partners**
  Students should see that two factors of a number remain together like partners, and can be found in the one multiplication fact, e.g. for \( 4 \times 2 = 8 \).
  \[ \begin{array}{c|c}
  2 & 8 \\\n  \hline
  \text{think} & 2 \text{ whats are 8?} \\
  \end{array} \]
  \[ 2 \text{ fours are 8.} \]
  Once again, these ideas can be reinforced using the calculator.

- **Using the calculator to make a family of facts**
  Students are given a fact, or a number with its factors, e.g. 
  \[ \begin{array}{c|c}
  2 & 12 \\\n  \hline
  \end{array} \]
  \[ 2 \text{ divided by 2.} \]
  \[ 2 \text{ whats are 12?} \]
  \[ 2 \text{ sixes are 12.} \]
  Students should be able to write the other division fact (the partner), using a calculator to verify the answer: 
  \[ \begin{array}{c|c}
  6 & 12 \\\n  \hline
  2 \text{ } \end{array} \]
Explaining strategies

Students can be given sets of division facts and be required to explain what strategy they used to determine the answer, e.g.

\[
\begin{array}{cccc}
2 & 16 & 8 & 64 \\
5 & 40 & 2 & 16 \\
9 & 36 & 1 & 8 \\
5 & 25 & \\
\end{array}
\]

Finding the incorrect answers

Ask the students to scan a group of division facts and find any which have incorrect answers. These should then be corrected, e.g.

\[
\begin{array}{cccc}
4 & 16 & 5 & 15 \\
6 & 24 & 7 & 28 \\
8 & 48 & 7 & 56 \\
8 & 64 & & \\
\end{array}
\]

Isolating 'eights'

Ask the children to find and circle as quickly as they can, the division facts which would have an answer of 8. They should explain what strategy they used to help them.

\[
\begin{array}{cccc}
2 & 16 & 4 & 12 \\
6 & 36 & 9 & 72 \\
8 & 64 & 1 & 8 \\
\end{array}
\]

Once both division facts are identified, the corresponding multiplication fact can be written, using calculators to check, e.g.

\[
\begin{array}{ccc}
6 & 2 \text{ what is 12?} & 6 \\
\text{think} & 2 \text{ sixers are 12.} & \times 2 \\
& & 12 \\
\end{array}
\]

\[
\begin{array}{ccc}
2 & 6 \text{ what is 12?} & 2 \\
\text{think} & 6 \text{ twos are 12.} & \times 6 \\
& & 12 \\
\end{array}
\]

Students should have a lot of practice saying and writing the facts belonging to the family. Being able to identify partner division facts will cut down the number of facts to be learned by half.
Using the inverse

Student should be encouraged to use the inverse operation to help work out the division facts, using materials, diagrams or calculators if necessary:

\[
\begin{array}{cccc}
7 & 3 & 9 & 6 \\
2 & 2 & 2 & * \end{array}
\]

Students could use the constant function on the calculator to find the answer to the following:

\[
2 \div 12
\]

**Key in**

<table>
<thead>
<tr>
<th>Key press</th>
<th>Display</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 \times 4 =</td>
<td>8</td>
</tr>
<tr>
<td>5 =</td>
<td>10</td>
</tr>
<tr>
<td>6 =</td>
<td>12</td>
</tr>
</tbody>
</table>

Discuss the relationship between multiplication and division.

**Beat the calculator**

The teacher writes a division fact on the blackboard, e.g.

\[
2 \div 16
\]

Some students use calculators to work out the answer while others perform the calculation mentally, with the aim of finding the answer faster than the calculator operators. Highlight the fact that mental calculations are often more efficient.

**Missing number flashcards**

\[
\begin{array}{cccc}
7 & 3 & 9 & 6 \\
2 & 2 & 2 & \ast \end{array}
\]

Students work in pairs or small groups, taking turns to answer or to show the flashcards. The corresponding multiplication facts should be written on the backs of the cards to help students work out the answers.

**Concentration**

Students match cards depicting completed division facts with cards showing the corresponding multiplication facts, e.g.

\[
\begin{array}{cc}
8 & 8 \\
2 \div 16 & \times 2 \\
& 16
\end{array}
\]
Students identify the numbers that come out of the machine after being operated on by 2.

Check answers with the calculator.

The teacher draws a circle with the numbers placed similarly to the ones below. One student points to the outside numbers in a random fashion, the others recall the relevant twos facts as quickly as possible. The answer can be called out in a group or class session, or by holding round-the-class speed competition.

(This activity can be adapted for any of the number facts.)
Specific Strategies

The sharing process
Students can develop understanding of the twos facts by using materials or objects to actually perform the sharing involved. The teacher can write a situation on the blackboard such as “2 blocks shared between 2” for students to model. The process can be recorded symbolically by both teacher and students. The rest of the twos facts can be built up in this way, until all have been covered:

- 2 blocks shared between 2
  \[ \frac{1}{2} \]
  \[ 2 \]
- 4 blocks shared between 2
  \[ \frac{2}{4} \]
  \[ 2 \]
- 6 blocks shared between 2
  \[ \frac{2}{6} \]
  \[ 2 \]
- 8, 10, 12, 14, 16 blocks...
- 18 blocks shared between 2
  \[ \frac{2}{18} \]
  \[ 2 \]

Students might like to try the sharing with other numbers not included in the list. It will be obvious that not all numbers of blocks can be shared out equally. Some blocks will be left over after the sharing is complete. Those numbers are not “multiples” of 2 and are therefore not included in the twos facts.

Linking division with the doubles
Students should link the twos facts in division with the doubles pictures which will have been used to develop the corresponding multiplication facts:

For \[ \frac{2}{6} \] think
- 2 threes are 6. So 6 divided by 2 is 3.

There may be other pictures that students find just as relevant for the fact above and they should be encouraged to draw them. The other “twos” facts should be dealt with in the same way, using appropriate doubles pictures.

Examples

<table>
<thead>
<tr>
<th>The Twos Facts and their partners</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 1 2 4 3 6 4 8 5 10 6 12 7 14 8 16 9 18</td>
</tr>
</tbody>
</table>

References:
Year 4 Sourcebook p. 144
Year 5 Sourcebook p. 47
Year 6 Sourcebook p. 29
Correct

<table>
<thead>
<tr>
<th>Corresponding multiplication facts</th>
<th>and its partner</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 whats are 6</td>
<td>3 whats are 6</td>
</tr>
<tr>
<td>2 threes are 6</td>
<td>3 twos are 6</td>
</tr>
<tr>
<td>So 6 divided by 2 is 3</td>
<td>So 6 divided by 3 is 2.</td>
</tr>
<tr>
<td><img src="2" alt="2x6" />→3→2)</td>
<td><img src="3" alt="3x6" />→2)</td>
</tr>
</tbody>
</table>

Incorrect

<table>
<thead>
<tr>
<th>How many twos in 6?</th>
<th>How many threes in 6?</th>
</tr>
</thead>
</table>

Examples:

<table>
<thead>
<tr>
<th>The Twos Facts and their partners</th>
<th>2</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
<th>10</th>
<th>12</th>
<th>14</th>
<th>16</th>
<th>18</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>6</td>
<td>8</td>
<td>10</td>
<td>12</td>
<td>14</td>
<td>16</td>
<td>18</td>
</tr>
</tbody>
</table>

References:
- Year 4 Sourcebook p. 144
- Year 5 Sourcebook p. 47
- Year 6 Sourcebook p. 29

The Fives Facts *(Introduce Year 4)*

Specific Strategies

- The sharing process

In a similar way to that used with the twos facts, students should develop understanding of all of the fives facts, using the sharing approach, e.g.

- 5 blocks shared among 5
  
  ![1](5)|5

- 10 blocks shared among 5
  
  ![2](5)|10

- 40 blocks shared among 5
  
  ![8](5)|40

- 45 blocks shared among 5
  
  ![9](5)|45

The examples which follow reinforce these facts, show other ways of developing understanding of the facts, and indicate links with corresponding multiplication facts.
The Twos Facts (Introduce Year 4)

Specific Strategies

The sharing process

Students can develop an understanding of the twos facts by using materials or objects to actually perform the sharing involved. The teacher can write a situation on the blackboard such as “2 blocks shared between 2” for students to model. The process can be recorded symbolically by both teacher and students. The rest of the twos facts can be built up in this way, until all have been covered:

<table>
<thead>
<tr>
<th>Blocks Shared</th>
<th>Division</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 blocks</td>
<td>1</td>
</tr>
<tr>
<td>4 blocks</td>
<td>2</td>
</tr>
<tr>
<td>6 blocks</td>
<td>3</td>
</tr>
<tr>
<td>8, 10, 12, 14, 16 blocks...</td>
<td></td>
</tr>
<tr>
<td>18 blocks</td>
<td>9</td>
</tr>
</tbody>
</table>

Students might like to try the sharing with other numbers not included in the list. It will be obvious that not all numbers of blocks can be shared out equally. Some blocks will be left over after the sharing is complete. Those numbers are not “multiples” of 2 and are therefore not included in the twos facts.

Linking division with the doubles

Students should link the twos facts in division with the doubles pictures which will have been used to develop the corresponding multiplication facts:

For \( 2 \div 6 \), think “2 threes are 6. So 6 divided by 2 is 3.”

There may be other pictures that students find just as relevant for the fact above and they should be encouraged to draw them. The other “twos” facts should be dealt with in the same way, using appropriate doubles pictures.

Language

Correct

<table>
<thead>
<tr>
<th>The Twos Facts</th>
<th>and their partners</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 divided by 2</td>
<td>3 divided by 6</td>
</tr>
<tr>
<td>6 shared among/between 2</td>
<td>6 shared among/between 3</td>
</tr>
</tbody>
</table>
Correct

<table>
<thead>
<tr>
<th>Corresponding multiplication facts</th>
<th>and its partner</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 whats are 6</td>
<td>3 whats are 6</td>
</tr>
<tr>
<td>2 threes are 6</td>
<td>3 twos are 6</td>
</tr>
<tr>
<td>So 6 divided by 2 is 3</td>
<td>So 6 divided by 3 is 2.</td>
</tr>
</tbody>
</table>
| \[
\begin{array}{c}
1 \\
2 \\
6
\end{array} \]
\[
\begin{array}{c}
3 \\
2 \\
6
\end{array} \] | \[
\begin{array}{c}
1 \\
3 \\
6
\end{array} \]
\[
\begin{array}{c}
2 \\
2
\end{array} \] |

Incorrect

<table>
<thead>
<tr>
<th>How many twos in 6?</th>
<th>How many threes in 6?</th>
</tr>
</thead>
</table>

Examples:

<table>
<thead>
<tr>
<th>The Twos Facts and their partners</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
</tr>
</tbody>
</table>

References:
Year 4 Sourcebook p. 144
Year 5 Sourcebook p. 47
Year 6 Sourcebook p. 29

The Fives Facts (Introduce Year 4)

Specific Strategies

- The sharing process

In a similar way to that used with the twos facts, students should develop understanding of all of the fives facts, using the sharing approach, e.g.

1 block shared among 5 \[
\begin{array}{c}
1 \\
5
\end{array} \]

2 blocks shared among 5 \[
\begin{array}{c}
2 \\
5
\end{array} \]

8 blocks shared among 5 \[
\begin{array}{c}
8 \\
5
\end{array} \]

9 blocks shared among 5 \[
\begin{array}{c}
9 \\
5
\end{array} \]

The examples which follow reinforce these facts, show other ways of developing understanding of the facts, and indicate links with corresponding multiplication facts.

Division Facts - Chapter 5: 16
Using the clock face

The clock face which was used to build the fives multiplication facts (see Multiplication Facts chapter) can be used for the corresponding division facts.

Example: For $5 \div 35$  
The numeral on the clock fact corresponding to 35 minutes after the hour is 7. So 7 fives are 35. So, 35 divided by 5 is 7.

Language

Correct

<table>
<thead>
<tr>
<th>The Fives Facts</th>
<th>and their partners</th>
</tr>
</thead>
<tbody>
<tr>
<td>9</td>
<td>5</td>
</tr>
<tr>
<td>$5 \mid 45$</td>
<td>$9 \mid 45$</td>
</tr>
<tr>
<td>45 divided by 5</td>
<td>45 divided by 9</td>
</tr>
<tr>
<td>45 shared among/between 5</td>
<td>45 shared among/between 9</td>
</tr>
</tbody>
</table>

Corresponding multiplication facts and its partner

- 5 whats are 45  
- 5 nines are 45  
- So 45 divided by 5 is 9

\[ \frac{1}{5} \div \frac{9}{45} = \frac{2}{9} \]

Incorrect

How many fives in 45?  
How many nines in 45?

Examples:

<table>
<thead>
<tr>
<th>The Fives Facts</th>
<th>and their partners</th>
</tr>
</thead>
<tbody>
<tr>
<td>$5 \mid 5$</td>
<td>$1 \mid 5$</td>
</tr>
<tr>
<td>$5 \mid 10$</td>
<td>$2 \mid 10$</td>
</tr>
<tr>
<td>$5 \mid 15$</td>
<td>$3 \mid 15$</td>
</tr>
<tr>
<td>$5 \mid 20$</td>
<td>$4 \mid 20$</td>
</tr>
<tr>
<td>$5 \mid 25$</td>
<td>$5 \mid 25$</td>
</tr>
<tr>
<td>$5 \mid 30$</td>
<td>$6 \mid 30$</td>
</tr>
<tr>
<td>$5 \mid 35$</td>
<td>$7 \mid 35$</td>
</tr>
<tr>
<td>$5 \mid 40$</td>
<td>$8 \mid 40$</td>
</tr>
<tr>
<td>$5 \mid 45$</td>
<td>$9 \mid 45$</td>
</tr>
</tbody>
</table>

Once students have developed confidence with the fives facts, the twos facts can be included in the activities thereby requiring students to discriminate between facts and strategies. Students should be encouraged to identify those facts that can be calculated by more than one strategy and decide which strategy/strategies they prefer to use in those cases. (Refer to grey shaded areas of Scope and Sequence - Division Facts in this chapter).

e.g. A child may prefer to learn $5 \mid 10$ than as a double (twos facts strategy)

References:
Year 4 Sourcebook pp. 148 – 149
Year 5 Sourcebook p. 47
Year 6 Sourcebook p. 29

Division Facts - Chapter 5 : 17
Specific Strategies

The new nines facts to be learned are related to the multiplication facts $9 \times 1$, $9 \times 3$, $9 \times 4$, $9 \times 6$, $9 \times 7$, $9 \times 8$ and $9 \times 9$. The facts relating to $9 \times 2$ and $9 \times 5$ will already have been learned in the twos and fives facts. The nines facts exhibit some unique features which make them one of the easier and more interesting sets to learn. *(Refer to part 4c of the Multiplication Facts chapter).*

- **The sharing process**

The students should develop knowledge of the nines division facts for themselves by actually sharing out blocks or counters. This continues the sharing idea of division as well as linking division to the multiplication facts in later examples, e.g.

<table>
<thead>
<tr>
<th>9 blocks shared among 9</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>9</td>
<td>9</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>18 blocks shared among 9</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>9</td>
<td>18</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>81 blocks shared among 9</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>9</td>
<td>81</td>
</tr>
</tbody>
</table>

- **Language**

**Correct**

<table>
<thead>
<tr>
<th>The Nines Facts</th>
<th>and their partners</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{3}{9} \mid 27$</td>
<td>$\frac{9}{3} \mid 27$</td>
</tr>
<tr>
<td>27 divided by 9</td>
<td>27 divided by 3</td>
</tr>
<tr>
<td>27 shared among/between 9</td>
<td>27 shared among/between 3</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Corresponding multiplication facts</th>
<th>and its partner</th>
</tr>
</thead>
<tbody>
<tr>
<td>9 whats are 27</td>
<td>3 whats are 27</td>
</tr>
<tr>
<td>9 threees are 27</td>
<td>3 fives are 27</td>
</tr>
<tr>
<td>So 27 divided by 9 is 3</td>
<td>So 27 divided by 3 is 9</td>
</tr>
<tr>
<td>$\frac{1}{9} \frac{3}{27} \frac{2}{2}$</td>
<td>$\frac{1}{3} \frac{9}{27} \frac{2}{2}$</td>
</tr>
</tbody>
</table>

**Incorrect**

<table>
<thead>
<tr>
<th>How many nines in 27?</th>
<th>How many threees in 27?</th>
</tr>
</thead>
</table>
Examples:

<table>
<thead>
<tr>
<th>The Nines Facts</th>
<th>9</th>
<th>9</th>
<th>9</th>
<th>9</th>
<th>9</th>
<th>9</th>
<th>9</th>
<th>9</th>
<th>9</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>and their</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>partner</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td>3</td>
<td>3</td>
<td>4</td>
<td>4</td>
<td>5</td>
<td>5</td>
</tr>
</tbody>
</table>

Once students have developed confidence with the nines facts, the twos and fives facts may be included in the activities along with the nines facts, thereby requiring students to discriminate between facts and strategies. Students should be encouraged to identify those facts that may be calculated by more than one strategy, and decide which strategy/strategies they prefer to use in those cases.

References:

Year 4 Sourcebook p. 110
Year 5 Sourcebook p. 47
Year 6 Sourcebook p. 29
Special care needs to be taken with the zero facts in division. Through the experiences below, students must realise that division by zero is not possible, because a number of things/objects cannot be shared to zero people or things.

### Specific Strategies

- **Acting out Situations**

  Have the students act out situations where there is nothing to share. E.g. There were 0 bones to be shared among 3 dogs. How many bones did each dog receive?

  The students can record the situation
  
  \[
  \begin{array}{l}
  0 \\
  3 \overline{\mid 0}
  \end{array}
  \]

  and should realise, after several examples, that when dividing zero by any number the answer is zero, because you cannot share zero things – each dog, person or other entity must get zero.

### General strategies

- **Corresponding multiplication facts**

  When the students are given a division fact, e.g. \( \frac{8}{0} \)

  they should think automatically of the matching multiplication fact:

  \[
  0 \times 8 = 0
  \]

- **Finding division partners**

  With other facts, the quotient and divisor have been interchangeable. Not so with zero! If students try to relate \( \frac{?}{0} \) to sharing, they should see that it's not possible to share out amongst no one. It can't be done.

  \[
  \frac{0}{17} \quad \frac{0}{3}
  \]

  Zero has no division partner!

- **Using a calculator to help make the fact families**

  **Example:**

  \[
  \begin{array}{l}
  \frac{0}{4} \\
  4 \overline{\mid 0}
  \end{array}
  \]

  Students could try other numbers instead of the 4 and see if they still end up with the zero answer.

  *Note:* Similar activities to those used for practising the other facts can be used to practise the zeros facts.

  Remember to omit 0 examples.
**Language**

**Correct**

<table>
<thead>
<tr>
<th>The Zeros Facts</th>
<th>partners</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>Division facts have no partners</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
</tr>
<tr>
<td>0 divided by 4</td>
<td>0 shared among/between 0</td>
</tr>
</tbody>
</table>

**Corresponding multiplication facts**

<table>
<thead>
<tr>
<th>partner</th>
</tr>
</thead>
<tbody>
<tr>
<td>Division facts have no partners</td>
</tr>
</tbody>
</table>

| 4 | 0 |
| 4 whats are 0 |
| 4 threees are 0 |
| So 0 divided by 4 is 0 |

**Incorrect**

| How many fours in 0? | Division facts have no partners |

**Examples**

<table>
<thead>
<tr>
<th>The Zeros Facts</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
</tr>
<tr>
<td>2</td>
</tr>
<tr>
<td>3</td>
</tr>
<tr>
<td>4</td>
</tr>
<tr>
<td>5</td>
</tr>
<tr>
<td>6</td>
</tr>
<tr>
<td>7</td>
</tr>
<tr>
<td>8</td>
</tr>
<tr>
<td>9</td>
</tr>
</tbody>
</table>

Note: no partners.

Once students have developed confidence with the zeros facts, the twos, fives and nines facts can be included in the activities along with the zeros facts, thereby requiring students to discriminate between facts and strategies.

**References:**
- Year 4 Sourcebook p. 152
- Year 5 Sourcebook p. 48
- Year 6 Sourcebook p. 29

---

**The Ones Facts (Introduce Year 4)**

**Specific Strategies**

It should not be assumed that all students will find ones facts as easy as they might appear initially. Once a correct pattern of thinking is established, however, students should have little trouble.

The facts relating to the multiplication facts $2 \times 1$, $5 \times 1$ and $9 \times 1$ will already have been studied.
Acting out situations

Ask the students to act out situations where only one person is involved in the sharing, e.g. Maria had 5 cakes that she ate by herself. How many did she eat? This situation can be recorded in the form

\[
\frac{1}{5}
\]

The students should realise, after a few examples, that when a number is being divided by one, it does not change. This should be related to the sharing concept.

Language

Correct

<table>
<thead>
<tr>
<th>The Ones Facts</th>
<th>and their partners</th>
</tr>
</thead>
<tbody>
<tr>
<td>8 1 8</td>
<td>1 8</td>
</tr>
<tr>
<td>8 divided by 1</td>
<td>8 divided by 8</td>
</tr>
<tr>
<td>8 shared among/between 1</td>
<td>8 shared among/between 8</td>
</tr>
</tbody>
</table>

Corresponding multiplication facts

<table>
<thead>
<tr>
<th>Corresponding multiplication facts</th>
<th>and its partner</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 what is 8</td>
<td>8 whats is 8</td>
</tr>
<tr>
<td>1 eight is 8</td>
<td>8 ones are 8</td>
</tr>
<tr>
<td>So 6 divided by 2 is 3</td>
<td>So 8 divided by 8 is 1.</td>
</tr>
<tr>
<td>(\frac{1}{8}) (\frac{3}{2})</td>
<td>(\frac{1}{8}) (\frac{1}{2})</td>
</tr>
</tbody>
</table>

Incorrect

<table>
<thead>
<tr>
<th>How many ones in 8?</th>
<th>How many eights in 8?</th>
</tr>
</thead>
</table>

Examples:

<table>
<thead>
<tr>
<th>The Ones Facts and their partners</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 1 2 3 4 5 6 7 8 9</td>
</tr>
<tr>
<td>1 2 3 4 5 6 7 8 9</td>
</tr>
</tbody>
</table>

Once students have developed confidence with the nines facts, the twos and fives facts may be included in the activities with the nines facts, thereby requiring students to discriminate between facts and strategies. Students should be encouraged to identify those facts that may be calculated by more than one strategy, and decide which strategy/strategies they prefer to use in those cases.

References:
Year 4 Sourcebook p. 151
Year 5 Sourcebook p. 47
Year 6 Sourcebook p. 29
The Remaining Facts

By encouraging children to colour or tick off facts studied on their division chart as they progress, it will become apparent that, at this stage, there are only 15 facts remaining. These facts fall within the threes, sixes, fours, eights and sevens facts and are as shown.

Students should have a good knowledge of the twos, fives, nines, ones and zeros facts and their partners before beginning a study of the remaining facts. It may be necessary in some cases to leave remaining facts until Year 5.

A sound knowledge of corresponding multiplication facts is essential. You will note that 5 of these 15 remaining facts are square number facts. For this reason you may may choose to teach a strategy for square numbers facts.

Since the very great majority of children should now be proficient with all the division facts other than these 15 remaining facts – the children should be encouraged to investigate and discuss some strategies of their own. It is possible that the childrens' strategies will be just as legitimate and user friendly as those advocated by the sourcebooks.

Remember, the child should be encouraged to use the strategy which he/she prefers. Also keep in mind that these strategies provide children with the means for calculating multiplication facts quickly and easily. It is important to note that these strategies are a means to an end and not an end in themselves. Instant recall from memory and understanding remains the ultimate goal. In addition to a strategy for square number facts, strategies for the four and seven facts are also outlined below. For most of the harder facts, general strategies and games are available.

The Square Number Facts

Students can build square numbers using gridpaper and see that the multiplication facts and division partners are the same.

References:
Year 4 Sourcebook p. 153
Year 5 Sourcebook p. 48
Year 6 Sourcebook p. 29

The Fours Facts

Dividing by 4 - halve and halve again.
Students who are proficient with halving even numbers to 36 can use the skill for any 4 \[ \underline{\times} \] fact – halve the number and halve it again.

References:
Year 4 Sourcebook p. 154
Year 5 Sourcebook p. 48
Year 6 Sourcebook p. 29
This fact relates to the multiplication fact which was represented as 3 weeks of 7 days. Students think '3 what's are 21' to find the answer.

<table>
<thead>
<tr>
<th>S</th>
<th>M</th>
<th>T</th>
<th>W</th>
<th>Th</th>
<th>F</th>
<th>S</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>7</td>
</tr>
<tr>
<td>8</td>
<td>9</td>
<td>10</td>
<td>11</td>
<td>12</td>
<td>13</td>
<td>14</td>
</tr>
<tr>
<td>15</td>
<td>16</td>
<td>17</td>
<td>18</td>
<td>19</td>
<td>20</td>
<td>21</td>
</tr>
<tr>
<td>22</td>
<td>23</td>
<td>24</td>
<td>25</td>
<td>26</td>
<td>27</td>
<td>28</td>
</tr>
<tr>
<td>29</td>
<td>30</td>
<td>31</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

References:
Year 4 Sourcebook p. 153
Year 5 Sourcebook p. 48
Year 6 Sourcebook p. 29

Some students might remember the number sequence 5, 6, 7, 8 to help build the family -

\[
\begin{array}{c}
56 \\
\hline
7 \quad 7
\end{array}
\quad \text{and} \quad
\begin{array}{c}
56 \\
\hline
8 \quad 7
\end{array}
\]

56 = 7 \times 8

Note: Ordinal sequence of numbers 5, 6, 7, 8.
It is important that teachers and students use correct and consistent language, models and symbolic representation from the introduction of the division concept right through the grades. The following language, models and symbolism is that which is advocated by the syllabus and should be used in all classrooms, in all schools throughout Queensland to promote continuity in each child’s learning experience, especially because of the high rate of student mobility between schools and because primary school children may be taught by many different teachers.

Initially in Years 3 and 4 division can be recorded as
12 “shared among” 4
12 “divided by” 4

When students are comfortable with the recognition of situations involving division, and with the drawing of diagrams to represent them, symbolic recording can be introduced. The language used should be quite informal, i.e.

\[ \frac{15}{3} \leftarrow \text{[The amount that each receives]} \]

[The number sharing] \[ \begin{array}{c} 3 \end{array} \leftarrow \text{[The amount to be shared]} \]

It is important to keep language consistent. The development of the written algorithm in Year 5 depends heavily on a consistent language pattern that explains the sharing out of physical materials.

It is inconsistent, for example to ask children to read \( \frac{15}{3} \) as “how many threes in 15?” This could lead to confusion when the written algorithm is developed.

After students have had considerable practice representing “sharing” situations with diagrams and symbols, “grouping” situations can be included. The students should realise that the recording is the same for both situations and that both can be solved by relating the division to known multiplication facts.

Division can be read using the related multiplication facts.

\[ \begin{array}{c} 6 \end{array} \leftarrow \text{say: } '6 \text{ whats are 30?}' \]

\[ \begin{array}{c} 30 \end{array} \leftarrow \text{say: } '6 \text{ fives are 30'} \]

\[ \begin{array}{c} 3 \end{array} \leftarrow \text{say: } '\text{So 30 divided by 6 is 5'} \]

Note that you read from left to top and then down.

\[ \begin{array}{c} 6 \end{array} \leftarrow \text{say: } '30 \text{ divided by 3'} \]

\[ \begin{array}{c} 30 \end{array} \leftarrow \text{say: } '30 \text{ shared among 3'} \]

Note that you read from right to left.

Although partition and quotition are the correct terms for division, the terms sharing and making groups are the terms used with children.

Partition is Sharing

Quotition is Making Groups
Language provides a link between children's manipulation of materials and their symbolic representations, and between their past and present experiences. Initially children will use their own everyday language to describe what they are thinking and doing, but as division becomes more internalized, the children's language should become the same as the formal language.

<table>
<thead>
<tr>
<th>Year 1</th>
<th>Year 2</th>
<th>Year 3</th>
<th>Year 4</th>
<th>Year 5</th>
<th>Year 6</th>
<th>Year 7</th>
</tr>
</thead>
<tbody>
<tr>
<td>each</td>
<td>as for Year 1</td>
<td>as for previous years</td>
<td>as for previous years</td>
<td>as for previous years</td>
<td>as for previous years</td>
<td></td>
</tr>
<tr>
<td>share</td>
<td>sharing</td>
<td>shared among</td>
<td>how many</td>
<td>divided by</td>
<td>making groups</td>
<td></td>
</tr>
<tr>
<td></td>
<td>shared between</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Related language**

<table>
<thead>
<tr>
<th>Year 2</th>
<th>Year 3</th>
<th>Year 4</th>
<th>Year 5</th>
<th>Year 6</th>
<th>Year 7</th>
</tr>
</thead>
<tbody>
<tr>
<td>odd/even</td>
<td>multiples facts</td>
<td>prime composite</td>
<td>square numbers</td>
<td>square root</td>
<td></td>
</tr>
</tbody>
</table>

*Division Facts - Chapter 5: 26*
Addition Algorithms

Contents

Scope and Sequence ........................................... 1
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  Year 2 Addition Facts & Algorithms ............... 1b
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  Year 5 Addition Algorithms .......................... 1e

Year Level Expectations ................................. 2

Glossary of Terms ......................................... 3

Reading and Recording ..................................... 4

Language ....................................................... 5
Since sequential development of the written algorithms and basic facts is desirable, it is important to ensure that each child is proficient with one stage before progressing to the next as knowledge of prior stages is usually a prerequisite.

The obligation to students is not to rush them through stages so they keep up with the peers, but ensure that they have the opportunity to develop understanding and skill before being asked to attempt a more difficult task.

### Year 1 Addition Number Facts

<table>
<thead>
<tr>
<th>YEAR 1 SCOPE AND SEQUENCE</th>
<th>EXAMPLE</th>
<th>REF.</th>
<th>For Teacher use</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>1st Addition Concept</strong> - using materials, pictures and language to model and explain addition.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>2nd Recording Addition Facts</strong> - using symbols to represent addition.</td>
<td>4 and 3</td>
<td>Figure A.</td>
<td></td>
</tr>
<tr>
<td><strong>Addition Facts</strong></td>
<td>4 + 3</td>
<td>Figure B.</td>
<td></td>
</tr>
<tr>
<td>- recording addition facts to 10</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>- developing thinking strategies to assist recall of addition facts to 10.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>3rd</strong> - Count on one</td>
<td>5 + 1 &amp; 1 + 5</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>4th</strong> - Count on two</td>
<td>5 + 2 &amp; 2 + 5</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>5th</strong> - Count on three</td>
<td>5 + 3 &amp; 3 + 5</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>6th</strong> - Count on zero</td>
<td>5 + 0 &amp; 0 + 5</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>7th</strong> - Doubles</td>
<td>4 + 4</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>8th</strong> - Addition facts to 10</td>
<td>4 + 6 &amp; 6 + 4</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: *Add* should be used until all students become familiar with the ‘+’ symbol.

REFERENCE: Year 1 Sourcebook pp. 169-181
<table>
<thead>
<tr>
<th>Year 2 Scope and Sequence</th>
<th>Example</th>
<th>REF.</th>
<th>For Teacher use</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st</td>
<td>Revise the Concept of Addition</td>
<td></td>
<td>Part 3</td>
</tr>
<tr>
<td>2nd</td>
<td>Revise the Addition facts to 10</td>
<td></td>
<td>Part 1a</td>
</tr>
<tr>
<td>3rd</td>
<td>Concept development of Addition facts beyond ten</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5th</td>
<td>- Near doubles</td>
<td>5 + 6</td>
<td></td>
</tr>
<tr>
<td>6th</td>
<td>- Near 10</td>
<td>3 + 8</td>
<td></td>
</tr>
<tr>
<td>7th</td>
<td>- Adding 9</td>
<td>9 + 5</td>
<td></td>
</tr>
<tr>
<td>8th</td>
<td>- Neighbours but One</td>
<td>3 + 5</td>
<td></td>
</tr>
<tr>
<td>9th</td>
<td>- Remaining Facts</td>
<td>7 + 4</td>
<td></td>
</tr>
<tr>
<td>10th</td>
<td>The Tens Facts</td>
<td>4 tens + 3 tens</td>
<td>Figure C</td>
</tr>
<tr>
<td>11th</td>
<td>The Addition Algorithm</td>
<td></td>
<td></td>
</tr>
<tr>
<td>11th</td>
<td>Two digit plus two digit, no regrouping</td>
<td>35 + 23</td>
<td>Figure D</td>
</tr>
<tr>
<td></td>
<td>• then with a zero</td>
<td>26 20 + 30 + 36</td>
<td></td>
</tr>
<tr>
<td></td>
<td>• then two digits plus one digit.</td>
<td>26 + 3</td>
<td></td>
</tr>
</tbody>
</table>
## Year 3 Addition Algorithms

<table>
<thead>
<tr>
<th>YEAR 3 SCOPE AND SEQUENCE</th>
<th>EXAMPLE</th>
<th>REF.</th>
<th>For Teacher Use</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st</td>
<td>Revise Year 2 work Continue work with algorithms</td>
<td></td>
<td>Part 1b</td>
</tr>
<tr>
<td>2nd</td>
<td>Three digit plus three digit, no regrouping</td>
<td>127+417</td>
<td></td>
</tr>
<tr>
<td></td>
<td>then with zeros</td>
<td>319+502+260+176</td>
<td></td>
</tr>
<tr>
<td></td>
<td>then three digit plus one or two digit</td>
<td>354+224+5+54</td>
<td></td>
</tr>
<tr>
<td>3rd</td>
<td>Three single digit addends, no regrouping</td>
<td>4+3+2</td>
<td></td>
</tr>
<tr>
<td>4th</td>
<td>Two digit plus two digit with regrouping in the ones</td>
<td>28+24</td>
<td></td>
</tr>
<tr>
<td></td>
<td>then two digit plus one digit</td>
<td>42+9</td>
<td></td>
</tr>
<tr>
<td>5th</td>
<td>Two digit plus two digit, with regrouping in the tens and ones</td>
<td>75+56</td>
<td>Figure E</td>
</tr>
<tr>
<td>6th</td>
<td>Three single digit addends, with regrouping</td>
<td>7+5+4</td>
<td></td>
</tr>
</tbody>
</table>

REFERENCE: Year 3 Sourcebook pp. 69-91
| 1st | Revise Year 3 work  
Continue work with algorithms | EXAMPLE | REF | For Teacher Use |
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>2nd</td>
<td>Three digit plus three digit, with regrouping in the tens and ones</td>
<td>358 + 276</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>• then with zeros</td>
<td>398 + 209</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>• three digit plus two digit</td>
<td>587 + 29</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3rd</td>
<td>Three digit plus three digit with regrouping in the hundreds</td>
<td>534 + 945</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>• then with zeros</td>
<td>504 + 940</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4th</td>
<td>Three digit plus three digit with regrouping in the hundreds, tens and ones, and a combinations of places</td>
<td>534 + 534 + 987 + 927</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>• then with zeros</td>
<td>530 + 984</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5th</td>
<td>Four digit plus four digit</td>
<td>3524 + 2345</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>• then with zeros</td>
<td>4053 + 2304</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>• four digit plus one, two and three digit numbers</td>
<td>4524 + 35</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6th</td>
<td>More than two addends</td>
<td>24 + 124 + 41 + 341</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>• then with zeros</td>
<td>20 + 104 + 32 + 230</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>• then with a mixture of one, two and three digit addends</td>
<td>23 + 232 + 32 + 41</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7th</td>
<td>More than two, two and three digit addends with regrouping in the ones</td>
<td>16 + 235 + 34 + 314</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>• then with zeros</td>
<td>20 + 207 + 19 + 410</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>• then with a mixture of one, two and three digit addends</td>
<td>24 + 365 + 5 + 6</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*Addition Algorithms - Chapter 6:5*
<table>
<thead>
<tr>
<th>YEAR 4 SCOPE AND SEQUENCE</th>
<th>EXAMPLE</th>
<th>REF.</th>
<th>For Teacher Use</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>8th</strong></td>
<td>More than two, two and three digit addends with regrouping in the tens and ones</td>
<td>46 247</td>
<td></td>
</tr>
<tr>
<td></td>
<td>+ 35 189</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>+ 34 231</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>• then with zeros</td>
<td>40 294</td>
<td></td>
</tr>
<tr>
<td></td>
<td>+ 9 107</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>+ 24 120</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>• then with a mixture of one, two and three digit addends</td>
<td>49 129</td>
<td></td>
</tr>
<tr>
<td></td>
<td>+ 8 32</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>+ 73 5</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>9th</strong></td>
<td>More than two, three digit addends with regrouping in the hundreds, tens and ones</td>
<td>459</td>
<td></td>
</tr>
<tr>
<td></td>
<td>+ 568</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>+ 987</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>• then with zeros</td>
<td>509</td>
<td></td>
</tr>
<tr>
<td></td>
<td>+ 687</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>+ 490</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>• then with a mixture of one, two and three digit addends</td>
<td>987</td>
<td></td>
</tr>
<tr>
<td></td>
<td>+ 126</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>+ 8</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

REFERENCE: *Year 5 Sourcebook pp. 67-70*
## Year 5 Addition Algorithms

### Year 5 Scope and Sequence
Year 5 Sourcebook pp. 67-70

<table>
<thead>
<tr>
<th>1st</th>
<th>Revise Year 4 work</th>
</tr>
</thead>
</table>

### Three digit numbers with regrouping

<table>
<thead>
<tr>
<th>2nd</th>
<th>Four digit plus four digit with regrouping in the tens and ones</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>4 329 + 3 284</td>
</tr>
<tr>
<td></td>
<td>• then with zeros</td>
</tr>
<tr>
<td></td>
<td>4 099 + 3 405</td>
</tr>
<tr>
<td></td>
<td>• four digit plus one, two or three digit number</td>
</tr>
<tr>
<td></td>
<td>4 359 + 4 876 + 287 + 97</td>
</tr>
</tbody>
</table>

### Four digit numbers with regrouping

<table>
<thead>
<tr>
<th>3rd</th>
<th>Four digit plus four digit with regrouping in the hundreds</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>7 435 + 1 822</td>
</tr>
<tr>
<td></td>
<td>• then with zeros</td>
</tr>
<tr>
<td></td>
<td>7 503 + 1 940</td>
</tr>
<tr>
<td></td>
<td>• then with four digit plus three digit</td>
</tr>
<tr>
<td></td>
<td>6 543 + 925</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>4th</th>
<th>Four digit plus four digit with regrouping in the hundreds, tens and ones</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>3 594 + 2 876</td>
</tr>
<tr>
<td></td>
<td>• then with zeros</td>
</tr>
<tr>
<td></td>
<td>3 094 + 2 947</td>
</tr>
<tr>
<td></td>
<td>• then with four digit plus three digit</td>
</tr>
<tr>
<td></td>
<td>3 594 + 876</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>5th</th>
<th>Four digit plus four digit with regrouping in the thousands</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>9 432 + 5 341</td>
</tr>
<tr>
<td></td>
<td>• then with zeros</td>
</tr>
<tr>
<td></td>
<td>8 243 + 9 535</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>6th</th>
<th>Four digit plus four digit with regrouping in the thousands, hundreds, tens, ones and a combination of places</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>• then with zeros</td>
</tr>
<tr>
<td></td>
<td>• four digit plus one, two or three digit with regrouping in the thousands, hundreds, tens, ones and in a combination of places</td>
</tr>
</tbody>
</table>

### Other

<table>
<thead>
<tr>
<th>7th</th>
<th>Other – as they arise in practical situations</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>62 741 + 688 + 24 653 + 4 289</td>
</tr>
<tr>
<td></td>
<td>+ 8 950 + 753 + 88 907 + 6 055</td>
</tr>
<tr>
<td></td>
<td>+ 211</td>
</tr>
</tbody>
</table>

---

*Addition Algorithms - Chapter 6: 7*
Recording Addition Facts (Introduce Year 1)

Begin with
Using words, for example:
'Four children joined by three more children makes seven children.'

Then use words and digits, for example:
\[
\begin{array}{c}
4 \\
\text{and}
\end{array}
\begin{array}{c}
3
\end{array}
\text{or}
\begin{array}{c}
4
\end{array}
\begin{array}{c}
\text{add}
\end{array}
\begin{array}{c}
3
\end{array}
\begin{array}{c}
7
\end{array}
\begin{array}{c}
7
\end{array}
\]

Note: The word 'add', should be used until the children become familiar with the '+' sign.

Even though children may experience both vertical and horizontal forms of recording calculations, vertical recording is recommended in preference to the traditional horizontal form. Using the vertical form helps children avoid problems associated with interpreting the equals sign and provides a firm basis for developing the written algorithm with larger numbers. When children encounter the equals sign on the calculator, it may be described as the answer key.

In Year 1 children are expected to formally record only the addition facts.

When children encounter the equal sign on the calculator, it may be described as the answer key.
Addition Facts (Introduce Year 1)

After children have explored the numbers to ten, have built up an understanding of the addition concept and are able to read and write addition statements, activities for developing recall of basic facts may begin.

A wide variety of addition situations should be investigated in Year 1. Teachers will find many examples of addition in the classroom, school, home, neighbourhood and natural environment. Classroom examples would include situations arising in other areas of mathematics, as well as other curriculum areas.

An example of a number story, in which two numbers are added together to find a total, may be: 'Kiki made two finger puppets yesterday and three more today. How many puppets has she made altogether?'

When discussing addition with children, teachers should use language which is familiar to the children. Teachers may repeat the problem to identify the numbers and action involved in the story – for example: 'Two and three more makes?'

Select children to predict the outcome of the problem – for example: 'How many puppets do you think she has made?'
'Does anyone have a different idea?'

Have children assist with representing the problem, as it is repeated by the teacher or the child who created it. This may be done by children:
• acting out the problem;
• using concrete materials to represent the people or objects in the story;
• using drawings or diagrams to represent the people or objects in the story.

When representing problems, use materials which are suited to the children's level of understanding, i.e. start with concrete materials and move on to more abstract representations. When introducing new concrete materials (for example, Unifix cubes which may have been used previously as discrete objects) children require precise instruction on their use (for example, measuring Unifix trains on the number ladder instead of counting individual cubes).

After representing the problem, ask the children to count the characters or objects to find the total number or 'how many altogether', and to validate their predictions of the outcome – for example: 'Who guessed that number?'
'Whose guess was close?'

A slow and careful development of recording addition is recommended to ensure complete understanding. Rushing through various forms of recording will confuse children. Each new example of recording addition may be explained to the children as a shorter or quicker way of writing something they already know – for example: 'Today I'm going to show you a quicker way of writing that.'

Record outcomes beneath or beside drawings or diagrams. The form of recording may be developed as follows:
• in words – e.g. 'Two and three more makes five altogether.'
• using a horizontal setting out with some words – e.g. '2 and 3 makes 5.'
• using a vertical setting out – e.g.

\[
\begin{array}{c}
2 \\
+ 3 \\
\hline
5
\end{array}
\quad \text{and/or} \quad \begin{array}{c}
\text{add} \\
2 \\
\hline
5
\end{array}
\]

• using the addition sign – e.g.

\[
\begin{array}{c}
2 \\
+ 3 \\
\hline
5
\end{array}
\] 
(Read as '2 and 3 is 5', '2 add 3 makes 5', or '2 plus 3 is 5'.

REFERENCE: Year 1 Sourcebook pp. 171-172
When the children have investigated the effect of making a number 10 more or less in Activity 1(i) of the topic 'Number study' Year 2, apply this knowledge of addition facts to adding together numbers of tens - for example:

\[
\begin{align*}
4 \text{ tens} & + 3 \text{ tens} \\
\hline
7 \text{ tens}
\end{align*}
\]

After children can calculate such examples which equal nine tens or less, introduce the number names for these amounts of tens - for example:

\[
\begin{align*}
40 & + 30 \\
\hline
70
\end{align*}
\]

Encourage children to explain any mental 'short-cuts' they may have discovered for the addition of groups of 10.

REFERENCE: Year 2 Sourcebook p. 83

If 4 and 3 makes 7, then 40 and 30 makes 70.
After investigating groups of 10, present children with a wide variety of addition examples involving single-digit and two-digit numbers but with no regrouping. Examples should involve adding:

- 2 two-digit numbers;
- zero in the top and/or the bottom number;
- a two-digit number and a single-digit number.

```
<table>
<thead>
<tr>
<th>T</th>
<th>Ones</th>
<th>T</th>
<th>Ones</th>
<th>T</th>
<th>Ones</th>
<th>T</th>
<th>Ones</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>6</td>
<td>4</td>
<td>0</td>
<td>1</td>
<td>5</td>
<td>8</td>
<td>2</td>
</tr>
<tr>
<td>+3</td>
<td>2</td>
<td>+2</td>
<td>7</td>
<td>+7</td>
<td>0</td>
<td>+5</td>
<td>2</td>
</tr>
<tr>
<td>+3</td>
<td>2</td>
<td>+2</td>
<td>7</td>
<td>+7</td>
<td>0</td>
<td>+5</td>
<td>0</td>
</tr>
</tbody>
</table>
```

As each type of example is investigated by the children, give sufficient time and practice for it to be fully understood before introducing a different type of example. Introduce each new example by giving the children the numbers in a story - for example:

'Two classes of Year 2 children went on a trip to Bay Park to study the animals and plants there. There were 25 children in Year 2A and 23 children in Year 2B. How many children's tickets needed to be bought?'

Ask the children to write the number sentence described in the problem.

In doing so, children are involved in the problem-solving skills of:

- searching for relevant information, and
- identifying and using the appropriate operation.

The children must search through the story for the numbers to be used in answering the question and decide which sign to use. By this stage, children would have two signs to choose from - addition and subtraction. Ask children to justify their choice of sign.

We have to add this number of children to find out how many tickets are needed.

Invite children to estimate the answer first. Direct their estimates with questions such as:

- Do you think the answer will be larger or smaller than 35? Why?
- About how many children do you think there will be? Why did you choose that number?

By discussing these ideas, children are encouraged to look critically at their results to see whether a reasonable answer has been reached.

To calculate answers, have the children represent numbers with the groups of tens and ones used in the previous activity. Explain that the ones must be calculated first, in case a new ten is formed. Then calculate the tens.

Children should compare their answers with their estimates to establish whether their estimates were close or not. Children may also check the accuracy of their answers with calculators.

Present the children with a number of addition problems to solve. Structure these problems so that children are required to use a variety of problem-solving skills. Reward the children's efforts by giving them the opportunity to present the problems they create, or solutions they have found, to the other children.

(Ref. Year 2 pp. 83-85)
Present an addition situation to children, for example:
"Bill used 35 litres of petrol in his car this week and Fiona used 23 litres in hers. How much petrol did they use altogether?"

Analyze the situation:
- What do we have to find out?
- What information do we need?
- How much petrol did Bill use?
- How many litres did Fiona use?
- How can we use that information to find the answer?

The following table shows the verbal, concrete and symbolic representations of the algorithm. The children should practise modelling the operation with MAB before they are asked to record the process.

<table>
<thead>
<tr>
<th>Verbal (child thinks or says)</th>
<th>Concrete (child demonstrates)</th>
<th>Symbolic (teacher or child writes)</th>
</tr>
</thead>
<tbody>
<tr>
<td>What operation is it? (Addition.)</td>
<td>T Ones 0000</td>
<td>T Ones 3 5 + 2 3</td>
</tr>
<tr>
<td>What numbers are we adding? (35 and 23.)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>What do we add first? (Ones.)</td>
<td>0000</td>
<td></td>
</tr>
<tr>
<td>How many ones are there? (5 ones plus 3 ones: 8 ones.)</td>
<td></td>
<td>T Ones 3 5 + 2 3</td>
</tr>
<tr>
<td>Have we enough ones to make a ten? (No.)</td>
<td></td>
<td>8</td>
</tr>
<tr>
<td>Where will we put the ones? (Ones' place.)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>What do we add next? (Tens.)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>How many tens are there? (3 tens plus 2 tens: 5 tens.)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Are there enough tens to make a hundred? (No.)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Where will we put the 5 tens? (Tens' place.)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>What is 35 plus 23? (58.)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Encourage the use of estimating to check the reasonableness of the answer.

REFERENCE: Year 3 Sourcebook p. 90
**Two digit plus two digit, with regrouping in the tens and ones (Introduce Year 3)**

Present an addition situation to the children.

"Year 2 at Calkeen State School collected 75 kg of aluminium cans and Year 3 collected 56 kg. How many kilograms of cans did Years 2 and 3 collect altogether?"

**Analyse the situation:**
- What do we have to find out?
- What information do we need?
- How many kilograms did Year 2 collect?
- How many kilograms did Year 3 collect?
- How may we use that information to find the answer?

The following table shows the verbal, concrete and symbolic format of the algorithm. The children should practise modelling the operation with MAB before they are asked to record the process.

<table>
<thead>
<tr>
<th>Verbal (child thinks or says)</th>
<th>Concrete (child demonstrates)</th>
<th>Symbolic (teacher or child writes)</th>
</tr>
</thead>
<tbody>
<tr>
<td>What operation is it? (Addition.)</td>
<td>[Symbolic representation]</td>
<td>$T \text{ Ones}$ [ 7 5 ] $+ \quad 5 6$</td>
</tr>
<tr>
<td>What numbers are we adding? (75 and 56).</td>
<td>[Concrete MAB representation]</td>
<td></td>
</tr>
<tr>
<td>What do we add first? (Omens.)</td>
<td>[Concrete MAB representation]</td>
<td>$T \text{ Ones}$ [ 7 5 ] $+ \quad 5 6$</td>
</tr>
<tr>
<td>How many ones are there? (5 ones plus 6 ones: 11 ones.)</td>
<td>[Concrete MAB representation]</td>
<td></td>
</tr>
<tr>
<td>Are there enough ones to make a ten? (Yes.)</td>
<td>[Concrete MAB representation]</td>
<td></td>
</tr>
<tr>
<td>What will we do? (Regroup, or trade, 10 ones for 1 ten.)</td>
<td>[Concrete MAB representation]</td>
<td></td>
</tr>
<tr>
<td>What do you have now? (1 ten and 1 one.)</td>
<td>[Concrete MAB representation]</td>
<td></td>
</tr>
<tr>
<td>Where will we put the 1 one? (Ones' place.)</td>
<td>[Concrete MAB representation]</td>
<td></td>
</tr>
<tr>
<td>Where do we put the 1 ten? (Tens' place. The carry figure should be normal size and placed above the ten.)</td>
<td>[Concrete MAB representation]</td>
<td></td>
</tr>
<tr>
<td>What do we add next? (Tens.)</td>
<td>[Concrete MAB representation]</td>
<td></td>
</tr>
<tr>
<td>How many tens are there? (1 ten plus 7 tens plus 5 tens: 13 tens.)</td>
<td>[Concrete MAB representation]</td>
<td>$H \text{ T \ Ones}$ [ 1 ] $7 5$ $+ \quad 5 6$</td>
</tr>
<tr>
<td>Are there enough tens to make a hundred? (Yes.)</td>
<td>[Concrete MAB representation]</td>
<td></td>
</tr>
<tr>
<td>What will we do? (Regroup, or trade, 10 tens for 1 hundred.)</td>
<td>[Concrete MAB representation]</td>
<td></td>
</tr>
<tr>
<td>What do you have now? (1 hundred and 3 tens.)</td>
<td>[Concrete MAB representation]</td>
<td></td>
</tr>
<tr>
<td>Where will we put the 3 tens? (Tens' place.)</td>
<td>[Concrete MAB representation]</td>
<td></td>
</tr>
<tr>
<td>Where will we put the 1 hundred? (Hundreds' place.)</td>
<td>[Concrete MAB representation]</td>
<td></td>
</tr>
<tr>
<td>What is 75 plus 56? (131.)</td>
<td>[Concrete MAB representation]</td>
<td>$1 \quad 3 \quad 1$</td>
</tr>
</tbody>
</table>

Encourage of use of estimation to check the reasonableness of the answer.

**REFERENCE:** *Year 3 Sourcebook* p. 89
Developing the algorithm with 3-digit numbers

The following algorithm is for a calculation which needs regrouping in the tens. The teacher needs to modify the language for other kinds of examples (e.g., regrouping in the ones also). It is recommended that students practise modelling the operation with MAB before they are asked to record the process.

**Example:**
Mike has collected 143 stickers and his sister has collected 284 stickers. How many stickers do they have altogether?

<table>
<thead>
<tr>
<th>Verbal (Student thinks or says)</th>
<th>Concrete (Student demonstrates)</th>
<th>Symbolic (Teacher or Student writes)</th>
</tr>
</thead>
<tbody>
<tr>
<td>What operation is it? (Addition)</td>
<td></td>
<td>H T Ones 143 +284</td>
</tr>
<tr>
<td>What numbers are we adding? (143 and 284)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>What do we add first? (Ones)</td>
<td></td>
<td>143 +284</td>
</tr>
<tr>
<td>How many ones are there? (3 ones plus 4 ones: 7 ones)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Have we enough ones to make a ten? (No)</td>
<td></td>
<td>7</td>
</tr>
<tr>
<td>Where will we put the ones? (Ones' place)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>What do we add next? (Tens)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>How many tens are there? (4 tens plus 8 tens: 12 tens)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Are there enough tens to make a hundred? (Yes)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>What will we do? (Change 10 tens for 1 hundred)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>What do you have now? (1 hundred and 2 tens)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Where will we put the 2 tens? (Tens' place)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Where do we put the 1 hundred? (Hundreds' place) The carry figure should be normal size and placed above the hundred.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
I paid $1 564 for a computer system and $3 788 for new furniture. How much money did I spend?

<table>
<thead>
<tr>
<th>Verbal (Student thinks or says)</th>
<th>Concrete (Student demonstrates)</th>
<th>Symbolic (Teacher or student writes)</th>
</tr>
</thead>
<tbody>
<tr>
<td>What operation is involved? (Addition)</td>
<td>![Concrete demonstration]</td>
<td>Th  H  T  Ones</td>
</tr>
<tr>
<td>What numbers are we adding? (1 564 and 3 788)</td>
<td>![Concrete demonstration]</td>
<td>1 5 6 4</td>
</tr>
<tr>
<td>What do we add first? (Ones)</td>
<td>![Concrete demonstration]</td>
<td>+3 7 8 8</td>
</tr>
<tr>
<td>How many ones are there? (4 ones plus 8 ones: 12 ones)</td>
<td>![Concrete demonstration]</td>
<td></td>
</tr>
<tr>
<td>Are there enough ones to make a ten? (Yes)</td>
<td>![Concrete demonstration]</td>
<td></td>
</tr>
<tr>
<td>What will you do? (Change 10 ones for a ten.)</td>
<td>![Concrete demonstration]</td>
<td></td>
</tr>
<tr>
<td>(Students exchange the ones for a ten from the bank of MAB.)</td>
<td>![Concrete demonstration]</td>
<td></td>
</tr>
<tr>
<td>Now what do we have? (1 ten and 2 ones)</td>
<td>![Concrete demonstration]</td>
<td>Th  H  T  Ones</td>
</tr>
<tr>
<td>Where will we put them? (The 2 ones stay in the one's place and the ten goes to the ten's place.)</td>
<td>![Concrete demonstration]</td>
<td>1 5 6 4</td>
</tr>
<tr>
<td>(The carry figure should be of normal size and be placed above the other tens.)</td>
<td>![Concrete demonstration]</td>
<td>+3 7 8 8</td>
</tr>
<tr>
<td>What do we add next? (Tens)</td>
<td>![Concrete demonstration]</td>
<td>2</td>
</tr>
<tr>
<td>How many tens do we have? (1 ten plus 6 tens plus 8: 15 tens.)</td>
<td>![Concrete demonstration]</td>
<td></td>
</tr>
<tr>
<td>Have we enough tens to make a hundred? (Yes)</td>
<td>![Concrete demonstration]</td>
<td></td>
</tr>
<tr>
<td>What will we do? (Change 10 tens for 1 hundred.)</td>
<td>![Concrete demonstration]</td>
<td></td>
</tr>
<tr>
<td>What do we have now? (1 hundred and 5 tens.)</td>
<td>![Concrete demonstration]</td>
<td></td>
</tr>
<tr>
<td>Where will we put them? (The 5 tens stay in the tens' place and the hundred goes to the hundreds' place.)</td>
<td>![Concrete demonstration]</td>
<td></td>
</tr>
<tr>
<td>(The carry figure should be of normal size and be placed above the other hundreds.)</td>
<td>![Concrete demonstration]</td>
<td></td>
</tr>
<tr>
<td>The pattern continues until the operation is complete.</td>
<td>![Concrete demonstration]</td>
<td></td>
</tr>
<tr>
<td>What is 1 564 + 3 788? (5 352)</td>
<td>![Concrete demonstration]</td>
<td></td>
</tr>
</tbody>
</table>

Addition Algorithms - Chapter 6:15
Have students use the algorithm in practical situations requiring the addition of amounts of money, distance, area and volume.

Some students may be able to use ‘grouping’ strategies as they add the numbers in each column in order to make the mental calculations easier for them. For example:

$445$
$\downarrow$
$17$
$569$
$\downarrow$
$235$

The fives in the ones place may be grouped to make 10 before the other two numbers are added. Similarly, the 4 and the 6 in the tens place could be grouped. An understanding of the associative property would enable students to change the order of numbers in a column as long as care was taken.

By the end of Year 5, nearly all students should have a sound understanding of addition and a fast and accurate recall of addition and subtraction facts.
In Year 1, children explore the addition concept and develop thinking strategies to recall addition facts to ten. Strategies taught include Count on 1, 2 and 0, Doubles and Addition facts to 10. Only addition facts are formally recorded in Year 1. The vertical format is used and 'add' or 'and' is written until children are familiar with the '+' symbol.

In Year 2, revision of addition facts to 10 occurs before all remaining addition facts are taught. Additional thinking strategies such as Near Doubles, Near 10, Adding 9 and Neighbours but one are taught. If the '+' symbol is understood it may be used. The vertical format for addition continues to be used until Year 5. Once the tens facts have been explored, work with the addition algorithm may begin.

In Year 2 algorithm work should progress as follows by adding:
- 2 two-digit numbers
- zero in the top and/or bottom number
- a two digit number and single digit number

Refer to the Scope and Sequence in brief and in detail for Year Expectations of Algorithms.

By the end of Year 5, nearly all students should have a sound understanding of addition and a fast and accurate recall of addition and subtraction facts.

Children record all addition calculations vertically until Year 5 when the horizontal format may be introduced. Both the horizontal and vertical format for addition may be practised from Year 5 onwards.

Remember to help students better understand the operations, they should represent each operation in as many ways as possible. Teachers are encouraged to present the full range of situations in activities which require students to identify, explain and represent each operation.

However, students who have a firm understanding of the processes involved in the algorithms may need to work with MAB for a short time only when examples with larger numbers are introduced. The prolonged use of concrete materials may be a hindrance to those students who are capable of a more abstract approach which relied on their understanding of numeration. Other students will need to work with MAB until they are confident enough to perform the algorithm without these.
A wide variety of activities aimed at increasing student's understanding of the four operations in relation to concepts, mental calculations (including number facts and estimation), number properties, written algorithms and use of calculators feature in the sourcebooks. It is the teacher's responsibility to provide a balance among these different areas and ensure that students receive extra practice and consolidation where needed.

YEAR 6 and 7
Consolidation and Maintenance
Year 6 Sourcebook pp. 33-34
By this stage students should be proficient with the use of the algorithm and calculator for addition. To consolidate and maintain these skills, have students undertake activities, particularly relating to everyday situations, and games involving problem solving.

Although mental computation and estimation should be encouraged from Year 1 when addition number fact strategies are introduced, they must be extremely active strategies in these years.
Addition involves joining numbers in order to find their total. Although there is only one way to model this joining it may be performed with a variety of materials in a range of situations as illustrated below -

<table>
<thead>
<tr>
<th>Type of addition</th>
<th>Word problem</th>
<th>Diagram</th>
</tr>
</thead>
<tbody>
<tr>
<td>‘joining’</td>
<td>There were 6 suitcases on one trolley and 3 suitcases on the other. How many suitcases were there altogether?</td>
<td></td>
</tr>
<tr>
<td>‘adding on’</td>
<td>Nick has 4 toy cars. He buys 2 more. How many cars does he have now?</td>
<td></td>
</tr>
<tr>
<td>‘comparison’</td>
<td>Penny has 6 felt pens. John has 2 more felt pens than Penny has. How many does John have?</td>
<td></td>
</tr>
</tbody>
</table>

Since sequential development of the written algorithm and basic facts is desirable, it is very important to ensure that each child is proficient with one stage before progressing to the next as knowledge of the prior stages is usually a prerequisite.

It is of little value, for example, to expect a child to regroup with three-digit numbers before proficiency has been attained with two-digit numbers. The obligation to students is not to rush them through the stages so they may keep up with their peers, but to ensure that they have the opportunity to develop understanding and skill before being asked to attempt a more difficult task.

Since considerable overlap occurs throughout addition activities children may be practising strategies for basic facts and estimation when they are also applying written algorithms, mental calculations, estimations and using calculators. These skills should not be taught in isolation of one another. They should be presented simultaneously throughout the program and not consecutively as isolated units.

To help students better understand the operations, they should represent each operation in as many ways as possible. Teachers are encouraged to present the full range of situations in activities which require students to identify, explain and represent each operation.
Real World Situation

e.g. – 'I drove 21 km from home to Mapleton, 24 km to Kenilworth from Mapleton and 38 km from Kenilworth to home. How far did I travel?'

Concrete
Modelling the situation using concrete materials such as M.A.B.

Symbolic
21
24
+ 38

Verbal
Explaining the situation and Acting out

Note: It is most important that students be able to analyse problems or tasks then identify and apply the mathematical operation involved rather than being presented with an isolated meaningless algorithm.

It is therefore important that children have many opportunities to apply and discuss a real life problem while working with concrete materials and making symbolic recordings. By the time only numbers are used, the method of working and thinking their way through the addition algorithm should be well established. While children are using concrete materials to develop and practice the algorithm, such materials should be available when assessment is carried out.

In all addition work, the children should be made aware of the commutative property of addition (that is, $7 + 3 = 3 + 7$), and that therefore the vertical format may be checked by adding from top to bottom. The associative property of addition, that is, $(6 + 8) + 9 = 6 + (8 + 9)$, should also be modelled where appropriate, such as in the last example above. The children may also use calculators to check addition when they have understood the process.

Never teach anything as a fact that will have to be untaught at a later date. So, if you teach “when you add you always get more” it will have to be untaught for –

\[
\begin{array}{c}
4 \\
+ 0 \quad and \quad 4 + (-2)
\end{array}
\]
Even though children may experience both vertical and horizontal forms of recording calculations, vertical recording is recommended in preference to the traditional form. Using the vertical form helps children avoid problems associated with interpreting the equal sign and provides a firm basis for developing the written algorithm with larger numbers.

When children encounter the equals sign on the calculator, it may be described as the answer key.

Addition algorithms are read from the ‘top down.’

\[
\begin{array}{c}
4 \\
+ 3 \\
\hline
7
\end{array}
\]

‘4 ones and 3 ones makes 7 ones altogether’

In the vertical format for addition, note in the following example the one ten which has been regrouped or traded is placed at the top of the tens and is written the same size as the existing tens digits. This is because the 1 ten traded represented tens no more or no less than the original tens and therefore should not be made smaller or larger - but the same size.

<table>
<thead>
<tr>
<th>CORRECT</th>
<th>INCORRECT</th>
<th>INCORRECT</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>75</td>
<td>75</td>
</tr>
<tr>
<td>75</td>
<td>+ 56</td>
<td>+ 56</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>+ 0.56</td>
</tr>
</tbody>
</table>

Children record addition calculations vertically until Year 5 when the horizontal format may be introduced. Both the horizontal and vertical format for addition may be practised from Year 5 onwards.
Language provides the link between children’s manipulation of materials and their symbolic representations, and between their past and present experiences. Initially, children will use everyday language to describe what they are thinking and doing. Gradually, teachers may include the following operational terms in discussions, explain their meaning and model their use. Children may be encouraged to use them appropriately within their own language patterns.

It is suggested that, in developing children’s understanding of addition, a variety of language patterns be modelled so that the children realise that terms like “altogether”, added to “joined” and “as well” apply addition in certain contexts.

REFERENCES: Year 2 Sourcebook p. 72

<table>
<thead>
<tr>
<th>YEAR 1</th>
<th>YEAR 2</th>
<th>YEAR 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quantitive</td>
<td>Addition</td>
<td>Addition</td>
</tr>
<tr>
<td>more</td>
<td>and</td>
<td>join together</td>
</tr>
<tr>
<td>pair</td>
<td>join together</td>
<td>more</td>
</tr>
<tr>
<td>double</td>
<td>more</td>
<td>too</td>
</tr>
<tr>
<td>one/two more than</td>
<td>also</td>
<td>also</td>
</tr>
<tr>
<td>one less than</td>
<td>as well</td>
<td>as well</td>
</tr>
<tr>
<td>Operational</td>
<td>another</td>
<td>another</td>
</tr>
<tr>
<td>count on</td>
<td>altogether</td>
<td>altogether</td>
</tr>
<tr>
<td>and/add</td>
<td>add</td>
<td>and</td>
</tr>
<tr>
<td>too/also/as well/another</td>
<td>plus</td>
<td>add</td>
</tr>
<tr>
<td>put/join together</td>
<td></td>
<td>plus</td>
</tr>
</tbody>
</table>

During Year 3 the language of addition will become more formalised. The children will use their own language when describing situations, but as addition becomes more internalised, the children’s language should become the same as the formal language.

Care must be taken to ensure that the words in this vocabulary list are used appropriately in context. For example, though the word “also” is often associated with the addition operation, children should not believe that “also” is a cue word that always means “add”. Similarly, the word “more” may be associated with any of four operations, depending on the sentence structure:

- “23 is 4 more than what number?”
- “I have 4 lollies and am given 15 more. How many do I have then?”
- “After the school camp, each of the 20 children had to pay $2.00 more to cover the cost of the bus. How much extra did the bus cost?”
- “Take 75 sticks and bundle them in tens. Will you have more than 7 bundles?”

The word “by” is one that will require careful analysis in context. Consider the following uses:

- “42 is larger than 10 by 32”
- “42 is larger than ... by 32”.

_Addition Algorithms - Chapter 6: 22_
Scope and Sequence ............................................... 1
   Year 1 Subtraction Concept ................................ 1a
   Year 2 Subtraction Facts Algorithms ................... 1b
   Year 3 Subtraction Algorithms .......................... 1c
   Year 4 Subtraction Algorithms ......................... 1d
   Year 5 Subtraction Algorithms .......................... 1e

Year Level Expectations ..................................... 2
Glossary of Terms .............................................. 3
Reading and Recording ....................................... 4
Language .......................................................... 5
Scope and Sequence
Subtraction Algorithms

Since sequential development of the written algorithms and basic facts is desirable, it is important to ensure that each child is proficient with one stage before progressing to the next as knowledge of prior stages is usually a prerequisite.

The obligation to students is not to rush them through stages to keep up with peers, but to ensure that there is the opportunity to develop understanding and skill before being asked to attempt a more difficult task.

### Year 1 Subtraction Concept

<table>
<thead>
<tr>
<th>SCOPE AND SEQUENCE</th>
<th>EXAMPLE</th>
<th>REF.</th>
<th>For Teacher use</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sourcebook pp. 169-181</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1st</td>
<td>Develop the subtraction concept - using materials, pictures and language to model and explain subtraction. Do not record or formalise in Year 1.</td>
<td></td>
<td>Figure A</td>
</tr>
</tbody>
</table>

Sourcebook pp. 180-181

### Year 2 Subtraction Facts Algorithms

<table>
<thead>
<tr>
<th>SCOPE AND SEQUENCE</th>
<th>EXAMPLE</th>
<th>REF.</th>
<th>For Teacher use</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sourcebook pp. 88-98</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1st</td>
<td>Revise and formalise the concept of subtraction - use materials, pictures and language to model and explain subtraction.</td>
<td></td>
<td>Figure A &amp; B</td>
</tr>
<tr>
<td>2nd</td>
<td>Record subtraction facts using symbols to represent subtraction. Develop thinking strategies to recall easy facts.</td>
<td>8 take 2</td>
<td>Figure B</td>
</tr>
<tr>
<td>3rd</td>
<td>Count on</td>
<td>4 take 3</td>
<td></td>
</tr>
<tr>
<td>4th</td>
<td>Count back</td>
<td>6 take 5</td>
<td></td>
</tr>
<tr>
<td>5th</td>
<td>Doubles</td>
<td>4 take 2</td>
<td></td>
</tr>
<tr>
<td>6th</td>
<td>Zeros</td>
<td>3 take 3 3 take 0</td>
<td></td>
</tr>
<tr>
<td>7th</td>
<td>Tens Facts</td>
<td>16 take 5</td>
<td></td>
</tr>
<tr>
<td>8th</td>
<td>Tens</td>
<td>3 tens take 1 ten</td>
<td></td>
</tr>
<tr>
<td>9th</td>
<td>The Subtraction Algorithm (a) Two digit take two digit, no regrouping</td>
<td>76 - 33</td>
<td>Figure C</td>
</tr>
<tr>
<td>(b) Then with zeros</td>
<td>68 - 20</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(c) Two digit take one digit, no regrouping</td>
<td>57 - 4</td>
<td>Figure D</td>
<td></td>
</tr>
</tbody>
</table>

NOTE: "Take" should be used until students become familiar with the "-" symbol.

Sourcebook pp. 93-110

Subtraction Algorithms - Chapter 7 : 2
### Year 3 Subtraction Algorithms

<table>
<thead>
<tr>
<th>SCOPE AND SEQUENCE</th>
<th>EXAMPLE</th>
<th>REF.</th>
<th>For Teacher use</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st Revise Year 2 work</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2nd Concept development of harder subtraction facts and recording of same. Develop thinking strategies to assist recall of harder subtraction facts.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3rd Doubles plus one</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4th Near doubles</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5th One more than a ten</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6th Take all the ones and one more</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7th Take 9 away from ten</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8th Take 8 away from ten</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>9th Others</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10th (a) Three digit take three digit, no regrouping</td>
<td>365 - 124</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(b) Then with zeros</td>
<td>426 567 682 -110 -203 -300</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(c) Three digit take two digit, with regrouping in the ones</td>
<td>268 567 - 46 - 3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>11th (a) Two digit take one or two digit, with regrouping in the ones</td>
<td>75 - 37</td>
<td>Figure E</td>
<td></td>
</tr>
<tr>
<td>(b) With zeros</td>
<td>50 - 36</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(c) Two digit, take one digit, with regrouping in the ones</td>
<td>76 - 9</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Sourcebook pp. 93-110
## Year 4 Subtraction Algorithms

<table>
<thead>
<tr>
<th>SCOPE AND SEQUENCE</th>
<th>EXAMPLE</th>
<th>REF.</th>
<th>For Teacher Use</th>
</tr>
</thead>
</table>
| **1st** | Revise Year 4 work  
Continue work with algorithms |       | Part 1c |
| **2nd** | (a) Three digit minus three digit, with regrouping in the ones  
- 472 \( \begin{array}{c} \text{ } \end{array} \) 520  
- 126 \( \begin{array}{c} \text{ } \end{array} \) 116 |       |       |
| (b) Three digit take one or two digits with regrouping in the ones | 347 \( \begin{array}{c} \text{ } \end{array} \) 347  
- 28 \( \begin{array}{c} \text{ } \end{array} \) 8 |       |       |
| **3rd** | Three digit take three digit with regrouping in the tens | 634 \( \begin{array}{c} \text{ } \end{array} \) 605  
- 251 \( \begin{array}{c} \text{ } \end{array} \) 273 |       |       |
| Three digit take two digit with regrouping in the tens | 634 \( \begin{array}{c} \text{ } \end{array} \) 605  
- 51 \( \begin{array}{c} \text{ } \end{array} \) 73 |       |       |
| **4th** | (a) Three digit take three digit with regrouping in the tens and ones | 762 \( \begin{array}{c} \text{ } \end{array} \) 720  
- 186 \( \begin{array}{c} \text{ } \end{array} \) 459 |       |       |
| (b) Three digit take two digit with regrouping in the tens and ones | 724 \( \begin{array}{c} \text{ } \end{array} \) 720  
- 59 \( \begin{array}{c} \text{ } \end{array} \) 59 |       |       |
| **5th** | Three digit take three digit with regrouping because of internal zero  
Note: Students need to understand how to rename numbers fully before 'internal zero' examples are introduced, as MAB materials are not entirely suitable for modelling these special cases unless base boards are used. | 402 \( \begin{array}{c} \text{ } \end{array} \) 400  
- 156 \( \begin{array}{c} \text{ } \end{array} \) 266 | Figure G |       |
| **6th** | (a) Four digit take four digit, no regrouping | 6 \( \begin{array}{c} \text{ } \end{array} \) 258  
- 1 \( \begin{array}{c} \text{ } \end{array} \) 127 |       |       |
| (b) With zeros | 3\( \begin{array}{c} \text{ } \end{array} \) 725 \( \begin{array}{c} \text{ } \end{array} \) 4\( \begin{array}{c} \text{ } \end{array} \) 394  
- 1\( \begin{array}{c} \text{ } \end{array} \) 410 \( \begin{array}{c} \text{ } \end{array} \) 3\( \begin{array}{c} \text{ } \end{array} \) 003 |       |       |
| (c) Four digit, take one, two or three digit numbers | 5\( \begin{array}{c} \text{ } \end{array} \) 987 \( \begin{array}{c} \text{ } \end{array} \) 6\( \begin{array}{c} \text{ } \end{array} \) 872  
- 3\( \begin{array}{c} \text{ } \end{array} \) 67 \( \begin{array}{c} \text{ } \end{array} \) 31 |       |       |

Sourcebook pp. 67-97

Subtraction Algorithms - Chapter 7:4
### Subtraction Algorithms

<table>
<thead>
<tr>
<th>SCOPE AND SEQUENCE</th>
<th>EXAMPLE</th>
<th>REF.</th>
<th>For Teacher Use</th>
</tr>
</thead>
</table>
| 1st                | Revise Year 4 work  
Continue work with algorithms |       | Part 1d |
| 2nd                | (a) Four digit numbers, with regrouping in the ones  
(b) Then with zeros  
(c) Four digit take one, two or three digit with regrouping in the ones | 5 746  4 340  
- 1 438 - 2 124  
5 746  
- 1 408  
5 746  5 732  
- 48 - 415 |      |
| 3rd                | (a) Four digit take four digit with regrouping in the tens  
(b) Then with harmless zeros  
(c) Four digit take one, two or three digit with regrouping in the ones | 6 249  
- 3 187  
6 249  
- 3 087  
6 249  6 249  
- 87 - 187 |      |
| 4th                | (a) Four digit take four digit with regrouping in the tens and ones  
(b) Then with harmless zeros  
(c) Four digit take one, two or three digit with regrouping in the tens and ones | 7 324  
- 4 187  
7 324  
- 4 087  
7 234  7 842  
- 298 - 65 |      |
| 5th                | (a) Four digit take four digit, with regrouping in the hundreds  
(b) Then with harmless zeros  
(c) Four digit take one, two or three digit numbers | 6 278  
- 3 544  
6 278  
- 3 504  
6 278  6 278  
- 544 - 44 |      |
| 6th                | (a) Four digit take four digit with regrouping in the hundreds, tens and ones  
(b) Then with harmless zeros  
(c) Four digit take three digit with regrouping in the hundreds, tens and ones | 3 247  
- 2 569  
7 523  
- 3 089  
8 743  
- 987 |      |
| 7th                | (a) Four digit take four digit with internal zeros (use number expanders)  
NOTE: Students need to understand how to rename numbers fully before these internal zero examples are introduced, as MAB are not entirely suitable for modelling these special cases unless base boards are used. | 705  3 062  
- 48 - 771  
8 008  
- 3 619 | Figure I |
Using materials, pictures and language to model and explain subtraction.

The concept of subtraction is introduced in Year 1 as the inverse of addition. Discuss examples of subtraction which arise in everyday situations with the children. Recording and memorising subtraction facts is emphasised in Year 2.

Extend the children’s experience with addition by involving them in:
• taking away one of the addends and finding what is left – e.g.
  "... so now there are nine birds altogether. How many birds would there be if these three flew away again?"
• covering one part of the whole and then finding out what is hidden or missing – e.g.
  "This is a picture of six cricket wickets"
  "How many can you see?"
  "How many am I hiding?"
  "Let’s see if you are right."
Using symbols to represent subtraction:

\[
\begin{align*}
5 & \text{ take } 3 \\
& \underline{-3} \\
& 2
\end{align*}
\]

(NOTE: “Take” should be used until students become familiar with the “−” symbol.)

**Subtraction Facts (Introduce Year 2)**

- Recording facts

\[
\begin{align*}
5 & \text{ take } 3 \\
& \underline{-3} \\
& 2
\end{align*}
\]

- Developing thinking strategies to assist recall

As numerous examples of take away, missing addend, and comparison are presented to Year 2 students, a gradual progression through the following forms of representation is recommended. The three types of subtraction differ until the word and digit representation is reached.

**Take away**

- Five boys were on the bus, then three got off. How many boys were left on the bus?

  - Five boys take away three, leaves two boys.

**Missing addend**

- Debbie needs five stickers for her collection, but only has three. How many more stickers does she need?

  - Five stickers are needed but she only has three, so we need two more stickers.

**Comparison**

- Jim had five blue marbles and three red marbles. How many more blue marbles than red marbles does Jim have?

  - Five marbles, take away three marbles leaves two marbles.

- Using words and digits

  “5 take 3 leaves 2”

- Using a vertical format – e.g.

\[
\begin{align*}
5 & \text{ take } 3 \\
& \underline{-3} \\
& 2
\end{align*}
\]

  - Makes 2/leaves 2/is 2

- Using the “−” sign

\[
\begin{align*}
5 & \text{ take } 3 \\
& \underline{-3} \\
& 2
\end{align*}
\]

  - Makes 2/leaves 2/is 2

For further detail please refer to pages 88 to 93 of the Year 2 Sourcebook.

*Subtraction Algorithms - Chapter 7:7*
Material: MAB materials for each child, place-value chart.

Some children have difficulties when all the digits do not line up and when some columns have no digits. With this type, for example

57
- 4

some children take the 4 from both columns.

You or the children may suggest a problem to be solved:

"Terri took 28 marbles to school. She gave away 6. How many does she have now?"

Analyse the situation:

- What do we have to find out?
- What information do we need?
- How many marbles did Terri have at first?
- How many did she give away?
- How many does she have now?
- What type of operation is it?

Have the children represent the operation with concrete materials.

Now introduce more complex examples.
Two digit take two digit, no regrouping (Introduce Year 3)

After checking that earlier ideas are well established, begin with two digit numbers with no regrouping. Either you or the children may suggest a problem to be solved, for example: Sally-Anne’s family has to travel 76 kilometres to see her grandparents. After travelling 33 kilometres, what distance does the family have left to drive?

Analyse the situation:
- What do we have to find out?
- What information do we need?
- How far is the family travelling?
- How far have they gone?
- How may we use that information to find the answer?

The following shows the verbal, concrete and symbolic formats for the algorithm. The children should practise modelling the operation with MAB before they are asked to record the process.

<table>
<thead>
<tr>
<th>Verbal (child thinks or says)</th>
<th>Concrete (child demonstrates)</th>
<th>Symbolic (teacher or child writes)</th>
</tr>
</thead>
<tbody>
<tr>
<td>What operation is involved? (Subtraction.)</td>
<td>T</td>
<td>Ones</td>
</tr>
<tr>
<td>What number do we start with? (76.)</td>
<td>7 6</td>
<td></td>
</tr>
<tr>
<td>How many are we taking away? (33.)</td>
<td>3 3</td>
<td></td>
</tr>
<tr>
<td>What do we take first? (Ones.)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>How many ones do we have? (6 ones.)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>How many ones are we taking away? (3 ones.)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Have we enough ones to take 3 away? (Yes.)</td>
<td>T</td>
<td>Ones</td>
</tr>
<tr>
<td>What do we now have? (3 ones.)</td>
<td>7 6</td>
<td></td>
</tr>
<tr>
<td>Where will we put the 3 ones? (Ones’ place.)</td>
<td>3 3</td>
<td></td>
</tr>
<tr>
<td>What do we take next? (Tens.)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>How many tens do we have? (7 tens.)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>How many tens are we taking away? (3 tens.)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Have we enough tens to take 3 away? (Yes.)</td>
<td>T</td>
<td>Ones</td>
</tr>
<tr>
<td>What do we now have? (4 tens.)</td>
<td>7 6</td>
<td></td>
</tr>
<tr>
<td>Where will we put the 4 tens? (Tens’ place.)</td>
<td>3 3</td>
<td></td>
</tr>
<tr>
<td>What is 76 take 33? (43.)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Encourage the use of estimation to check the reasonableness of the answer. The children may also use calculators to check their answers. The inverse operation (addition) may be used as a double check, that is, “Does 43 and 33 add to 76?”
A sound understanding of place value is essential to an understanding of the 'decomposition' method of algorithm. The children should continue to explore numbers to assist with decomposition.

e.g.: 63
- 6 tens and 3 ones
- 5 tens and 13 ones
- 4 tens and 23 ones

Number expanders may be used to explore these ideas.

<table>
<thead>
<tr>
<th>Verbal (child thinks or says)</th>
<th>Concrete (child demonstrates)</th>
<th>Symbolic (teacher or child writes)</th>
</tr>
</thead>
<tbody>
<tr>
<td>What operation is involved?</td>
<td>T</td>
<td>Ones</td>
</tr>
<tr>
<td>(Subtraction.)</td>
<td>7</td>
<td>5 7</td>
</tr>
<tr>
<td>What number do we start with?</td>
<td>T</td>
<td>Ones</td>
</tr>
<tr>
<td>(75.)</td>
<td>6</td>
<td>13</td>
</tr>
<tr>
<td>How many are we taking away?</td>
<td>T</td>
<td>Ones</td>
</tr>
<tr>
<td>(37.)</td>
<td>4</td>
<td>23</td>
</tr>
<tr>
<td>How many ones do we have?</td>
<td>T</td>
<td>Ones</td>
</tr>
<tr>
<td>(5 ones.)</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>How many ones are we taking away?</td>
<td>T</td>
<td>Ones</td>
</tr>
<tr>
<td>(7 ones.)</td>
<td>6</td>
<td>13</td>
</tr>
<tr>
<td>Have we enough ones to take 7 away? (No.)</td>
<td>T</td>
<td>Ones</td>
</tr>
<tr>
<td>(The children take 1 ten from the 7 tens and exchange it for 10 ones at the bank of MAB.)</td>
<td>1 6 13</td>
<td>- 3 7</td>
</tr>
<tr>
<td>What do we have now? (6 tens and 10 ones.)</td>
<td>T</td>
<td>Ones</td>
</tr>
<tr>
<td>Where will we put the ones? (Ones’ place.)</td>
<td>1 5</td>
<td>13</td>
</tr>
<tr>
<td>How many ones do we have? (10 ones and 5 ones: 15 ones.)</td>
<td>T</td>
<td>Ones</td>
</tr>
<tr>
<td>15 ones take 7 ones? (8 ones. The 7 ones being taken away are removed.)</td>
<td>6</td>
<td>15</td>
</tr>
<tr>
<td>What do we take next? (Tens.)</td>
<td>T</td>
<td>Ones</td>
</tr>
<tr>
<td>How many tens do we have? (6 tens.)</td>
<td>5</td>
<td>13</td>
</tr>
<tr>
<td>How many tens are we taking away? (3 tens.)</td>
<td>T</td>
<td>Ones</td>
</tr>
<tr>
<td>Have enough tens to take 3 tens away? (Yes.)</td>
<td>1 5 13</td>
<td>- 3 7</td>
</tr>
<tr>
<td>What do we have now? (3 tens.)</td>
<td>T</td>
<td>Ones</td>
</tr>
<tr>
<td>Where will we put the tens? (Tens’ place.)</td>
<td>6</td>
<td>15</td>
</tr>
<tr>
<td>What is 75 take 37? (38.)</td>
<td>T</td>
<td>Ones</td>
</tr>
</tbody>
</table>

Encourage the use of estimation to check the reasonableness of the answer. The children may also use calculators to check their answers. The inverse operation (addition) may be used as a double check, that is, 'Does 38 and 37 add to 75?'

Subtraction Algorithms - Chapter 7 : 10
Example: Our family has to travel 762 km to get to Southport on the Gold Coast. We have driven 186 kms. How much further do we have to travel?

<table>
<thead>
<tr>
<th>Verbal</th>
<th>Concrete</th>
<th>Symbolic</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Student thinks or says)</td>
<td>(Student demonstrates)</td>
<td>(Teacher/student writes)</td>
</tr>
<tr>
<td>• What operation is involved? (Subtraction)</td>
<td></td>
<td>HT T Ones</td>
</tr>
<tr>
<td>• What number do we start with? (762)</td>
<td></td>
<td>7 6 2</td>
</tr>
<tr>
<td>• How many are we taking away? (186)</td>
<td></td>
<td>- 1 8 6</td>
</tr>
<tr>
<td>• What do we take first? (Ones)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>• How many ones do we have? (2 ones)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>• How many ones are we taking away? (6 ones)</td>
<td></td>
<td>5 1 2</td>
</tr>
<tr>
<td>• Have we enough ones to take 6 away? (No)</td>
<td></td>
<td>7 6 2</td>
</tr>
<tr>
<td>• What will we do? (Change one ten into ones)</td>
<td></td>
<td>- 1 8 6</td>
</tr>
<tr>
<td>(Students take 1 ten from the 6 tens and exchange it for 10 ones at the bank of MAB.)</td>
<td></td>
<td>7 6</td>
</tr>
<tr>
<td>• What do we have now? (5 tens and 10 ones)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>• Where will we put the ones? (One's place)</td>
<td></td>
<td>HT T Ones</td>
</tr>
<tr>
<td>• How many ones do we have? (10 ones and 2 ones: 12 ones)</td>
<td></td>
<td>6 1 5 1 2</td>
</tr>
<tr>
<td>• 12 ones, take 6 ones? (6 ones) (The 6 ones being taken away are removed.)</td>
<td></td>
<td>7 6 2</td>
</tr>
<tr>
<td>• What do we take next? (Tens)</td>
<td></td>
<td>- 1 8 6</td>
</tr>
<tr>
<td>• How many tens do we have?</td>
<td></td>
<td>7 6</td>
</tr>
<tr>
<td>• How many tens are we taking away? (8 tens)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>• Have we enough tens to take 8 away? (No)</td>
<td></td>
<td>HT T Ones</td>
</tr>
<tr>
<td>• What will we do? (Change one hundred into tens)</td>
<td></td>
<td>6 1 5 1 2</td>
</tr>
<tr>
<td>(Students take 1 hundred from the 7 hundred and exchange it for 10 tens at the bank of MAB.)</td>
<td></td>
<td>7 6 2</td>
</tr>
<tr>
<td>• What do we have now? (6 hundred and 10 tens)</td>
<td></td>
<td>- 1 8 6</td>
</tr>
<tr>
<td>• Where will we put the tens? (Ten's place)</td>
<td></td>
<td>7 6</td>
</tr>
<tr>
<td>• How many tens do we have altogether? (10 tens and 5 tens: 15 tens)</td>
<td></td>
<td>HT T Ones</td>
</tr>
<tr>
<td>• 15 tens, take 8 tens? (7 tens) (The 8 tens being taken away are removed.)</td>
<td></td>
<td>6 1 5 1 2</td>
</tr>
<tr>
<td>• Where will we put the tens? (Ten's place)</td>
<td></td>
<td>7 6 2</td>
</tr>
<tr>
<td>• What do we take next? (Hundreds)</td>
<td></td>
<td>- 1 8 6</td>
</tr>
<tr>
<td>• How many hundreds are we taking away? (1 hundred)</td>
<td></td>
<td>7 6</td>
</tr>
<tr>
<td>• Have we enough hundreds to take 1 hundred away? (Yes)</td>
<td></td>
<td>HT T Ones</td>
</tr>
<tr>
<td>• 6 hundreds take 1 hundred? (5 hundreds)</td>
<td></td>
<td>6 1 5 1 2</td>
</tr>
<tr>
<td>(The 1 hundred being taken away is removed.)</td>
<td></td>
<td>7 6 2</td>
</tr>
<tr>
<td>• Where do we put the 5 hundreds? (Hundred's place)</td>
<td></td>
<td>- 1 8 6</td>
</tr>
<tr>
<td>• What is 762 take 186? (576)</td>
<td></td>
<td>5 7 6</td>
</tr>
</tbody>
</table>

Encourage the use of estimation to check the reasonableness of the answer. Students may also use calculators to check their answers. The inverse operation (addition) may be used as a double check. (i.e. Does 186 and 576 add to 762?)

The above dialogue should be used to introduce the algorithm to students, but may later be reduced or shortened as students become familiar with the procedure. Students should practice modelling the operation with MAB before they are asked to record the process.

Subtraction Algorithms - Chapter 7:11
For this type of subtraction students should have a very good understanding of the written algorithm (to this stage), and should be able to model the subtraction with MAB as well as explain the process involved.

A zero in the tens place, i.e. as in \( 402 - 156 \), causes regrouping from the hundreds to the ones, and the MAB materials confuse rather than help the development. The materials use a two-step procedure that required the hundreds to be regrouped as tens, and then one of those tens is regrouped as 10 ones. As all subtraction to this stage has used a single-step procedure, it is preferable to maintain consistency, and use a single-step procedure for these examples as well.

Before these 'middle zero' cases are introduced, students, relying on their understanding of numeration, should be able to say, for example, that 4 hundreds is the same as 40 tens, and if one ten was changed into ones, then there would be 39 tens and 10 ones. They could then proceed directly to recording:

If students are unable to explain this regrouping, they should concentrate on developing understanding using MAB. A 'number expander' can be used in conjunction with the algorithm to demonstrate this process as follows:

<table>
<thead>
<tr>
<th>Verbal (Student thinks or says)</th>
<th>Concrete (Student demonstrates)</th>
<th>Symbolic (Student writes)</th>
</tr>
</thead>
<tbody>
<tr>
<td>What operation is involved? (Subtraction)</td>
<td>(402 is shown on the number expander)</td>
<td>H T Ones 4 0 2</td>
</tr>
<tr>
<td>What number do we start with? (402)</td>
<td></td>
<td>- 1 5 6</td>
</tr>
<tr>
<td>How many are we taking away? (156)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>What do we take first? (Ones)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>How many ones do we have? (2 ones)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>How many ones are we taking away? (6 ones)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Have we enough ones to take 6 away? (No)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>What will we do? (Change one ten for ten ones)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>How many tens do we have? (40 tens)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(Students must realise that although there are no tens in the ten's place, 4 hundred can be renamed as 40 tens, as shown on the number expander.)

(Change the 40 tens to 39 tens and 10 ones. 10 ones and 2 ones make 12 ones.)

12 ones, take 6 ones? (6 ones)

Where will we put the 6 ones? (One's place)

Once this recording has been done, the calculation is straightforward. The 5 tens are taken from 9 tens leaving 4 tens, and 1 hundred is taken from the 3 hundreds, leaving 2 hundreds.

The answer can be checked using estimation, calculators and the inverse operation (i.e. does 246 and 156 add to 402?)
Example: I had $3247 in the bank and spent $1569 on a holiday overseas. How much money do I have left?

<table>
<thead>
<tr>
<th>Verbal (Student thinks or says)</th>
<th>Concrete (Student demonstrates)</th>
<th>Symbolic (Teacher or student writes)</th>
</tr>
</thead>
<tbody>
<tr>
<td>What operation is involved? (Subtraction)</td>
<td>![Concrete Demonstration]</td>
<td>$3 \begin{array}{c} 2 \text{ } 4 \text{ } 7 \ \text{Th H T Ones} \end{array} - \begin{array}{c} 1 \text{ } 5 \text{ } 6 \text{ } 9 \ \text{Th H T Ones} \end{array}$</td>
</tr>
<tr>
<td>What number do we start with? (3247)</td>
<td>![Concrete Demonstration]</td>
<td></td>
</tr>
<tr>
<td>What are we taking away? (1569)</td>
<td>![Concrete Demonstration]</td>
<td></td>
</tr>
<tr>
<td>What do we subtract first? (Ones)</td>
<td>![Concrete Demonstration]</td>
<td>$3 \begin{array}{c} 2 \text{ } 4 \text{ } 7 \ \text{Th H T Ones} \end{array}$ - $1 \begin{array}{c} 5 \text{ } 6 \text{ } 9 \ \text{Th H T Ones} \end{array}$</td>
</tr>
<tr>
<td>What do we have in the ones? (7 ones take 9 ones)</td>
<td>![Concrete Demonstration]</td>
<td></td>
</tr>
<tr>
<td>Do we have enough ones to take 9 away? (No)</td>
<td>![Concrete Demonstration]</td>
<td></td>
</tr>
<tr>
<td>What will we do? (Change 1 ten for 10 ones and put them with the 7 ones)</td>
<td>![Concrete Demonstration]</td>
<td></td>
</tr>
<tr>
<td>(Students exchange from the MAB bank)</td>
<td>![Concrete Demonstration]</td>
<td></td>
</tr>
<tr>
<td>How many tens do we have now? (3)</td>
<td>![Concrete Demonstration]</td>
<td></td>
</tr>
<tr>
<td>Do we have enough ones to take 9 away? (Yes)</td>
<td>![Concrete Demonstration]</td>
<td></td>
</tr>
<tr>
<td>How many ones will be left? (17 ones take 9 ones: 8 ones)</td>
<td>![Concrete Demonstration]</td>
<td></td>
</tr>
<tr>
<td>(Students take 9 ones away and put them aside)</td>
<td>![Concrete Demonstration]</td>
<td></td>
</tr>
<tr>
<td>Where do we put the ones that are left? (In the one’s place)</td>
<td>![Concrete Demonstration]</td>
<td></td>
</tr>
<tr>
<td>What do we take next? (Tens)</td>
<td>![Concrete Demonstration]</td>
<td>$3 \begin{array}{c} 2 \text{ } 4 \text{ } 7 \ \text{Th H T Ones} \end{array}$ - $1 \begin{array}{c} 5 \text{ } 6 \text{ } 9 \ \text{Th H T Ones} \end{array}$</td>
</tr>
<tr>
<td>What do we have in the tens? (3 tens take 6 tens)</td>
<td>![Concrete Demonstration]</td>
<td></td>
</tr>
<tr>
<td>Have we enough tens to take 6 away? (No)</td>
<td>![Concrete Demonstration]</td>
<td></td>
</tr>
<tr>
<td>What will we do? (Change 1 hundred for 10 tens and put them with the 3 tens)</td>
<td>![Concrete Demonstration]</td>
<td></td>
</tr>
<tr>
<td>(Students exchange from the MAB bank)</td>
<td>![Concrete Demonstration]</td>
<td></td>
</tr>
<tr>
<td>How many hundreds do we have now? (1 hundred)</td>
<td>![Concrete Demonstration]</td>
<td></td>
</tr>
<tr>
<td>How many tens do we have? (10 tens and 3 tens: 13 tens)</td>
<td>![Concrete Demonstration]</td>
<td></td>
</tr>
<tr>
<td>Now do we have enough tens to take 6 away? (Yes)</td>
<td>![Concrete Demonstration]</td>
<td></td>
</tr>
<tr>
<td>How many tens will be left? (13 tens take 6 tens: 7 tens)</td>
<td>![Concrete Demonstration]</td>
<td></td>
</tr>
<tr>
<td>(Students take 6 tens away and put them aside)</td>
<td>![Concrete Demonstration]</td>
<td></td>
</tr>
<tr>
<td>Where do we put the tens? (Ten's place)</td>
<td>![Concrete Demonstration]</td>
<td></td>
</tr>
</tbody>
</table>

The pattern continues until the operation is complete.

What is $3247 - 1569$? (1678)

Students may use addition to check the answer on their calculators, or check the reasonableness of the answer by estimation.
Regrouping with middle zeros

When zeros cause regrouping that involves three or more places, for example:

\[
\begin{array}{c}
705 \\
48
\end{array} \quad \begin{array}{c}
3062 \\
771
\end{array} \quad \begin{array}{c}
8008 \\
3619
\end{array}
\]

MAB can no longer be used to model the process efficiently. The procedure is time-consuming and tedious, especially in the third example above, where blocks need to be changed from thousands to hundreds, to tens, then ones, before 9 ones can be subtracted.

If students understand the renaming of numbers – for example that 7 hundreds are equivalent to 70 tens and 7 thousands are equivalent to 700 tens or 70 hundreds – then they can regroup in a one-step procedure that is consistent with the regrouping for other examples. In the example:

\[
\begin{array}{c}
3062 \\
771
\end{array}
\]

the 3 thousands can be renamed as 30 hundreds. One of these hundreds is changed into 10 tens (Leaving 29 hundreds). The 10 tens are with the other 6 tens to make 16 tens. Students can proceed directly to the recording and complete the algorithm.

Any students who do not understand these renaming procedures should work with number expanders, MAB and calculators before beginning this type of subtraction. The Number Study topics in Years 4 and 5 provide suggestions about the types of relevant experiences. The following explains the procedure and language for subtraction using the number expander. Have students verify the answer by using the inverse operation, addition, and calculators. (Does 3 619 + 4 389 equal 8 008?)

---

### Example: A truck driver travelled 8 008 km in two months. He travelled 3 619 km in the first month. What distance did he travel in second month?

<table>
<thead>
<tr>
<th>Verbal</th>
<th>Concrete</th>
<th>Symbolic</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Student thinks or says)</td>
<td>(Student demonstrates)</td>
<td>(Student writes)</td>
</tr>
<tr>
<td>What operation is involved? (Subtraction)</td>
<td>(8 008 is shown on the number expander)</td>
<td>Th H T Ones</td>
</tr>
<tr>
<td>What number do we have? (8 008)</td>
<td>8 0 0 8</td>
<td>8 0 0 8</td>
</tr>
<tr>
<td>How many are we taking away? (3 619)</td>
<td>-3 6 1 9</td>
<td>-3 6 1 9</td>
</tr>
<tr>
<td>What do we take first? (Ones)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>How many ones do we have? (2 ones)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>What do we have in the ones? (8 ones take 9 ones)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Have we enough ones to take 9 away? (No)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>What will we do? (There are no tens or hundreds so rename 8 thousand as 800 tens and change 1 ten into 10 ones)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>How many tens do we have now? (799 tens)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>How many ones do we have? (10 ones and 8 ones: 18 ones)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Now do we have enough ones to take 9? (Yes)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>How many ones will be left? (18 ones take 9 ones: 9 ones)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Where do we put the ones? (One's place)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>What do we take next? (Tens)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Note: once the recording on the top line has been done, the subtraction is straightforward and the student continues until the operation is complete.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>What is 8 008 take 3 619? (4 389)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

---

Subtraction Algorithms - Chapter 7: 14
Concept Development

*Year 1 Sourcebook pp. 180-181*

The concept and associated language of subtraction is introduced in Year 1. Subtraction is shown as the inverse of addition. When two numbers were added together, one was occasionally taken away again to leave the other addend. In this way, the conservation of number was also reinforced. Subtraction facts are not formally recorded in Year 1.

**YEAR 2**

Introduction to some recall strategies

*Year 2 Sourcebook pp. 88-98*

In Year 2, the concept of subtraction is formalised. Activities involve children in examining the three aspects of subtraction – namely take away, missing addend and comparison – through real-life problems. Each of these dimensions involves children in using different language and thinking, so it is recommended that new dimensions are only introduced when children clearly understand the previous ones. Both addition and subtraction facts are formally recorded in Year 2. The vertical format is used and ‘take’ is used until students become familiar with the ‘–’ symbol.

\[ \begin{align*}
9 & \quad \text{take} \quad 2 \\
\hline
7 & \\
\end{align*} \]

In Year 2, subtraction recall strategies are introduced and include Count On, Count Back, Doubles, Zeros and Tens Facts.

**YEAR 3**

All subtraction facts are taught

*Year 3 Sourcebook pp. 93-110*

In Year 3, the childrens' repertoire of subtraction memory recall strategies are expanded to include those that will assist with harder facts so that all subtraction facts are learned. Once the tens facts have been explored, with the subtraction algorithm may begin.

In Year 3, algorithm work should progress by subtracting:
- two digit numbers, no regrouping
- three digit numbers, no regrouping
- two digit numbers, regrouping in the ones

**YEAR 4 and 5**

Sequential development in complexity

*Year 4 Sourcebook pp. 67-99
Year 5 Sourcebook pp. 67-75*

Refer to the ‘Scope and Sequence in Brief and in Detail’ for Year expectations of algorithms. Since students have been formally experiencing subtraction as a mathematical concept since Year 2, they should be able to recognise subtraction problems or tasks quite readily by Year 4. However, they will still need practice at modelling, explaining and representing addition in more complex situations and continue development of the concept. Students are required to have fast and accurate recall of subtraction facts by the end of Year 5. It is important to help students isolate the facts they do not know so they may concentrate on those. Some students may need to practise many of the subtraction facts. It is suggested that a plan such as the following be used with those students.
A plan to teach new facts involves:

1. Identifying a strategy to work out the set of facts to be learned.
2. Finding the corresponding addition facts.
3. Matching the subtraction partners.
4. Building the family of facts.
5. Practising the facts.

Children record all addition calculations vertically until Year 5 when the horizontal format may be introduced. Both the horizontal and vertical format for addition may be practised from Year 5 onwards.

Remember to help students to achieve a better understanding of the operations, they should represent each operation in as many ways as possible. Teachers are encouraged to present the full range of situations in activities which require students to identify, explain and represent each operation.

However students who have a firm understanding of the processes involved in the algorithms may need to work with MAB for a short time only when examples with larger numbers are introduced. The prolonged use of concrete materials may be a hindrance to those students who are capable of a more abstract approach which relied on their understanding of numeration. Other students will need to work with MAB until they are confident enough to perform the algorithm without these.

A wide variety of activities aimed at increasing student’s understanding of the four operations in relation to concepts, mental calculations (including number facts and estimation), number properties, written algorithms and use of calculators feature in the sourcebooks. It is the teacher’s responsibility to provide a balance among these different areas and ensure that students receive extra practice and consolidation where needed.

**YEAR 6 and 7**

Consolidation and Maintenance

*Year 6 Sourcebook pp. 34-36*

As with addition, most students should be proficient with the use of the algorithm and calculator for subtraction. Similarly, students should undertake problem-solving activities, particularly those related to everyday situations, and games to consolidate and maintain these skills.

Although mental computation and estimation for subtraction should be encouraged from Year 2 when subtraction recall strategies are introduced, they must be extremely active strategies in these years.
Subtraction situations may be identified by the need to compare the values of the numbers. The three different types of situations that represent subtraction are called 'take away', 'comparison' and 'missing addend'. Children need to know that subtraction concepts may be represented in these three ways. The term 'missing addend' need not be used by the children.

<table>
<thead>
<tr>
<th>Type of subtraction</th>
<th>Word problem</th>
<th>Diagram</th>
</tr>
</thead>
<tbody>
<tr>
<td>'take away'</td>
<td>There were 8 people at a party and 3 went home. How many people were left?</td>
<td>[Diagram of 8 people and 3 crossed out]</td>
</tr>
<tr>
<td>'comparison'</td>
<td>Janice has 9 books and Marni has 3. How many more books does Janice have?</td>
<td>[Diagram of 9 books] [Diagram of 3 books]</td>
</tr>
<tr>
<td>'missing addend'</td>
<td>I need 7 rolls of wallpaper for the lounge walls. I only have 4 rolls. How many more do I need?</td>
<td>[Diagram of 7 rolls] [Diagram of 3 rolls]</td>
</tr>
</tbody>
</table>

**Missing Addend/Subtraction Facts**

Before a subtraction fact is introduced, children must know the corresponding addition fact. Both the conceptual understanding and mastery of the subtraction facts depend on an understanding of the relationship between addition and subtraction.

Because many subtraction facts are learned by relating them to known addition facts, the 'missing addend' model is often used. Knowledge of addition facts is important because the strategy of thinking of the related addition fact is a most useful aid to memorisation.

For example: 12 -7 think of the addition fact 7 + 5 are 12 7 and what are 12? or what can I add to 7 to get 12?
When the algorithm is being taught and/or revised, the children must rely on the 'take away' model and associated language to build the process and to find the solution. The operation is represented symbolically in a vertical format and read from the top down, for example:

\[
\begin{array}{c}
7 \\
-3 \\
\end{array}
\]

\[\text{takeaway/take 3}
\]

\[\text{makes 4, or leaves 4, or is 4.}\]

Introduce the subtraction concept by using stories about the children's classroom and the outside environment.

Since sequential development of the written algorithm and basics facts is desirable, it is very important to ensure that each child is proficient with one stage before progressing to the next as knowledge of the prior stages is usually a prerequisite. The obligation to students is not to rush them through stages so that they may keep up with their peers, but to ensure that they have the opportunity to develop understanding and skill before being asked to attempt a more difficult task.

Since considerable overlap occurs throughout subtraction activities, children may be practising strategies for basic facts and estimation when they are also applying written algorithms, mental calculations, estimations and using calculators. Their skills should not be taught in isolation of one another. They should be presented simultaneously throughout the program and not consecutively to isolated units.

To help students better understand the operations, they should represent each operation in as many ways as possible. Teachers are encouraged to present the full range of situations in activities which require students to identify, explain and represent each operation.

**Real World Situation**

e.g. “Kerry saved 402 tokens. She used 156 on a school trip. How many tokens does she have left?”

**Concrete**

Modelling the situation using concrete materials e.g. MAB

**Symbolic**

\[
\begin{array}{c}
402 \\
-156 \\
\end{array}
\]

**Verbal**

Explaining the situation and acting it out.

Note: this is most important that students be able to analyse problems or tasks then identify and apply the mathematical operation involved rather than being presented with an isolated, meaningless algorithm.

It is therefore important that children have many opportunities to apply and discuss a real life problem while working with concrete materials and making symbolic recordings. By the time only numbers are used, the method of working and thinking their way through the subtraction algorithm should be well established.
Even though children may experience both vertical and horizontal forms of recording calculations, vertical recording is recommended in preference to the traditional form. Using the vertical form helps children avoid problems in association with interpreting the equal sign and provides a firm basis for developing the written algorithm with larger numbers.

When children encounter the equals sign on the calculator, it may be described as the answer key.

As for addition algorithms, subtraction algorithms are read from the 'top down.'
The dialogue presented in the detailed scope and sequence should be used to introduce the algorithm to students, but later may be reduced or shortened to the following as students become more familiar with the procedure.

e.g.  

\[
\begin{array}{c}
265 \\
-172 \\
\end{array}
\Rightarrow \begin{array}{c}
\text{take away/take 2 ones} \\
\text{makes/leaves/is 3 ones} \\
\end{array}
\]

In the vertical format for subtraction, note in the following example the regroupings or tradings have been placed at the top and are written the same size as the existing digits.

\[
\begin{array}{cccccc}
2 & 1 & 7 \\
\underline{3} & 6 \\
-1 & 8 & 3 \\
\hline
5 & 3 \\
\end{array}
\Rightarrow \begin{array}{cccc}
2 & 1 & 1 & 3 & 17 \\
3 & 2 & 4 & 7 \\
-1 & 5 & 6 & 9 \\
\hline
1 & 6 & 7 & 8 \\
\end{array}
\]

Children record addition calculations vertically until Year 5 when the horizontal and vertical format for subtraction may be practised.

Children record addition calculations vertically until Year 5 when the horizontal format may be introduced. Both the horizontal and vertical format for addition may be practised from Year 5 onwards.
In Year 2, the word 'take' is used until children become familiar with the '−' symbol.

The word regroup is used to describe the procedure of remaining numbers. For example, 15 tens may be renamed (regrouped as) 1 hundred and 5 tens. The children may use the words 'trade', 'snap', 'change', 'carry' or any other that conveys the same meaning.
Multiplication Algorithms

Contents

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### YEAR 3 SCOPE AND SEQUENCE

**Example:**
- **1st:** Revise and formalise the concept of multiplication - use language, unstructured and structured materials, pictures and diagrams to model and explain multiplication.

**Ref.:**
- **2nd:** Use numbers and everyday language to represent multiplication.
- **3rd:** Introduce the symbol for multiplication and use it to record multiplication.

### Easier Facts

**4th:** Twos facts and their turnarounds
- Examples: 
  - $2 \times 3$ & $2 \times 2$
- Reference: Chap. 4

**5th:** Fives facts and their turnarounds
- Examples: 
  - $5 \times 3$ & $5 \times 5$
- Reference: Chap. 4

**6th:** Nines facts and their turnarounds
- Examples: 
  - $9 \times 3$ & $9 \times 9$
- Reference: Chap. 4

**7th:** Zeros facts and their turnarounds
- Examples: 
  - $0 \times 3$ & $0 \times 9$
- Reference: Chap. 4

**8th:** Ones facts and their turnarounds
- Examples: 
  - $1 \times 3$ & $1 \times 1$
- Reference: Chap. 4

### Harder Facts - Year 3 for some children, Year 4 for others.

**9th:** Square number facts
- Examples: 
  - $6 \times 6$
- Reference: Chap. 4

**10th:** Three facts and their turnarounds
- Examples: 
  - $3 \times 6$ & $3 \times 3$
- Reference: Chap. 4

**11th:** Sixes facts and their turnarounds
- Examples: 
  - $6 \times 3$ & $6 \times 6$
- Reference: Chap. 4

**12th:** Fours facts and their turnarounds
- Examples: 
  - $4 \times 3$ & $4 \times 4$
- Reference: Chap. 4

**13th:** Eights facts and their turnarounds
- Examples: 
  - $8 \times 3$ & $8 \times 8$
- Reference: Chap. 4

**14th:** Sevens facts and their turnarounds
- Examples: 
  - $7 \times 3$ & $7 \times 7$
- Reference: Chap. 4

**15th:** The tens facts
- Examples: 
  - $1 \times 3$ & $1 \times 1$
- Reference: Fig 1

### YEAR 4 SCOPE AND SEQUENCE

**Example:**
- **1st:** Consolidate all of the easy facts.

**2nd:** Determine if the harder facts have been taught. Consolidate if they have, or refer to steps 9 to 14 of the Year 3 Scope and Sequence if they have not.

**3rd:** Revise and consolidate the tens facts.

**Introduce two digit x single digit**

**4th:** Two digit number x single digit number with:
- **no regrouping**
- **regrouping in the ones**
- **regrouping in the tens**
- **regrouping in the tens and ones**
- Examples: 
  - $23 \times 3$
  - $25 \times 3$
  - Reference: Fig 2, Fig 3

---

*Multiplication Algorithms - Chapter 8:2*
<table>
<thead>
<tr>
<th>YEAR 5 SCOPE AND SEQUENCE</th>
<th>EXAMPLE</th>
<th>REF.</th>
<th>For Teacher use</th>
</tr>
</thead>
</table>

**Three digit x single digit**

5th
- Three digit x single digit with:
  - no regrouping
  - some empty houses
  - Example:
    - 234
    - 264

**Introduce four digit x single digit**

6th
- Four digit x single digit with:
  - no regrouping
  - some empty houses
  - Example:
    - 2224
    - 2080

**Return to three digit x single digit to begin regrouping**

7th
- Three digit x single digit with:
  - regrouping in the ones
  - regrouping in the tens
  - regrouping in the tens and ones
  - zeros
  - Example:
    - 224
    - 242
    - 246
    - 206

**Return to four digit x single digit to begin regrouping**

8th
- Four digit x single digit with:
  - regrouping in the ones and tens
  - regrouping in the hundreds
  - regrouping in the hundreds tens and ones
  - regrouping in the thousands
  - regrouping throughout
  - regrouping in a combination of places
  - regrouping with zeros
  - Example:
    - 1254
    - 2723
    - 1754
    - 5251
    - 5754
    - 5251
    - 5034

**Introduce multiplying by a multiple of ten. A readiness concept for the introduction of two digit x two digit numbers**

9th
- Multiplying by a multiple of ten
  - identifying the pattern:
  - tens by ones gives tens
  - tens by tens gives hundreds
  - Example:
    - 0
    - 0

**Two digit x two digit number**

10th
- Two digit x two digit with:
  - no regrouping
  - regrouping with the ones multiplier
  - regrouping with the tens multiplier
  - regrouping with zeros in the partial products
  - regrouping with both the ones and the tens multipliers
  - Example:
    - 23
    - 43
    - 43
    - 45
    - 38

*Multiplication Algorithms - Chapter 8 : 3*
<table>
<thead>
<tr>
<th>YEAR 6 SCOPE AND SEQUENCE</th>
<th>EXAMPLE</th>
<th>REF.</th>
<th>For Teacher use</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st</td>
<td>Revise three digit x two digit with regrouping throughout.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Three digit x two digit number**

<table>
<thead>
<tr>
<th>2nd</th>
<th>Three digit x two digit with:</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>• no regrouping</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>• regrouping with the ones multiplier</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>• regrouping with the tens multiplier</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>• regrouping with zeros in the partial products</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>• regrouping with both the ones and the tens multipliers</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Explore**

- Napier's Bones
- Multiplication Patterns
It is important for the children to understand patterns with ones and tens if they are to apply their knowledge to mental calculation, estimation and, in Year 4, the written algorithm.

- **Patterns with ones**

  *Materials: MAB, calculators.*

  Involve the children in making the following facts with MAB and then recording them:

  $$\begin{align*}
  4 \times 2 &= 8 \\
  3 \times 2 &= 6
  \end{align*}$$

  Help the children see that all single-digit whole numbers represent ones. Write in the names of the numbers, for example:

  $$\begin{align*}
  4 \text{ ones} \\
  \times 2 &= 8 \text{ ones}
  \end{align*}$$

  and ask the children to verbalise other examples of ones by ones. To investigate whether ones by ones always gives an answer involving ones, model other calculations with MAB and check answers with calculators, for example:

  $$\begin{align*}
  5 \text{ ones} \\
  \times 2 &= 10 \text{ ones}
  \end{align*}$$

  *Note: These answers are obtained in ones before they are renamed.*

  Children can be led to the generalisation that ones multiplied by ones gives ones.

- **Patterns with tens and ones**

  A similar activity can be performed with tens and ones. Involve the children in modelling

  $$\begin{align*}
  30 \times 2 &= 60
  \end{align*}$$

  With the help of the children, record the calculations on the chalkboard as:

  $$\begin{align*}
  30 \times 2 &= 60 \\
  3 \text{ tens} \\
  \times 2 &= 6 \text{ tens}
  \end{align*}$$

  The children could model other calculations involving tens and ones, for example:

  $$\begin{align*}
  4 \text{ tens} \\
  \times 2 \text{ ones} &= 8 \text{ tens}
  \end{align*}$$

  and explain the answers. The pattern of tens by ones can be investigated with larger numbers, for example:

  $$\begin{align*}
  7 \text{ tens} \\
  \times 4 \text{ ones} &= 28 \text{ tens}
  \end{align*}$$

  and answers can be verified with calculators. A calculation of 7 tens by 4 ones would give the answer of 28 tens before the answer is renamed, that is, '4 sevens are 28, so 4 lots of 7 tens are 28 tens'.

  Help the children to see the generalisation that tens multiplied by ones gives tens, and investigate the pattern with turnarounds.
Example: Mum bought three bags of oranges. Each bag contained 23 oranges. How many oranges did she buy altogether?

<table>
<thead>
<tr>
<th>Verbal (Student thinks or says)</th>
<th>Concrete (Student demonstrates)</th>
<th>Symbols (Teacher or student writes)</th>
</tr>
</thead>
<tbody>
<tr>
<td>What number are we multiplying? (23)</td>
<td>[Illustration of 23 ones]</td>
<td>T Ones 2 3 x 3</td>
</tr>
<tr>
<td>What are we multiplying by? (3)</td>
<td>[Illustration of 3 ones]</td>
<td></td>
</tr>
</tbody>
</table>

What will we multiply first? (Ones)
How many ones do we have? (3 lots of 3 ones; 3 by 3 ones)
How many ones altogether? (9 ones)
(The ones are grouped together).
Have we enough ones to make a ten? (No)
Where will be put the 9 ones? (Ones place)

What will we multiply next? (Tens)
How many tens do we have? (3 lots of 2 tens; 3 by 2 tens)
How many tens altogether? (6 tens)
(Group the tens together).
Do we have enough tens to make a hundred? (No)

Note: Again, students should be given the opportunity of working with the materials, and developing the language pattern, before recording is introduced.

Multiplication Algorithms - Chapter 8 : 6
Example: The green grocer bought three 25 kg bags of potatoes from the markets. What is the total number of kilograms of potatoes he bought?

<table>
<thead>
<tr>
<th>Verbal (Student thinks or says)</th>
<th>Concrete (Student demonstrates)</th>
<th>Symbols (Teacher or student writes)</th>
</tr>
</thead>
<tbody>
<tr>
<td>What number are we multiplying? (25)</td>
<td>![Concrete Representation]</td>
<td>T Ones 25 x 3</td>
</tr>
<tr>
<td>What are we multiplying by? (3)</td>
<td>![Concrete Representation]</td>
<td></td>
</tr>
<tr>
<td>What will we multiply first? (Ones)</td>
<td>![Concrete Representation]</td>
<td></td>
</tr>
<tr>
<td>How many ones do we have? (3 lots of 5 ones; 3 by 5 ones)</td>
<td>![Concrete Representation]</td>
<td>T Ones 25 x 3</td>
</tr>
<tr>
<td>How many ones altogether? (15 ones)</td>
<td>![Concrete Representation]</td>
<td></td>
</tr>
<tr>
<td>Have we enough ones to make a ten? (Yes)</td>
<td>![Concrete Representation]</td>
<td></td>
</tr>
<tr>
<td>What will we do? (change the ten ones for 1 ten).</td>
<td>![Concrete Representation]</td>
<td></td>
</tr>
<tr>
<td>(The students use the bank of MAB to make this change).</td>
<td>![Concrete Representation]</td>
<td></td>
</tr>
<tr>
<td>How many ones are left in the ones place? (5 ones).</td>
<td>![Concrete Representation]</td>
<td></td>
</tr>
<tr>
<td>Where will be put the ten? (Tens place)</td>
<td>![Concrete Representation]</td>
<td>T Ones 125 x 3</td>
</tr>
<tr>
<td>(Note: the carry figure should be of normal size and placed above the tens).</td>
<td>![Concrete Representation]</td>
<td></td>
</tr>
<tr>
<td>What will we multiply next? (Tens)</td>
<td>![Concrete Representation]</td>
<td></td>
</tr>
<tr>
<td>How many tens do we have? (3 lots of 2 tens and 1 ten; 3 by 2 tens and 1 ten)</td>
<td>![Concrete Representation]</td>
<td>T Ones 125 x 3</td>
</tr>
<tr>
<td>How many tens altogether? (7 tens)</td>
<td>![Concrete Representation]</td>
<td></td>
</tr>
<tr>
<td>(Group the tens together).</td>
<td>![Concrete Representation]</td>
<td></td>
</tr>
<tr>
<td>Do we have enough tens to make a hundred? (No)</td>
<td>![Concrete Representation]</td>
<td></td>
</tr>
<tr>
<td>What is 25 multiplied by 3? (75)</td>
<td>![Concrete Representation]</td>
<td></td>
</tr>
</tbody>
</table>

*Multiplication Algorithms - Chapter 8*
The following shows how MAB can be used to develop the algorithm for a three-digit number multiplied by a one-digit number where regrouping occurs in the ones and tens. The language pattern should be maintained until students are familiar with the process. Encourage the use of estimation to check the reasonableness of the answer.

Example: A farmer sold 246 bags of potatoes. Each bag had a mass of 3 kg. What was the total mass of those potatoes?

<table>
<thead>
<tr>
<th>Verbal (Student thinks or says)</th>
<th>Concrete (Student demonstrates)</th>
<th>Symbols (Teacher or student writes)</th>
</tr>
</thead>
<tbody>
<tr>
<td>What operation is involved? (Multiplication).</td>
<td>![Concrete demonstration]</td>
<td>H T Ones</td>
</tr>
<tr>
<td>What number are we multiplying? (246)</td>
<td>![Concrete demonstration]</td>
<td>2 4 6</td>
</tr>
<tr>
<td>What number are we multiplying by? (3)</td>
<td>![Concrete demonstration]</td>
<td>x 3</td>
</tr>
<tr>
<td>(Students set out 3 lots of 246).</td>
<td></td>
<td></td>
</tr>
<tr>
<td>What will we multiply first? (Ones)</td>
<td>![Concrete demonstration]</td>
<td>H T Ones</td>
</tr>
<tr>
<td>How many ones do we have? (3 lots of 6 ones or 6 ones by 3: 18 ones).</td>
<td>![Concrete demonstration]</td>
<td>1</td>
</tr>
<tr>
<td>(Students group the ones together).</td>
<td></td>
<td>2 4 6</td>
</tr>
<tr>
<td>Do we have enough ones to make a ten? (Yes).</td>
<td>![Concrete demonstration]</td>
<td>x 3</td>
</tr>
<tr>
<td>What will we do? (Change ten ones for 1 ten)</td>
<td>![Concrete demonstration]</td>
<td>8</td>
</tr>
<tr>
<td>(The students use the bank of MAB to make this change).</td>
<td></td>
<td></td>
</tr>
<tr>
<td>How many ones are left in the ones place? (8 ones)</td>
<td>![Concrete demonstration]</td>
<td></td>
</tr>
<tr>
<td>Where will we put the ten? (Tens place)</td>
<td>![Concrete demonstration]</td>
<td></td>
</tr>
<tr>
<td>(The carry figure should be placed above the tens and should be of normal size).</td>
<td></td>
<td></td>
</tr>
<tr>
<td>What do we multiply next? (Tens)</td>
<td>![Concrete demonstration]</td>
<td>H T Ones</td>
</tr>
<tr>
<td>How many tens do we have? (3 lots of 4 tens and 1 more ten: 13 tens)</td>
<td>![Concrete demonstration]</td>
<td>1</td>
</tr>
<tr>
<td>(Students group the tens together).</td>
<td>![Concrete demonstration]</td>
<td>2 4 6</td>
</tr>
<tr>
<td>Do we have enough tens to make a hundred? (Yes)</td>
<td>![Concrete demonstration]</td>
<td>x 3</td>
</tr>
<tr>
<td>What will we do? (Change 10 tens for 1 hundred).</td>
<td>![Concrete demonstration]</td>
<td>3 8</td>
</tr>
<tr>
<td>(The students use the bank of MAB to make this change).</td>
<td></td>
<td></td>
</tr>
<tr>
<td>How many tens are left in the tens place? (3 tens)</td>
<td>![Concrete demonstration]</td>
<td></td>
</tr>
<tr>
<td>Where will we put the hundred? (Hundreds place)</td>
<td>![Concrete demonstration]</td>
<td></td>
</tr>
<tr>
<td>(The carry figure should be placed above the hundreds and should be of normal size).</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Continue until the algorithm is complete.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>What is 246 multiplied by 3? (738)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*Multiplication Algorithms - Chapter 8: 8*
Identifying prerequisite understandings for two-digit multiplication

Before students are introduced to the algorithm for multiplying by two-digit numbers (other than multiples often), it is very important for them to have:

- a sound understanding of the algorithm for single-digit multiplication;
- understanding of the patterns: - ones x tens
  - tens x ones
  - tens x tens;
- the ability to multiply by a multiple of 10;
- an understanding that the different parts of numbers can be multiplied separately then added to find the answer (distributive property).

Many difficulties experienced with the algorithm relate to multiplying by the tens digit. Students are less likely to make mistakes with this part of the algorithm if they understand the process of multiplying by multiples of 10.

Use the following activities to help students develop this understanding.

### Multiplying by 10 on the calculator

Have students explore the process of multiplying by 10 using the calculator.

<table>
<thead>
<tr>
<th>6</th>
<th>16</th>
<th>26</th>
<th>84</th>
<th>34</th>
<th>14</th>
</tr>
</thead>
<tbody>
<tr>
<td>x 10</td>
<td>x 10</td>
<td>x 10</td>
<td>x 10</td>
<td>x 10</td>
<td>x 10</td>
</tr>
</tbody>
</table>

Ask students to explain what has happened. The 'add a zero' notion should be discouraged as this type of thinking is not consistent when decimal fractions are involved.

Using place-value charts, the students should demonstrate that the numbers move one place to the left and the empty ones place is filled with a zero.

### Using the calculator for multiplying by multiples of 10 up to 90

Begin with numbers where the pattern is obvious and related to basic facts. Have students multiply by some multiples of 10 and investigate the pattern.

<table>
<thead>
<tr>
<th>8</th>
<th>12</th>
<th>9</th>
<th>11</th>
<th>7</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>x 20</td>
<td>x 20</td>
<td>x 40</td>
<td>x 40</td>
<td>x 30</td>
<td>x 30</td>
</tr>
</tbody>
</table>

Some larger but manageable numbers could be used:

<table>
<thead>
<tr>
<th>23</th>
<th>31</th>
<th>22</th>
<th>32</th>
<th>21</th>
<th>41</th>
</tr>
</thead>
<tbody>
<tr>
<td>x 20</td>
<td>x 20</td>
<td>x 30</td>
<td>x 30</td>
<td>x 50</td>
<td>x 50</td>
</tr>
</tbody>
</table>

The students should discuss their findings and be encouraged to look for other ways to achieve the same result. Relate multiplication by multiples of 10 to earlier experiences of multiplying by 10. Students should arrive at the conclusion that multiplying by 40 is the same as multiplying by 10 and then by 4 (or by 4 then by 10).

<table>
<thead>
<tr>
<th>16</th>
<th>16</th>
<th>160</th>
</tr>
</thead>
<tbody>
<tr>
<td>x 40</td>
<td>x 10</td>
<td>x 4</td>
</tr>
</tbody>
</table>

Using the number slide, have students explore this idea further.

Given 23 x 60

- mentally, multiply 23 by 10 (23 x 10 = 230)
- use the calculator to complete the calculation (230 x 6 = 1380)

Note: While this calculation would have been possible in a single step using the calculator, leading students through these two steps helps them develop understanding.

### Recording the operation

Show the students how recording can be shown in one line.

**Step 1**

Multiplying by 10: Record a zero in the lines place to show you are multiplying by 10.

**Step 2**

Multiplying by 2. (24 is multiplied by 2.)

### Practising the operation

Have students practise multiplying by multiples of 10 up to 90 until they can do so unassisted.
### Recording two-digit multiplication

Before this algorithm is introduced, students should have gained the understandings outlined in Fig __. Teachers are advised to refer to this section if they have not already done so. As students should be very familiar with the language and procedure for single-digit multiplication, the language can be simplified. The main steps focus on:
- multiplying by the ones digit
- multiplying by the tens digit
- adding the numbers [similar to the investigations in 9(a)].

Answers can be checked by using estimation, the inverse operation and calculators. (Is 988 + 26 equal to 38?)

Example: There were 38 rows of pine trees planted, with 26 trees in each row. How many trees were there altogether?

<table>
<thead>
<tr>
<th>Verbal (Student thinks or says)</th>
<th>Symbolic (Teacher or student writes)</th>
</tr>
</thead>
<tbody>
<tr>
<td>What operation is it? (Multiplication)</td>
<td>T Ones</td>
</tr>
<tr>
<td>What number are we multiplying? (38)</td>
<td>3 8</td>
</tr>
<tr>
<td>What are we multiplying by? (26)</td>
<td>x 2 6</td>
</tr>
<tr>
<td>What do we multiply first? (38 tens by 6 ones)</td>
<td>T Ones</td>
</tr>
<tr>
<td>What is 38 multiplied by 6? (228)</td>
<td>4</td>
</tr>
<tr>
<td>• 8 by 6 is 48</td>
<td>3 8</td>
</tr>
<tr>
<td>• 3 (tens) by 6 is 18 (tens) and 4 more (tens) makes 22 (tens)</td>
<td>x 2 6</td>
</tr>
<tr>
<td>What is multiplied next? (38 x 20)</td>
<td>2 2 8</td>
</tr>
<tr>
<td>• Record a zero in the ones place to show I'm multiplying by 10.</td>
<td></td>
</tr>
<tr>
<td>• Now multiply 38 by 2.</td>
<td></td>
</tr>
<tr>
<td>What is 38 x 20? (760)</td>
<td>7 6 0</td>
</tr>
<tr>
<td>How do we find the answer?</td>
<td>T Ones</td>
</tr>
<tr>
<td>(Add 228 and 760)</td>
<td>1 4</td>
</tr>
<tr>
<td>What is 228 + 760? (988)</td>
<td>3 8</td>
</tr>
<tr>
<td>What is 38 x 26? (988)</td>
<td>x 2 6</td>
</tr>
<tr>
<td></td>
<td>2 2 8</td>
</tr>
<tr>
<td></td>
<td>7 6 0</td>
</tr>
<tr>
<td></td>
<td>9 8 8</td>
</tr>
</tbody>
</table>

The language pattern should be maintained until students are familiar with the process. Encourage the use of estimation to check the reasonableness of the answer.

*Multiplication Algorithms - Chapter 8* : 10
Year 1

In Year 1 children investigate the numbers to 10 by adding, subtracting, multiplying and dividing these numbers in order to compare and analyse the number's size. Children also create their own number stories involving these numbers and operations. However, addition is the only operation which is formally recorded and developed in Year 1.

Year 2

Informal Exploration of the Multiplication Concept

In the Year 2 topic 'Number study', children arrange materials into small groups and then into groups of tens and ones to study the place value of numbers to 99. These grouping activities give children experience with multiplication and division processes.

In Year 2, problems from school, home and environmental situations are investigated. The focus is on developing a variety of strategies to solve such problems, discussing the problems using the appropriate language and representing them accurately with materials. Even though no formal development of multiplication is intended until Year 3 children may wish to record these problems and their solutions using a combination of pictures, words and digits.

The associated concept of odd and even will be introduced in Year 2.

Year 3

Formal Development of Multiplication

The formal development of multiplication and the introduction of the 'x' sign takes place in Year 3. The Year 3 activities give the children opportunities to investigate the concept and associated language of multiplication by identifying and representing multiplication symbol and multiplication facts and playing games to consolidate multiplication skills. Work on the associated concept of odd and even will continue in Year 3.

Although the limit set on multiplication facts for Year 3 is $9 \times 9 = 81$ some children may not be able to recall all facts to $9 \times 9$ by the end of the year.

The following sequence is recommended for developing multiplication in Year 3.

[The multiplication concept is dealt with informally in Years 1 & 2].

The following sequence is recommended for developing multiplication:

1. Multiplication concept
   - Use language, unstructured and structured materials, pictures and diagrams to model and explain multiplication.

2. Recording multiplication
   - Use symbols and everyday language to represent multiplication, for example: 3 rows of 4; 3 groups of 4.
   - Introduce the symbol for multiplication:
     \[
     \begin{array}{c}
     4 \\
     \times 3 \\
     \hline
     12
     \end{array}
     \]
3. Multiplication facts

- Use the ‘x’ sign to record multiplication: 6

\[
\begin{array}{c}
\text{x} \\
2 \\
\hline
12
\end{array}
\]

- Develop thinking strategies to assist recall.

A suggested sequence for developing the multiplication facts in Year 3 is:

**Easier Facts**
- twos facts
- fives facts
- nines facts
- zero facts
- ones facts

**Harder Facts**
- Remaining facts from the threes, sixes, fours, eights, and sevens.

*Refer to Chapter 4 for further details*

Year 3 multiplication algorithm work goes beyond multiplication facts to include the tens facts. Refer to Fig 1.

**Year 4**

Consolidate multiplication number facts, Introduce the written algorithm for multiplication.

*Year 4 Sourcebook pp. 122-130*

This is the first year that students will have experienced the written multiplication algorithm. They should therefore have ample opportunity to model the operation with appropriate materials such as MAB and bundling sticks before they are required to record the process. Students can be encouraged to estimate answers to calculations as they are learning the procedure for the algorithm.

Year 4 multiplication algorithm work is confined to two digit x single digit and ranges from no regrouping to regrouping throughout. This does not represent a lot of content because the number facts will need consolidating before this work can begin along with much conceptual development and hands on work as the algorithm is grappled with.

Work on associated concepts of odd and will continue along with the introduction of multiples and factors.

**Year 5**

Introduce horizontal to promote mental computation in +, - and x.

*Year 5 Sourcebook pp. 75-89*

To this stage, students have recorded all calculations vertically. In Year 5, students may be introduced to the horizontal format for recording operations and may practise both formats. As the division algorithm is new to students in Year 5, teachers are advised to emphasise the vertical recording of division facts and other division calculations so as to provide a consistent model for students.

A wide variety of activities aimed at increasing students’ understanding of multiplication relation to concepts, mental calculations (including number facts and estimation), number properties, written algorithms and use of calculators is proposed for Year 5. It is the teacher’s responsibility to provide a balance among these different areas and ensure that students receive extra practice and consolidation where needed.

It is not so much the number of digits in a multiplication algorithm that creates increments in difficulty, rather the amount of regrouping which is involved. For this reason Year 5 algorithm work examines three digit by a single digit then four digit by a single digit is later revisited to begin work on regrouping. Of most significance to Year 5 is the conceptual work on multiplying...
by a multiple of ten in readiness for multiplication by a two digit multiplier. The need for use of concrete materials and correct questioning should not be underestimated.

To help students better understand the operations, they should represent each operation in as many ways as possible. Teachers are encouraged to present the full range of situations in activities which require students to identify, explain and represent each operation.

Work on associated concept such as odd, even, multiples and factors will also continue along with the introduction of prime, composite and square numbers and the investigation of the 'Sieve of Eratosthenes' p. 23.

**Year 6**

Consolidation of multiplication involving a two digit multiplier.

Investigation of multiplication patterning and Napier's bones.

*Year 6 Sourcebook pp. 36-37*

The concept and algorithm for multiplication should be formerly established for most students by Year 6. Complete competence in these operations includes proficiency with mental calculations, as well as the traditional algorithm.

Up to Year 6, the multiplication algorithm will have been used to multiply two digit x two digit numbers with full regrouping. During Year 6, students should develop proficiency with the written algorithm when multiplying up to three digit by two digit with regrouping throughout. Investigation of Napier's Bones and multiplication patterns may be of interest to Year 6 students and mental computation strategies will benefit from an understanding of the associative, commutative and distributive properties for operations with whole numbers.

Activities presented in Year 6 should be linked to everyday use of the operations, such as with monetary transactions and measurement.

Work on associated concepts such as odd, even, factors, multiples, prime, composite and square numbers will continue along with the introduction of factor tree work.

**Year 7**

Exercise appropriate and effective estimation, mental computation and calculator skills.

*Year 7 Sourcebook pp. 27-28*

At this stage students should be proficient with all three methods of calculating with whole numbers - mental, pen and paper, and calculator. The skill of being able to identify the most appropriate method for each situation should continue to be emphasised and developed throughout Year 7.

The syllabus requires as many students as possible to operate with pen and paper for multipliers greater than two digits and with divisors greater than one digit. However, time should be spent developing the appropriate estimation skills, with the calculator being used to find precise answers in these situations.

To provide practice with the four operations, employ a range of problem-solving activities in preference to requesting students to complete lists of practice examples. In fact, these activities could serve a dual purpose: providing practice in applying the four operations and developing and consolidating specific problem-solving skills.

Work on associated concepts such as odd, even, factors, multiples, prime composite, square numbers and factor trees will continue along with the introduction of the square root and divisibility rules.
Three Models of Multiplication

There are three different ways of modelling multiplication: grouping, length and area.

1. **Grouping (set) model**
   
   *Example: Show 4 teams with 5 people in each team.*

2. **Linear or length model** (sometimes called a measurement model)
   
   *Example: There were 4 skipping ropes each 2 m long.*

3. **Area (array) model**
   
   *Example: Show 4 rows of chairs with 6 chairs in each row.*

It is important that students model and explain multiplication involving all three types of situations in order to develop a full understanding of the concept - however continued emphasis should be placed on the array model and the word ‘by’ as both emphasis commutativity (4 x 3 = 3 x 4) much better than the other models.

Teachers are encouraged to use a consistent approach that involves the same language, model and symbolic representation from the beginning of the concept through to the final stages of the algorithm.

Whatever model is used, the children should see and hear it in a variety of forms before the symbol ‘x’ is introduced. Thus, when discussing the number of tyres on three motor cars, the question may be expressed in these ways:

4
4
+4
12

3 groups of 4;

4 put out 3 times; 3 fours.

Although the children will be exposed to the three different multiplication models, continued emphasis may be placed on the array model and the work ‘by’. Both emphasis commutativity (4 x 3 = 3 x 4) much better than the other models. The work ‘times’ does not provide as meaningful a context as ‘by’.

Introducing the Multiplication Symbol (*Introduce Year 3*)

Children are formally introduced to the multiplication symbol (x) in Year 3, although many of them will already be familiar with it.

Discuss with the children the difficulty of writing out all the words they use to describe an arrangement. Include in the discussion a description of how the symbols + and - relate to the addition and subtraction concepts. Introduce the use of the multiplication symbol (x) to replace the words.
You may like to discuss the origin of the symbol with the children. In the late fifteenth century, an Italian mathematician, Pacioli, introduced a method called cross multiplication. For example, \(76 \times 28\) was set out as:

\[
\begin{array}{c}
7 \\
6 \\
2 \\
8
\end{array}
\]

Ask the children if they may see how the multiplication symbol came to be used; you may explain that, because setting crossed lines was difficult, printers often used the letter \(x\) instead.

The history of mathematics provides a wealth of ideas for discussion at all Year levels.

When recording the problem, use the vertical setting out, with the amount given (that is, the number in each row) being the top number and the number of groups (that is, the number of rows) the bottom number. Avoid the use of the word ‘times’.

When the children read the recording, it is preferable if they say, for example:

\[
\begin{array}{c}
2 \\
\cdot \\
\cdot \\
\cdot \\
5 \\
\cdot \\
\cdot \\
\cdot
\end{array}
\]

\[
\begin{array}{c}
2 \text{ in each row} \\
\cdot \\
\cdot \\
\cdot \\
\cdot \\
5 \text{ rows}
\end{array}
\]

‘2 multiplied by 5’

or

‘2 by 5’

or

‘5 twos’

The children will need to develop familiarity with the symbolisation. The following activities will help them do this.

- Match a picture with a symbolisation.
  ‘Two groups with three butterflies in each group’.

  The symbolisation would be: \(3 \times 2\)

  (2 threes or 3 multiplied by 2).

- Draw a picture for this symbolisation:

  \[
  \begin{array}{c}
  4 \\
  \cdot \\
  \cdot \\
  \cdot \\
  \cdot
  \end{array}
  \]

  \[
  \begin{array}{c}
  \cdot \\
  \cdot \\
  \cdot \\
  \cdot
  \end{array}
  \]

**Developing the written algorithm (Introduce Year 4)**

Figures 1 to 5 in the Scope and Sequence of this chapter explain the procedures and language for developing an algorithm.

A word story may be given to students which contains the appropriate numbers in context. Students can identify the operation involved, and explain the calculation that is needed. For practical reasons, the larger number is usually recorded on the top line. If students understand commutativity, they should have little difficulty accepting this convention. Students should be given the opportunity of working with the materials, and developing the language pattern, before recording is introduced.
What to do with the regrouping digits

The digits generated by the regrouping should not be written smaller than, to the side of or on the line below the original digits of the algorithm. Instead they must be written the same size as and placed on the line above, the original digit.

<table>
<thead>
<tr>
<th>Correct</th>
<th>Incorrect</th>
</tr>
</thead>
<tbody>
<tr>
<td>3 2</td>
<td>3 254</td>
</tr>
<tr>
<td>2 5 4</td>
<td>x 7</td>
</tr>
<tr>
<td>x 7</td>
<td>x 7</td>
</tr>
<tr>
<td>1 7 7 8</td>
<td>1 778</td>
</tr>
</tbody>
</table>

Associative/Commutative & Distributive Properties for Multiplication Calculations (Introduce Year6)

Students should understand that the techniques they use to find products of problems such as 5 x 70 or 30 x 40 are based on the associative, commutative and distributive properties for operations with whole numbers. The associative property is used to find the product of 5 x 70.

5 x 70  think
5 x (7 x 10)
(5 x 7) x 10
35 x 10 is 350

The associative and commutative properties are used to find the product of 30 x 40.

30 x 40  think
(i3 x 10) x (4 x10)
(3 x 4) x (10 x 10)
12 x 100 is 1 200

The distributive property is used to find the product of 123 x 4.

123  x 4  think
4 (100 + 20 + 3)
(4 x 100) + (4 x 20) + (4 x 3)
123
x 4
12 3 x 4
80 20 x 4
400 100 x 4
492

Napier's Bones (Introduce Year6)

Investigating 'Napier's Bones'

John Napier was a 17th century mathematician who invented a set of rods to help solve multiplication problems. The rods were made from whale bones and became known as 'Napier's Bones'.

Have the students make their own set of 'bones' using stiff cardboard or paddle pop sticks as in Figure A and discuss the patterns on the 'bones'.

Multiplication Algorithms - Chapter 8 : 16
To multiply 468 and 36, the 4, 6 and 8 'bones' are selected and placed alongside the index 'bone' as shown in Figure b.

Look down the index 'bone' in Figure B to 3 and write the numbers in the squares alongside on the 4, 6 and 8 'bones' in a 3 x 2 lattice, as shown in Figure C. Look down the index 'bone' to 6 and write the numbers in the squares alongside 'bones' in the lattice.

Add the numbers in the lattice diagonally from the right. If there is more than 10 in any diagonal, add 1 to the next diagonal and subtract 10 from the original diagonal total as shown in Figure C.

Write down the numbers from each diagonal, starting from the top left of the lattice to the bottom right of the lattice, to find the product of 468 x 36.

Involve students in multiplying a variety of numbers using Napier's method.

### Multiplication Patterns (Introduce Year 7)

**Consecutive number patterns**

Have students use the following procedures to find products and explain the patterns they find.

- Select four consecutive numbers, 3, 4, 5 and 6, then use the following procedure:
  1. Find the product of the inside numbers. (4 x 5 = 20)
  2. Find the product of the outside numbers. (3 x 6 = 18)
  3. Repeat steps 1 and 2 with other numbers, such as those in the table following.
  4. Explain the pattern you discovered. (The inside product is two greater than the outside product).

- Select four consecutive even numbers, such as 4, 6, 8 and 10.
  1. Repeat steps 1 and 2 in the previous example five times.
  2. Explain the pattern you discovered. (The inside product is 8 greater than the outside product).

- Have students investigate a number of cases involving odd numbers.

- As an extension activity, students can investigate other patterns such as the inside and outside products for four consecutive multiples of 3 or 4. Have the students try to predict patterns before trialling sets of numbers.
Square number patterns

Students may square numbers ending in different numerals to see if they can determine a pattern and state a rule. For example, square any number ending in 5:

\[
egin{align*}
25 \times 25 & = 625 = (20 \times 30) + 25 \\
35 \times 35 & = 1225 = (30 \times 40) + 25 \\
45 \times 45 & = 2025 = (40 \times 50) + 25 \\
55 \times 55 & = 3025 = (50 \times 60) + 25 \\
65 \times 65 & = 4225 \\
75 \times 75 & = 5625
\end{align*}
\]

Ask students to explain these square number patterns:

\[
21 \times 21 = 441 \\
31 \times 31 = 961
\]

Note the characteristics of both:
- they end in 1 (as expected);
- the tens digits of the factors are added to form the tens digit of the product;
- the tens digits of the factors are multiplied to form the hundreds digit of the product.

Do they think that the same thing happens with other numbers?

\[
egin{align*}
41 \times 41 & = 1681 \\
51 \times 51 & = 2601 \\
61 \times 61 & = 3721
\end{align*}
\]

When does the pattern become less obvious? Why?
The need for consistency

It is important that the teachers and students use correct and consistent language, models and symbolic representation from the introduction of the multiplication concept right through the grades in both multiplication number facts and multiplication algorithm work.

Refer to Chapter 4 part 6 for the correct use of language with multiplication facts.

Language, representation and symbolism for multiplication

The language, representations and symbolism for multiplication algorithms, as advocated by the syllabus is present in Figure 1 to 5 in the Scope and Sequence section of this chapter. This language, representation and symbolism should be used by all classrooms throughout Queensland to promote continuity in each child's learning experience, especially because of the high rate of mobility between schools.

Although the multiplication algorithm may be read downwards as:

24 multiplied by 3

it is calculated upwards as:

24 3 lots or 4 ones
\times 3 \text{ or } 3 \text{ by } 4 \text{ ones}

In the development stage ask:

“What number are we multiplying?” (24)

“What are we multiplying by?”

Avoid the use of times in both multiplication number fact and multiplication algorithm work.

Vertical and/or horizontal presentation ???

In Year 4, students record all calculations vertically. In Year 5, students can be introduced to the horizontal format for recording operations and may practice both formats for multiplication. Research suggests the vertical presentation of calculations facilities / encourages mental computation.

When to introduce the equal sign?

The introduction of the horizontal presentation in Year 5 is the first time students need to use the equal sign. Therefore there is no real reason why the abstract symbolism of equivalence needs to be introduced any earlier than Year 5.

Biggest number on top

For practical reasons, the larger number is usually recorded on the top line. If students understand commutativity, they should have little difficulty accepting this convention.
What to do with the regrouping digits

The digits generated by the regrouping should not be written smaller than, to the side of or on the line below the original digits of the algorithm. Instead they must be written the same size as and placed on the line above, the original digit.

<table>
<thead>
<tr>
<th>Correct</th>
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<td>3 2</td>
<td>3254</td>
</tr>
<tr>
<td>2 5 4</td>
<td>254</td>
</tr>
<tr>
<td>× 7</td>
<td>× 7</td>
</tr>
<tr>
<td>1 7 7 8</td>
<td>1 7 7 8</td>
</tr>
</tbody>
</table>
Language and Multiplication Algorithms

Language provides a link between children's manipulation of materials and their symbolic representations, and between their past and present experiences.

Initially children will use their own everyday language to describe what they are thinking and doing, but as addition becomes more internalized, the children's language should become the same as the formal language.

<table>
<thead>
<tr>
<th>Year 2</th>
<th>Year 3</th>
<th>Year 4</th>
<th>Year 5</th>
<th>Year 6</th>
<th>Year 7</th>
</tr>
</thead>
<tbody>
<tr>
<td>groups of</td>
<td>as for Year 1</td>
<td>as for previous years</td>
<td>as for previous years</td>
<td></td>
<td></td>
</tr>
<tr>
<td>lots of</td>
<td>arrange</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>sets of</td>
<td>rows of</td>
<td>9 twos, etc</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>each</td>
<td>bundles</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>every</td>
<td>multiply by</td>
<td></td>
<td>For the correct use of language when working with multiplication algorithms refer to Figures 1 to 5 in the Scope and Sequence section of this chapter.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>equal</td>
<td>put out used</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Please note:** The word ‘times’ does not provide as meaningful a context as ‘by’ and is discouraged by the syllabus.

### Related language

<table>
<thead>
<tr>
<th>Year 2</th>
<th>Year 3</th>
<th>Year 4</th>
<th>Year 5</th>
<th>Year 6</th>
<th>Year 7</th>
</tr>
</thead>
<tbody>
<tr>
<td>odd/even</td>
<td></td>
<td>multiples facts</td>
<td>prime composite</td>
<td>square numbers</td>
<td>square root</td>
</tr>
</tbody>
</table>
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\[
\begin{align*}
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35 \times 35 &= 1225 = \underbrace{(30 \times 40)}_{1200} + 25 \\
45 \times 45 &= 2025 = \underbrace{(40 \times 50)}_{2000} + 25 \\
55 \times 55 &= 3025 = \underbrace{(50 \times 60)}_{3000} + 25 \\
65 \times 65 &= 4225 \\
75 \times 75 &= 5625
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Ask students to explain these square number patterns:

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Do they think that the same thing happens with other numbers?

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When does the pattern become less obvious? Why?
Contents

Scope and Sequence ........................................ 1
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  Year 6
  Year 7

Year Level Expectations ................................. 2

Glossary of Terms ........................................ 3
  The division concept ................................... 3a
  Two models of division ................................ 3b
  Short division ........................................... 3c
  Long division ............................................. 3d
  Expressing remainders as decimal fractions

Reading and Recording ................................. 4

Language ................................................... 5
For a scope and sequence for division from Year 1 to Year 4 refer to the Division Number facts in Chapter 5.

**Year 5 Division Algorithm Scope and Sequence**

<table>
<thead>
<tr>
<th>Year 5 Sourcebook pp. 80-88</th>
<th>Example</th>
<th>Ref.</th>
<th>School Use</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st</td>
<td>Consolidate division number fact work</td>
<td></td>
<td>Chap 5</td>
</tr>
</tbody>
</table>

**Introduction of the division algorithm.**

2nd | Develop the pattern with the tens facts |

**Introduce two digit dividend with a single digit divisor**

<table>
<thead>
<tr>
<th>3rd</th>
<th>Two digit dividend, single digit divisor, no regrouping:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>• with a multiple of ten:</td>
</tr>
<tr>
<td></td>
<td>• without a remainder</td>
</tr>
<tr>
<td></td>
<td>• with a remainder</td>
</tr>
</tbody>
</table>

|     | 4180 | Fig 1 |
|     | 3169 | Fig 2 |
|     | 5157 |

**Introduce three digit dividend with a single digit divisor**

<table>
<thead>
<tr>
<th>4th</th>
<th>Three digit dividend, single digit divisor, no regrouping:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>• with a multiple of ten:</td>
</tr>
<tr>
<td></td>
<td>• without a remainder</td>
</tr>
<tr>
<td></td>
<td>• with a remainder</td>
</tr>
</tbody>
</table>

|     | 51100 |
|     | 31366 |
|     | 61668 |

**Return to two digit dividend with a single digit divisor for regrouping**

<table>
<thead>
<tr>
<th>5th</th>
<th>Two digit dividend, single digit divisor with regrouping:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>• without a remainder, include multiple of ten examples</td>
</tr>
<tr>
<td></td>
<td>• with a remainder, include multiple of ten examples</td>
</tr>
</tbody>
</table>

|     | 3154 | Fig 3 |
|     | 4197 |

**Return to three digit dividend with a single digit divisor for regrouping**

<table>
<thead>
<tr>
<th>6th</th>
<th>Two digit dividend, single digit divisor with regrouping:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>• without a remainder, include multiple of ten examples</td>
</tr>
<tr>
<td></td>
<td>• with a remainder, include multiple of ten examples</td>
</tr>
</tbody>
</table>

|     | 31675 | Fig 4 |
|     | 81257 |
### Year 6 Division Algorithm Scope and Sequence

<table>
<thead>
<tr>
<th>Year 6 Sourcebook pp.36-41</th>
<th>Example</th>
<th>Ref</th>
<th>School Use</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st</td>
<td>Consolidate Year 5 work</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Introduce four digit dividends with a single digit divisor**

<table>
<thead>
<tr>
<th>2nd</th>
<th>Four digit dividend, single digit divisor, no regrouping:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>• with a multiple of ten:</td>
</tr>
<tr>
<td></td>
<td>• without a remainder</td>
</tr>
<tr>
<td></td>
<td>• with a remainder</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Example</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>2nd</td>
<td></td>
</tr>
<tr>
<td>414840</td>
<td></td>
</tr>
<tr>
<td>313396</td>
<td></td>
</tr>
<tr>
<td>616668</td>
<td></td>
</tr>
</tbody>
</table>

**Introduce three digit dividend with a single digit divisor**

<table>
<thead>
<tr>
<th>3rd</th>
<th>Four digit dividend, single digit divisor, with regrouping:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>• without a remainder, include multiple of ten examples</td>
</tr>
<tr>
<td></td>
<td>• with a remainder, include multiple of ten examples</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Example</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>3rd</td>
<td></td>
</tr>
<tr>
<td>313345</td>
<td></td>
</tr>
<tr>
<td>818257</td>
<td></td>
</tr>
</tbody>
</table>

---

Division Algorithm - Chapter 9:3
### Long Division - Optional Year 6 and 7 Work
The following stages are only for those students who are experiencing little to no difficulty with division algorithms to date or would benefit from long division for some individual reason.

**Introduce long division involving a two/three and four digit dividends and a single digit divisor.**

<table>
<thead>
<tr>
<th>Stage</th>
<th>Description</th>
<th>Example</th>
<th>Ref</th>
<th>School Use</th>
</tr>
</thead>
</table>
| 4th   | Two digit dividend, single digit divisor, no regrouping:  
- with a multiple of ten:  
- without a remainder  
- with a remainder | 4 | 80 |  |  |
|       | Three digit dividend, single digit divisor, no regrouping:  
- with a multiple of ten:  
- without a remainder  
- with a remainder | 5 | 1100 |  |  |
|       | Four digit dividend, single digit divisor no regrouping:  
- with a multiple of ten:  
- without a remainder  
- with a remainder | 4 | 1480 |  |  |

**Introduce two/three/four digit dividend, single digit divisor with regrouping in a long division algorithm**

<table>
<thead>
<tr>
<th>Stage</th>
<th>Description</th>
<th>Example</th>
<th>Ref</th>
<th>School Use</th>
</tr>
</thead>
</table>
| 7th   | Two digit dividend, single digit divisor with regrouping:  
- without a remainder, include multiple of ten examples  
- with a remainder, include multiple of ten examples | 3 | 154 |  |  |
|       | Three digit dividend, single digit divisor with regrouping:  
- without a remainder, include multiple of ten examples  
- with a remainder, include multiple of ten examples | 4 | 197 |  |  |
|       | Four digit dividend, single digit divisor with regrouping:  
- without a remainder, include multiple of ten examples  
- with a remainder, include multiple of ten examples | 3 | 1479 | Fig 5 |  |

*Division Algorithm - Chapter 9:4*
<table>
<thead>
<tr>
<th>Year 7 Sourcebook pp. 27-30</th>
<th>Example</th>
<th>Ref</th>
<th>School Use</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st</td>
<td>Revise short division algorithms from Year 6 work.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2nd</td>
<td>Revise optional long division algorithms from Year 6 work for selected students.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3rd</td>
<td>Identify any students who were not ready to be introduced to long division when it was initially taught in Year 6 but who appear to be ready now. Refer to previous page for correct sequence.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4th</td>
<td>Consolidate long division of a single digit divisor with those students who were introduced to long division in Year 6. Refer to previous page for correct sequence.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Optional Year 7 work

*Introduce long division with a two/three/four digit dividend and a two digit divisor.*

| 5th                             | Introduce long division with a two digit divisor to those students who are displaying readiness |       |            |
| 6th                             | Long division involving a two digit dividend, a two digit divisor:  
  • without a remainder. Begin with easy examples where the divisor and dividend are multiples of ten  
  • with a remainder. Begin with easy examples where the divisor is a multiple of ten |       |            |
| 7th                             | Long division involving a three digit dividend, a two digit divisor:  
  • without a remainder. Begin with easy examples where the divisor and dividend are multiples of ten  
  • with a remainder. Begin with easy examples where the divisor is a multiple of ten  
  • with/without a remainder but has working that involves subtraction with regrouping | 14 \[168 \] | Fig 6 |
| 8th                             | Long division involving a four digit dividend a two digit divisor:  
  • without a remainder. Begin with easy examples where the divisor and dividend are multiples of ten  
  • with a remainder. Begin with easy examples where the divisor is a multiple of ten  
  • with/without a remainder but has working that involves subtraction with regrouping | 31 \[7254 \] | Fig 7 |
| 9th                             | For those students who are proficient with the division algorithm (short and/or long) introduce expressing remainders as decimal fractions:  
  • which are finite  
  • which are infinite/repeating. Answers to the nearest hundredth or thousandth will usually suffice. | 24 \[714 \] | Fig 8 |
Example: 80 parcels are to be put into 4 lucky-dip boxes. How many parcels will go into each box?

<table>
<thead>
<tr>
<th>Verbal (Student thinks or says)</th>
<th>Concrete (Student demonstrates)</th>
<th>Symbolic (Teacher or student writes)</th>
</tr>
</thead>
</table>
| What operation is it? (Division) |                                 | \[ T \text{ Ones} \]
| How many are to be shared? (80) | \[ = = = \] \[ = = = \] \[ = = = \] \[ = = = \] | 4 \[ \underline{180} \] |
| How many are sharing? (4)       |                                 | \[ T \text{ Ones} \]
| How many tens are to be shared? (8) |                                | \[ 4 \[ \underline{180} \] \] |
| Where will we put the 2 tens? (Tens place) | | \[ T \text{ Ones} \]
| Are there any tens left to share? (No) | | \[ 4 \[ \underline{180} \] \] |
| Is there anything left to share? (No) | \[ = = \] \[ = = \] \[ = = \] \[ = = \] | \[ T \text{ Ones} \]
| What is 80 divided by 4? (20)    |                                 | \[ 4 \[ \underline{180} \] \] |
| Fill in the ones place with a zero. | | |

Division Algorithm - Chapter 9:6
Figure 2 ......Two digit dividend, single digit divisor, no regrouping without a remainder
(Introduce Year 5)

Example: Three pairs of running shoes cost a total of $69. How much did each pair of shoes cost if they were exactly the same price?

<table>
<thead>
<tr>
<th>Verbal (Student thinks or says)</th>
<th>Concrete (Student demonstrates)</th>
<th>Symbolic (Teacher or student writes)</th>
</tr>
</thead>
<tbody>
<tr>
<td>What operation is it? (Division) How many are to be shared? (69) (Ask students to make an estimate of the size of the answer before they begin).</td>
<td></td>
<td>T Ones 3 ( \overline{69} )</td>
</tr>
<tr>
<td>What will be shared first? (Tens) How many tens are to be shared? (6 tens) 6 tens shared among 3. What is the greatest number of tens each would get? think 3 twos (tens) (2 tens) Where will be put the 2 tens? (Tens place)</td>
<td></td>
<td>T Ones 2 ( \overline{69} )</td>
</tr>
<tr>
<td>How many tens are left to be shared? (0) What will be shared next? (Ones) How many ones are to be shared? (9) How many are sharing? (3) 9 ones shared among 3. What is the greatest number of ones each would get? think 3 threes Where will we put the 3 ones? (Ones place)</td>
<td></td>
<td>T Ones 23 ( \overline{69} )</td>
</tr>
</tbody>
</table>
**Figure 3** Two digit dividend, single digit divisor, with regrouping without a remainder  
*(Introduce Year 5)*

Example: 3 students shared 54 stickers. How many stickers did they each receive?

<table>
<thead>
<tr>
<th>Verbal (Student thinks or says)</th>
<th>Concrete (Student demonstrates)</th>
<th>Symbolic (Teacher or student writes)</th>
</tr>
</thead>
<tbody>
<tr>
<td>What operation is it? (Division)</td>
<td></td>
<td><strong>T Ones</strong></td>
</tr>
<tr>
<td>How many are to be shared? (54)</td>
<td></td>
<td>3 <strong>54</strong></td>
</tr>
<tr>
<td>How many are sharing? (3)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(Students could make an estimate of the answer before they begin).</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>What will be shared first? (Tens)</td>
<td></td>
<td><strong>T Ones</strong></td>
</tr>
<tr>
<td>How many tens are to be shared? (5 tens)</td>
<td></td>
<td>1 <strong>54</strong></td>
</tr>
<tr>
<td>5 tens shared among 3. What is the greatest number of tens each would get?</td>
<td>3 tens</td>
<td>3 <strong>54</strong></td>
</tr>
<tr>
<td><img src="image" alt="3 tens" /></td>
<td><strong>3</strong></td>
<td><strong>54</strong></td>
</tr>
<tr>
<td>3 what are 5 (tens)? think 3 ones (tens) with (2 tens) left</td>
<td>3 <strong>54</strong></td>
<td>3 <strong>54</strong></td>
</tr>
<tr>
<td>1 ten</td>
<td><strong>54</strong></td>
<td><strong>54</strong></td>
</tr>
<tr>
<td>Where will be put the 1 ten? (Tens place)</td>
<td></td>
<td>3 <strong>54</strong></td>
</tr>
<tr>
<td>How many tens are left to be shared? (2 tens)</td>
<td></td>
<td>3 <strong>54</strong></td>
</tr>
<tr>
<td>Can we share the 2 tens left? (Yes, if we change them into ones).</td>
<td></td>
<td>3 <strong>54</strong></td>
</tr>
<tr>
<td><em>(Students make the change at the bank of MAB. The value of the 2 tens carries over to the ones place and is recorded).</em></td>
<td>3 <strong>54</strong></td>
<td><strong>1</strong> <strong>54</strong></td>
</tr>
<tr>
<td>How many ones are to be shared? (24)</td>
<td></td>
<td>3 <strong>54</strong></td>
</tr>
<tr>
<td>24 ones shared among 3. What is the greatest number of ones each would get?</td>
<td></td>
<td>3 <strong>54</strong></td>
</tr>
<tr>
<td><img src="image" alt="3 eights" /></td>
<td><strong>3</strong></td>
<td><strong>54</strong></td>
</tr>
<tr>
<td>3 what are 24? think 3 eights (8 ones)</td>
<td>3 <strong>54</strong></td>
<td><strong>3</strong> <strong>54</strong></td>
</tr>
<tr>
<td>Where will we put the 3 ones? (Ones place)</td>
<td></td>
<td>3 <strong>54</strong></td>
</tr>
<tr>
<td>How many ones are left? (0)</td>
<td></td>
<td>3 <strong>54</strong></td>
</tr>
<tr>
<td>Is there anything else to share? (No)</td>
<td></td>
<td>3 <strong>54</strong></td>
</tr>
<tr>
<td>What is 54 divided by 3? (18)</td>
<td></td>
<td>3 <strong>54</strong></td>
</tr>
<tr>
<td><strong>Remainders:</strong> With all division examples, stress to students that they share as many as possible each time.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>When the ones have been shared, any that are left are called the remainder. They are recorded as shown:</td>
<td><strong>T Ones</strong></td>
<td>18 <strong>rem 2</strong></td>
</tr>
<tr>
<td>In this case there are 2 ones left unshared.</td>
<td>3 <strong>54</strong></td>
<td><strong>2</strong> <strong>54</strong></td>
</tr>
</tbody>
</table>
**Figure 4: Three digit dividend, single digit divisor, with regrouping, without a remainder (Introduce Year 5)**

Example: Five people shared a prize of $675. How much did each person receive?

<table>
<thead>
<tr>
<th>Verbal (Student thinks or says)</th>
<th>Concrete (Student demonstrates)</th>
<th>Symbolic (Teacher or student writes)</th>
</tr>
</thead>
<tbody>
<tr>
<td>What operation is it? (Division)</td>
<td>[Image of 675]</td>
<td>H T Ones 1675</td>
</tr>
<tr>
<td>How many are to be shared? (675)</td>
<td></td>
<td>5 675</td>
</tr>
<tr>
<td>How many are sharing? (5)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>'Students could make an estimate of the answer before they begin.'</td>
<td></td>
<td></td>
</tr>
<tr>
<td>What will be shared first? (Hundreds)</td>
<td>[Image of 600]</td>
<td>H T Ones 1675</td>
</tr>
<tr>
<td>How many hundreds are to be shared? (6 hundreds - 6 hundreds shared among 5)</td>
<td></td>
<td>1 675</td>
</tr>
<tr>
<td>What is the greatest number of hundreds each would get? (1 hundred)</td>
<td>[Image of 100]</td>
<td></td>
</tr>
<tr>
<td>Where will I record the 1 hundred? (Hundreds place)</td>
<td>[Image of 1]</td>
<td>5 675</td>
</tr>
<tr>
<td>How many hundreds are left to be shared? (1 hundred)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Can I share the 1 hundred left? (Yes, if I exchange them for tens).</td>
<td>[Image of 10]</td>
<td>H T Ones 1675</td>
</tr>
<tr>
<td>How many tens are to be shared altogether? (17 tens)</td>
<td></td>
<td>13 675</td>
</tr>
<tr>
<td>What is the greatest number of tens each gets? (3 tens)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Where will I record the 3 tens? (Tens place - 3 tens)</td>
<td></td>
<td>3 675</td>
</tr>
<tr>
<td>How many tens are left to be shared? (2 tens)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Can we share the 2 tens left? (Yes, if we change them into ones).</td>
<td>[Image of 20]</td>
<td>H T Ones 1675</td>
</tr>
<tr>
<td>How many ones are to be shared? (25)</td>
<td></td>
<td>13 675</td>
</tr>
<tr>
<td>What is the greatest number of ones each would get? (5 ones)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Where will I record the 5 ones? (Ones place)</td>
<td></td>
<td>3 675</td>
</tr>
<tr>
<td>How many ones are left? (0)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Is there anything else to share? (No)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>What is 675 divided by 5? (135)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Three digit dividend, single digit divisor, regrouping with a remainder (Introduce Year 5)

Example: Three Year 6 students share 479 marbles. How many does each receive?

<table>
<thead>
<tr>
<th>Verbal (Student thinks or says)</th>
<th>Concrete (Student demonstrates)</th>
<th>Symbolic (Teacher or student writes)</th>
</tr>
</thead>
<tbody>
<tr>
<td>What operation is it? (Division)</td>
<td></td>
<td>H T Ones 3 4 7 9</td>
</tr>
<tr>
<td>How many are to be shared? (479)</td>
<td></td>
<td>(Students could make an estimate of the answer before they begin).</td>
</tr>
<tr>
<td>How many are sharing? (3)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>What will be shared first? (Hundreds)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>How many hundreds are to be shared? (4 hundreds - 4 hundreds shared among 3)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>What is the greatest number of hundreds each would get? (1 hundred)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Where will I record the 1 hundred? (Hundreds place)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>How many hundreds are left to be shared? (1 hundred)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Can I share the 1 hundred left? (Yes, if I exchange them for tens).</td>
<td></td>
<td></td>
</tr>
<tr>
<td>How many tens are to be shared altogether? (17 tens)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>What is the greatest number of tens each gets? (5 tens)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Where will I record the 5 tens? (Tens place)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>How many tens are left to be shared? (2 tens)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Can we share the 2 tens left? (Yes, if we change them into ones).</td>
<td></td>
<td></td>
</tr>
<tr>
<td>How many ones are to be shared? (29)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>What is the greatest number of ones each would get? (9 ones)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Where will I record the 9 ones? (Ones place)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>How many ones were used? (27)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>How many ones are left? (2 ones)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Division Algorithm - Chapter 9:10
**Figure 6** Long division involving a three digit dividend, a two digit divisor, without a remainder (Optional Year 7)

Example: A syndicate of 14 people won $168 in Lotto. What is each person's share?

<table>
<thead>
<tr>
<th>Verbal (Student thinks or says)</th>
<th>Concrete (Student demonstrates)</th>
<th>Symbolic (Teacher or student writes)</th>
</tr>
</thead>
<tbody>
<tr>
<td>What operation is it? (Division)</td>
<td></td>
<td>H T Ones</td>
</tr>
<tr>
<td>How many are to be shared? (168)</td>
<td></td>
<td>14 1 6 8</td>
</tr>
<tr>
<td>How many are sharing? (14)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(Students could make an estimate of the answer before they begin).</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>What will be shared first? (The hundred)</td>
<td></td>
<td>H T Ones</td>
</tr>
<tr>
<td>Can you share 1 hundred between 14? (No)</td>
<td></td>
<td>14 1 6 8</td>
</tr>
<tr>
<td>What can you do? (Change the 1 hundred to tens)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>How many tens are to be shared? (16 tens)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>What is the greatest number of tens each of the 14 people will get? (1 ten)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Where will I record the 1 ten? (Tens place)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Can you share these? (Only by changing them to ones)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>How many ones to share altogether? (28 ones)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>What is the greatest number of ones each will get? (2)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Where do you record the 2? (Ones place)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>How many ones were shared? (28)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>How many ones are left to be shared? (0)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>How much will each person get? ($12)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*Division Algorithm - Chapter 9: 11*
**Figure 7** Long division involving a four digit dividend, a two digit divisor, without a remainder (Optional Year 7)

Example: For the month of October (31 days), a nursery sold $7254 worth of plants. What were the mean daily sales if the nursery opens 7 days a week?

<table>
<thead>
<tr>
<th>Language (Student thinks or says)</th>
<th>Symbolic (Student writes)</th>
</tr>
</thead>
</table>
| Share 72 hundreds. (2 thirty-ones are 62). | \[
\begin{array}{c}
234 \\
317254 \\
62 \\
10 \\
\end{array}
\] |
| Share 105 tens. (3 thirty-ones are 93) | \[
\begin{array}{c}
234 \\
317254 \\
62 \\
105 \\
93 \\
12 \\
\end{array}
\] |
| Share 124 ones. (4 thirty-ones are 124 and no ones remain) | \[
\begin{array}{c}
234 \\
317254 \\
62 \\
105 \\
93 \\
124 \\
124 \\
0 \\
\end{array}
\] |

The average daily sales for the plant nursery are $234.
YEAR 1

In Year 1 children’s investigations of the numbers to 10 lead them to adding, subtracting, multiplying and dividing these numbers in order to compare and analyse the numbers’ size. Children also create their own number stories involving these numbers and operations. In Year 1 however, addition is the only operation which is formally recorded and developed.

YEAR 2

Informal Exploration of the Division Concept
Year 2 Sourcebook pp. 99 – 100

In the Year 2 topic “Number Study”, children arrange materials into small groups and then into groups of tens and ones to study the place value of numbers to 99. These grouping activities give children experience with multiplication and division processes.

In Year 2, number problems from school, home and environmental situations are investigated. The focus is on developing a variety of strategies to solve such problems, discussing the problems using the appropriate language and representing them accurately with materials. Even though no formal development of division is intended until Year 3, children may wish to record these problems and their solutions using a combination of pictures, words and digits. The associated concept of odd and even will be introduced in Year 2.

YEAR 3

Formal Development of the Division Concept
Year 3 Sourcebook pp. 134 – 136

In Year 3, children are given considerable opportunity to develop a solid understanding of the division concept by representing and exploring division using physical materials and diagrams. Many children find division more difficult than addition, subtraction and multiplication. This is because they do not have a clear understanding of the division concept before they are introduced to division symbolism and the algorithm. Other reasons for their difficulty include failure to make links between multiplication and division, a too rapid move to symbolism and the use of language that does not convey a clear meaning (for example, “6 goes into 12 ...”). Year 3 teachers must be most diligent in their efforts to facilitate full understanding of the division concept. Introduction to the division algorithm and use of the symbol is Year 4 work.

In Year 3 division can be recorded as 12 shared among 4.

Although division in the real world can be categorised as two main types: partition (sharing) and quotition (masking groups) Year 3 exploration activities focus upon partition because:

- it occurs more often in real life and is therefore more meaningful;
- it can be demonstrated more effectively with concrete materials;
- it is used as the basis for further development of the concept in Year 4.

(Refer to Part 3, Glossary of Terms in this chapter).

Year 3 activities focus on giving the children ample opportunity to use a variety of materials to explore the concept and language associated with division.
Formal Development of Division Facts

Year 4 activities continue the development of the division concept through the use of physical materials, diagrams and language.
Once students are comfortable with the recognition of situations involving division, and with the drawing of diagrams to represent them, the following symbolic recording can be introduced.

[The number sharing] \[ \frac{5}{3} \] \[ 15 \] [The amount to be shared]

\[ 3 \div 15 \] can be read as:
15 divided by 3; or
15 shared among 3

To work out answers to these situations, the students will need a strategy which enables them to interpret and understand the given problem. They should be advised to change the division into a multiplication which involves known facts, e.g. say “3 what are 15?”
In doing this, the students are relating division to multiplication, a relationship which will always be very useful.

It is important to keep language consistent. The development of the written algorithm in Year 5 depends heavily on a consistent language pattern that explains the sharing out of physical materials. It is inconsistent, for example to ask children to read

\[ 3 \div 15 \] as “how many threes in 15?” This could lead to confusion when the written algorithm is developed.

After students have had considerable practice representing “sharing” situations with diagrams and symbols, “grouping” situations can be included. The students should realise that the recording is the same for both situations and that both can be solved by relating the division fact to the known multiplication facts. (Refer to Part 3, Glossary of Terms, in this chapter).

Year 4 students can be practising strategies for learning division facts. The main strategy for learning a division fact is to relate it to known multiplication fact, e.g.

say: ’6 what are 30?’
For \[ \frac{6}{30} \]
’6 fives are 30’
’S0 30 divided by 6 is 5’

Another strategy is to rely on counting, i.e. in the above example, counting off groups of six until 30 is reached, but in the long-term, counting is too slow. It could however, be used as a back-up to the preferred method.

There is little value in asking students to learn division facts unless the corresponding multiplication facts are known. Some students, therefore, may not be introduced to division facts until later in the year, when they should have a good knowledge of multiplication facts. Students have until the end of Year 7 before they must be proficient with division facts (although it is expected that many will show proficiency well before then).

The following sequence for the introduction and development of division facts has been designed by the author and varies slightly from that presented in the sourcebooks. This is hopefully a preferred sequence as it may be necessary in some cases to leave some or all of the harder facts until Year 5. It is hoped that this will ease possible confusion created by the conflicting sequences presented on page 135 of the Year 3 sourcebook and page 138 of the Year 4 sourcebook. Endorsement of this scope and sequence will be required at school level.
1. Division Concept
Using materials, pictures and language to represent division; the sharing process is emphasised.

2. Recording Division
Initially division can be recorded as 12 'shared among' 4

3. Recording division
12 'divided by' 4.

4. Division Facts - • Recording the facts e.g. $4 \div 12$
   4a Easier Facts • Developing thinking strategies to assist recall
   The Twos Facts
   The Fives Facts
   The Nines Facts
   The Zeros Facts
   The Ones Facts

4b Harder Facts
   The Square Facts
   The Threes Facts
   The Sixes Facts
   The Fours Facts
   The Eights Facts
   The Sevens Facts

A. Wait until the children know all the multiplication facts well before beginning division facts; or
B. After children are proficient with one set of multiplication facts, practice the corresponding division facts.

The different methods may suit different situations, but it is important to have consistency throughout the school. At ............... State School it is school policy to use Method A/B.

The facts are presented in a sequence progressing from the easier facts to the more difficult ones. The entire sequence is shown in Part 1 of this chapter. The matching multiplication Scope and Sequence can be found in Part 1 of the Multiplication Facts chapter.
Continuation of work on harder facts or consolidation of facts.
*Year 5 Sourcebook pp. 46-49*

Introduction of division algorithm
*Year 5 Sourcebook pp. 80-88*

- **Division Number Facts**
  
  Since students have a good knowledge of the twos, fives, nines, ones and zero facts and their partners before beginning a study of the remaining facts, it may be necessary for some children to leave the remaining facts until Year 5. A sound knowledge of the corresponding multiplication facts is essential.

  Although mastery of multiplication and division facts is not expected until Year 7, some children may obtain mastery by Year 5.

- **Division Number Algorithms**
  
  The division algorithm is introduced in Year 5. To maintain consistency, students should model the division algorithm using the sharing process. In Year 5, students are not required to show the working below the dividend, i.e. long division in Year 5. Long division is optional from Year 6.

  There is a set sequence used to develop division and this is outlined in the Scope and Sequence. Students should have plenty of opportunities to model division before recording is introduced. See Figure 1. Encourage students to use the inverse operation to check the answer and say, “20 x 4 (4 twenties are 80)”, which is the number that was shared.

- **Associated Concepts**
  
  Work on associated concepts such as odd, even, factors, multiples, prime, composite and square numbers will continue along with the investigation of the Sieve of Eratosthenes. Refer to page 23 of the Year 5 Sourcebook.

- **Consolidation of division number facts**
  
  *Year 6 Sourcebook pp. 29-21*

  Optional exploration of long division
  *Year 7 Sourcebook pp. 37-41*

- **Division Facts**
  
  Year 6 is a consolidation year for multiplication and division number facts with view to Year 7 being a mastery year for same.

- **Short Division**
  
  Students are introduced to the written algorithm for division in Year 5. To maintain the language pattern from Year 5, students should model the division algorithm using the “sharing” process. The oral language used in carrying out the steps in the algorithm for four-digit numbers is the same as that already used for two and three digit numbers. Ensure students are proficient with the division of three digits before progressing to four-digit numbers. Figures and show the language material and recording procedure appropriate for the division of three digits by a single digit.

- **Long Division**
  
  The recording procedure in Figure ___ illustrates the traditional setting out of “long division”. The teaching of long division must be optional, not at a school or classroom level but an individual student level and must not be considered as core content. It is appropriate to teach long division to those students who are experiencing little to no difficulty with the short division method and would benefit from extension work. The teaching of long division with a single digit divisor for some Year 6 students will mean these students will most likely be taught long division with a two digit divisor in Year 7.
Before teaching long division, ask yourself how often you use a long division algorithm in your everyday life outside school. If the answer is rarely, be very selective about which students can afford to dedicate previous learning time and effort to acquire such a rarely used skill.

Children who are not proficient with short division do not need to be subjected to long division, especially by two digit divisors. Instead they should be referred to a calculator to achieve the same end. In this way their self-esteem and a problem solving ability will not be inhibited by their inability to produce a long division algorithm.

In some instances however, children may be able to understand long division more readily than short division. For this reason it may be fruitful to explore the potential of long division for an individual before referring him/her to a calculator as their only means of deriving an answer to certain division operations.

**Associated Concepts**

Work on associated concepts such as odd, even, factors, multiples, prime, composite and square numbers will continue along with the introduction of factor tree work.

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**YEAR 7**

**Mastery of Division facts**

*Year 7 Sourcebook pp. 17-18*

Optional long division by a two digit divisor

*Year 7 Sourcebook pp. 27-30*

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**Division Number Facts**

Mastery of addition and subtraction facts is expected by Year 4 and mastery of multiplication and division facts is expected by Year 7.

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**Division Algorithms**

Most students have been introduced to all four written algorithms in previous years and by the beginning of Year 7 should be proficient with division of four digits by a single digit divisor with regrouping and remainders.

Practice with the four operations should be integrated with problem-solving activities—students should not simply complete lists of practice examples.

---

**Long division with two digit divisors**

Note: Teachers of Year 7 students should read the above Year Level Expectation for Year 6.

Those children who have been able to handle division well enough either in Year 6 or Year 7 to warrant exploration of long division of a single digit divisor, can be introduced to division with a two digit divisor which requires a long division format.

Detailed information about the setting out and developing the algorithms can be found in the figures in the Scope and Sequence section of this chapter. To maintain a consistent language from previous years, it is recommended that students model the algorithm using the steps explained in these figures. The sharing of MAB blocks between more than 10 people can become unwieldy, but by careful selection of numbers this can be overcome for initial examples. Fortunately, a sound development of the associated language structures will generally provide the understanding needed to perform the algorithm correctly and efficiently with larger numbers.

It should be noted that proficiency with the algorithm for two-digit divisors depends on:

- understanding division with single digit divisors and
- facility with the multiplication and subtraction algorithms.

If the students do not have these skills it is pointless to introduce the written algorithm at this stage.
Dividing into four or more digits

For some students the algorithm can be extended to include dividends with four or more digits. At this stage, students will be working without concrete materials. (See Figure ___.)

Note: This same procedure applies to division calculations involving divisors of three (or more) digits. In identifying the quotient figure for each stage of division, rounding the divisor to the nearest 100 would be a useful technique to draw to students' attention. Specific attention may also have been given to examples which have a zero in the dividend.

Expressing remainders as decimal fractions

Until now, students have expressed remainders as whole numbers. Point out to students that the division process can continue on through the tenths, hundredths and thousandths places if renaming procedures are continued. Explain that in some instances, the division may continue for many places to the right of the decimal point. In these instances, it is important that students know how far to go, then they round to that place. For the fast majority of examples, an answer to the nearest hundredth will suffice.

Associated Concepts

Work on associated concepts such as odd, even, factors, multiples, prime, composite and square numbers will continue along with the introduction of the square root and the exploration of divisibility rules.
The Division Concept

Many students find the division algorithm more difficult than the algorithms for additions, subtraction and multiplication. Reasons often given include the following:

- Many students do not have a good understanding of the division concept before they are introduced to the algorithm;
- Students are often asked to “learn” the algorithm in an abstract manner which does not reinforce understanding;
- Approaches and language used vary from teacher to teacher;
- Much of the language used does not convey meaning (e.g. 6 goes into 12)?

For these reasons, students should be given the opportunity to develop a good understanding of the division concept by representing and explaining division using physical materials and diagrams before they are introduced to the algorithm. When students are introduced to the written algorithm in the Year 5 Sourcebook, emphasis is placed on understanding the procedure through the use of physical materials and appropriate language.

![Diagram]

- Real-world problem
- Physical representation
- Verbal representation
- Representation in mathematical symbols

Two Models of Division – Partition & Quotition

Introduce Partition Initially

Introduce Quotition Year 4

Although there is one global view of division, i.e. division is the breaking up of a number quantity into equal parts, division in the real world is of two main types: partition (sharing) and quotition (making groups).

### Partition (sharing)

**Summary:** The number of groups is given. You have to find the number in each group.

**Situation:** I have 12 boys to put into 3 equal teams

**Question:** How many in each team?

**Diagram:**

```
  ||   ||   ||
 o  o  o
```

**Language:** 12 shared among 3

**Answer:** 4 boys in each team

### Quotition (making groups)

**The number in each group is given. You have to find the number of groups.**

I have 12 boys to be put into teams of 3.

**How many teams can I make?**

```
  ||   ||   ||
  ||   ||   ||
```

4 teams.
Note: With quotation examples, the “names” in the situation are always the same (e.g. 12 boys, teams of 3 boys).

In Year 2, 3 and initially Year 4, greater emphasis is placed on the partition form because:
• it is closer to “real life” and therefore more meaningful for students;
• it can be demonstrated more effectively with concrete materials; and
• it is used to develop the algorithm and associated language in Year 5.

After students have had considerable practice representing “sharing” situations with diagrams and symbols, “grouping” situations can be included. The students should realise that the recording is the same for both situations and that both can be solved by relating the division to known multiplication facts.

Throughout students should be provided with opportunities to explain situations involving division, and to draw appropriate diagrams. These kinds of activities play a crucial role in helping students firstly to recognise situations involving division, and secondly to become familiar with the different types of diagrams necessary to illustrate the “sharing” and “marking groups” situations.

The following word problems may be used as models for the construction of both partitive (“sharing”) and quotitive (“group”) examples:
• In my exercises, I intend to run 36 kms over 4 days. How far should I run every day so that I run the same number of kilometres each day? (“Sharing”)  
• Mario caught 12 fish that he packed into two ice boxes. How many in each box? (“Sharing”)  
• Susan had to pack 24 apples into bags, each holding 6. How many bags did she need? (“Groups”)  
• Stickers are 5c each. How many can Lea buy for 30c? (“Groups”)  
• 21 children were selected for 3 tunnel ball teams. How many in each team? (“Sharing”)  
• Mum had to cut pieces of ribbon 8 cm from a 56 cm length. How many pieces can Mum cut? (“Groups”)  
• Farmer Cho planted 24 lettuces in 4 rows. How many were in each row? (“Sharing”)  
• The tyre market had 28 tyres to fit some cars. Each car has 4 tyres. How many cars can have new tyres? (“Groups”)  
• There are 30 calculators to share evenly among 5 classes. How many should each class be given? (“Sharing”)  
• I was given an allowance of $18 which had to last for 3 weeks. How much is that per week? (“Sharing”)  
• 24 children are being taken to sport in 6 cars of the same size. How many children should go in each car? (“Sharing”)  

**Short Division (Introduce Year 5)**

The division algorithm is introduced in Year 5. There is a set sequence used to develop division and this is outlined in the Scope and Sequence.

**Year 5**
The algorithm is introduced and examples built in complexity up to three digit dividends, single digit divisor, regrouping with remainders of whole numbers.

**Year 6**
The short division algorithm work is extended to include four digit dividends, single digit divisors, regrouping with remainders expressed as whole numbers. Long division work is optional as it is appropriate for some students only.

**Year 7**
Remainders may be expressed as decimal fractions and long division work is extended for some students to include two digits divisors.
Ensure students are proficient with each stage before progressing to the next. Students should have plenty of opportunities to model division before recording is introduced. To maintain consistency, students should model the division algorithm using the sharing process. In Year 5, students are not required to show the working below the dividend, i.e. long division. Long division is optional from Year 6.

**Long Division (Optional work introduced in Year 6 or 7)**

The teaching of long division must be optional, not at a school or classroom level but an individual student level and must not be considered as core content. It is appropriate to teach long division to those Year 6 and 7 students who are experiencing little to no difficulty with the short division methods and would benefit from extension work.

The teaching of long division with a single digit divisor may begin in Year 6 or Year 7 for some students while others are referred to a calculator to achieve the same end. Students who begin long division work in Year 6 will continue long division work in Year 7 to include two digit divisors and decimal remainders expressed as decimal numbers rather than whole numbers.

Before teaching long division, ask yourself how many times you have used a long division in your everyday life outside school. If the answer is rarely, be very selective about which students can afford to dedicate previous learning time and effort to acquire such a rarely used skill.

Children who are not proficient with short division to not need to be subjected to long division, especially by two digit divisors. Instead they should be referred to a calculator to achieve the same end. In this way their self-esteem and problem solving ability will not be inhibited by their inability to produce a long division algorithm.

In some instances however, children may be able to understand long division more readily than short division. For this reason it may be fruitful to explore the potential of long division for an individual before referring him/her to a calculator as their only means of deriving an answer to certain division operations.

**Expressing remainders as decimal fractions (Introduce Year 7)**

Until Year 7 students have been expressing remainders as whole number. Once students have become proficient with the division algorithm (short and/or long) instruction can begin on expressing remainders as whole numbers.

Point out to students that the division process can continue on through the tenths, hundredths and thousandths places if renaming procedures are continued. Explain that in some instances, the division may continue for many places to the right of the decimal point.

In these instances, it is important that students know how far to go, then they round to that place. For the vast majority of examples, an answer to the nearest hundredth will suffice.
It is important that teachers use correct and consistent language, modelling and symbolic representation from the introduction of the division concept right through the development of number fact and algorithm work.

The language, representation and symbolic recording for short division and long division can be found in the figures attached to the Scope and Sequence part of this chapter. To maintain consistency with the number fact work, the division algorithm should be modelled using the sharing process.

\[
\begin{array}{c}
\text{The number} \quad \frac{52}{\text{sharing}} \quad \rightarrow \quad 3 \mid 156 \quad \leftarrow \quad \text{The amount to be shared}
\end{array}
\]

\[
3 \mid 156
\]

*can be read as:*

1. 156 divided by 3; or
2. 156 shared among 3

It is inconsistent, for example to ask students to read \(3 \mid 156\) as "How many threes in 15?" This can lead to considerable confusion.

For detailed information on the correct reading and recording of number facts, refer to Part 5 of Chapter 5.